Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/48-1.2.3.4-f-x-^m-d+e-x^n-^q-a+b-x^n+c-x^-2-n-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [156]. This is test number [48].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (156)	0.00 (0)
Mathematica	94.23 (147)	5.77 (9)
Maple	87.82 (137)	12.18 (19)
Fricas	82.69 (129)	17.31 (27)
Mupad	78.21 (122)	21.79 (34)
Giac	70.51 (110)	29.49 (46)
Sympy	50.00 (78)	50.00 (78)
Maxima	42.31 (66)	57.69 (90)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

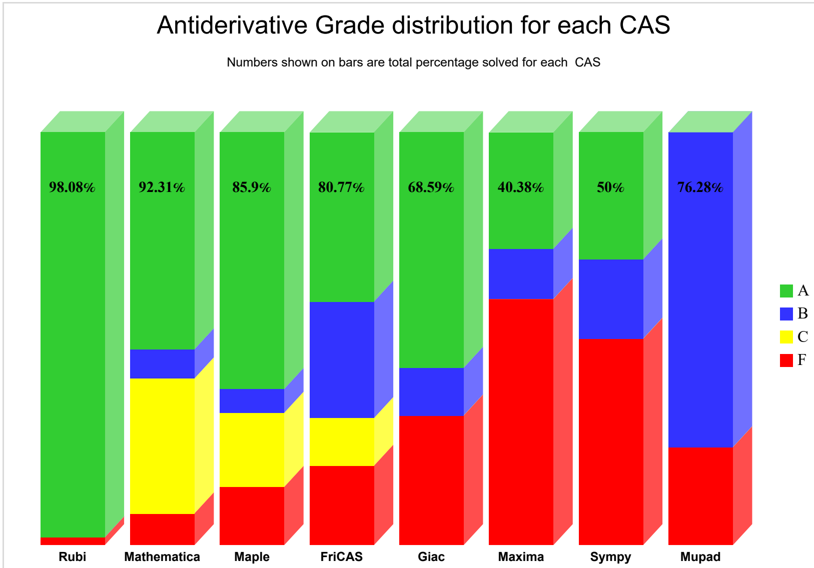
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

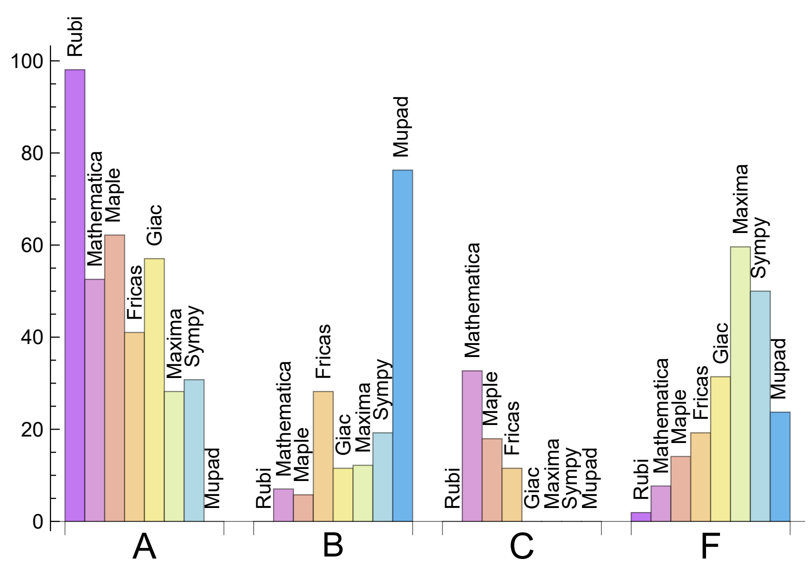
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.077	0.000	0.000	1.923
Maple	62.179	5.769	17.949	14.103
Giac	57.051	11.538	0.000	31.410
Mathematica	52.564	7.051	32.692	7.692
Fricas	41.026	28.205	11.538	19.231
Sympy	30.769	19.231	0.000	50.000
Maxima	28.205	12.179	0.000	59.615
Mupad	0.000	76.282	0.000	23.718

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	9	100.00	0.00	0.00
Maple	19	100.00	0.00	0.00
Fricas	27	70.37	29.63	0.00
Mupad	34	0.00	100.00	0.00
Giac	46	78.26	13.04	8.70
Sympy	78	11.54	80.77	7.69
Maxima	90	68.89	0.00	31.11

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Giac	0.45
Rubi	0.45
Maple	1.72
Mathematica	2.23
Fricas	5.71
Sympy	10.21
Mupad	10.21

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	202.40	0.99	153.50	1.00
Maxima	215.95	10.05	35.50	1.06
Sympy	216.04	9.03	93.50	1.02
Giac	221.92	3.24	68.00	1.00
Maple	258.39	2.34	44.00	0.95
Mathematica	436.46	1.95	80.00	0.99
Fricas	1431.50	9.27	240.00	1.45
Mupad	3286.97	17.19	295.00	3.15

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

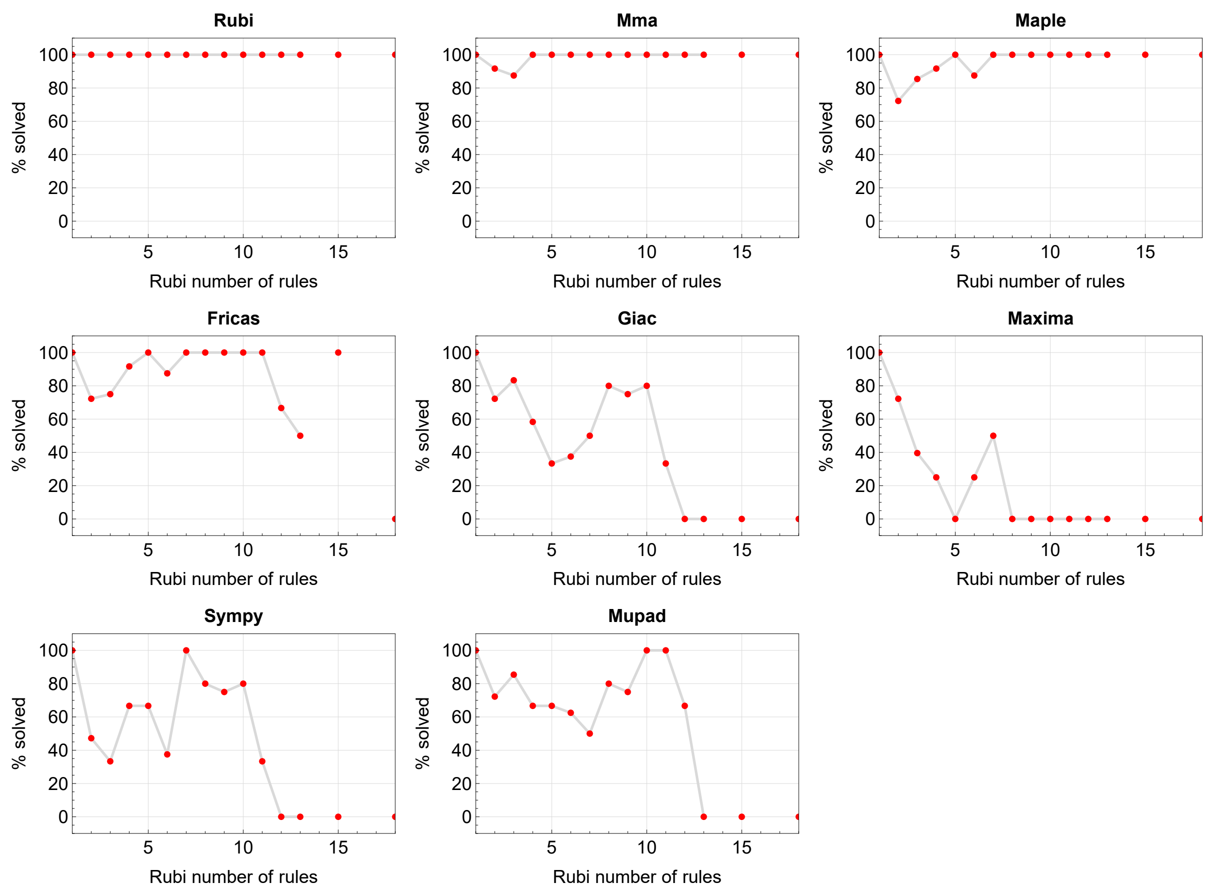


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

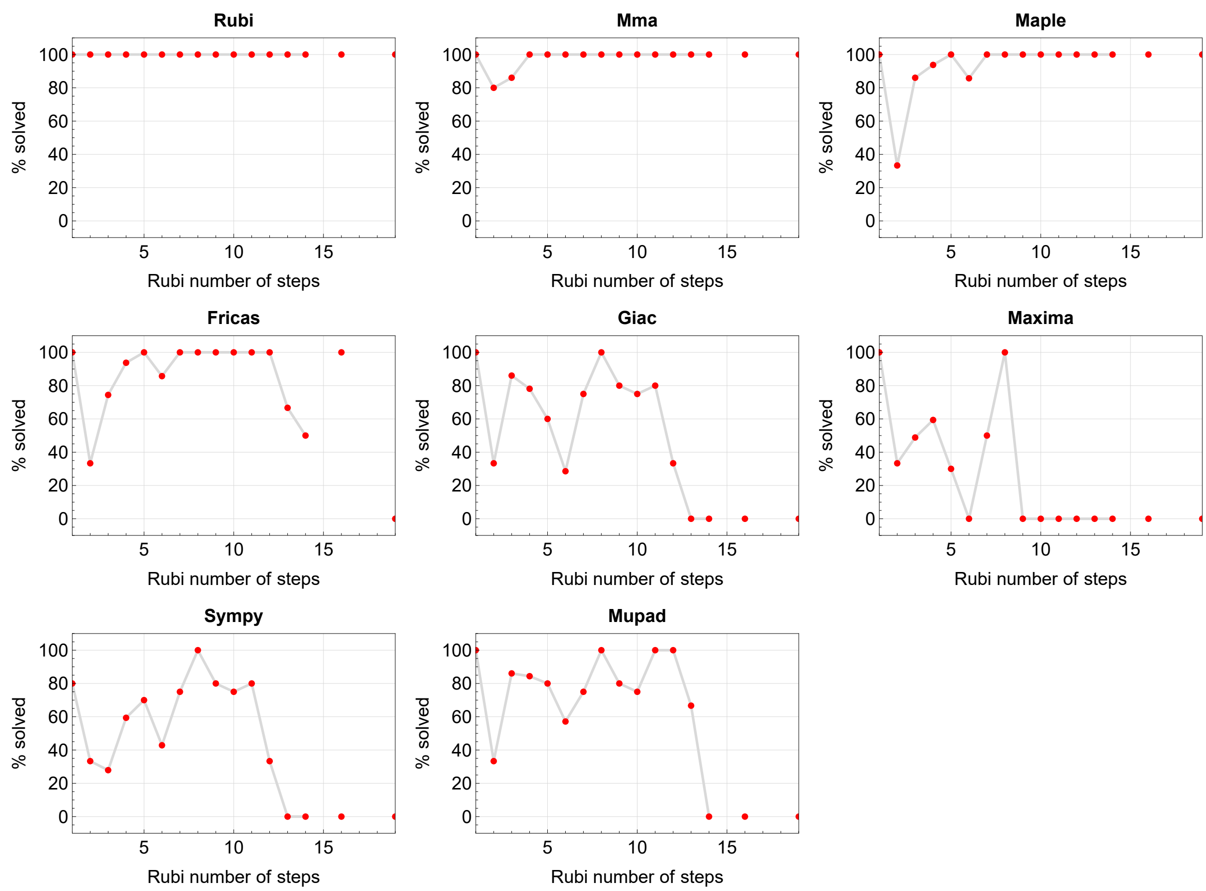


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

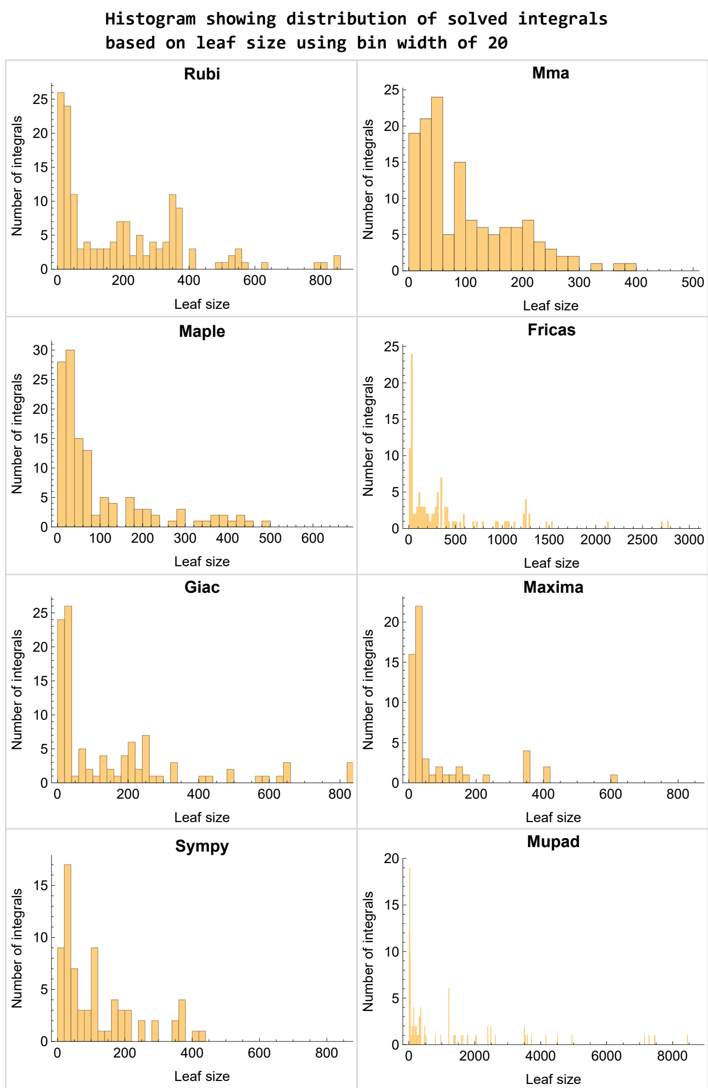


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

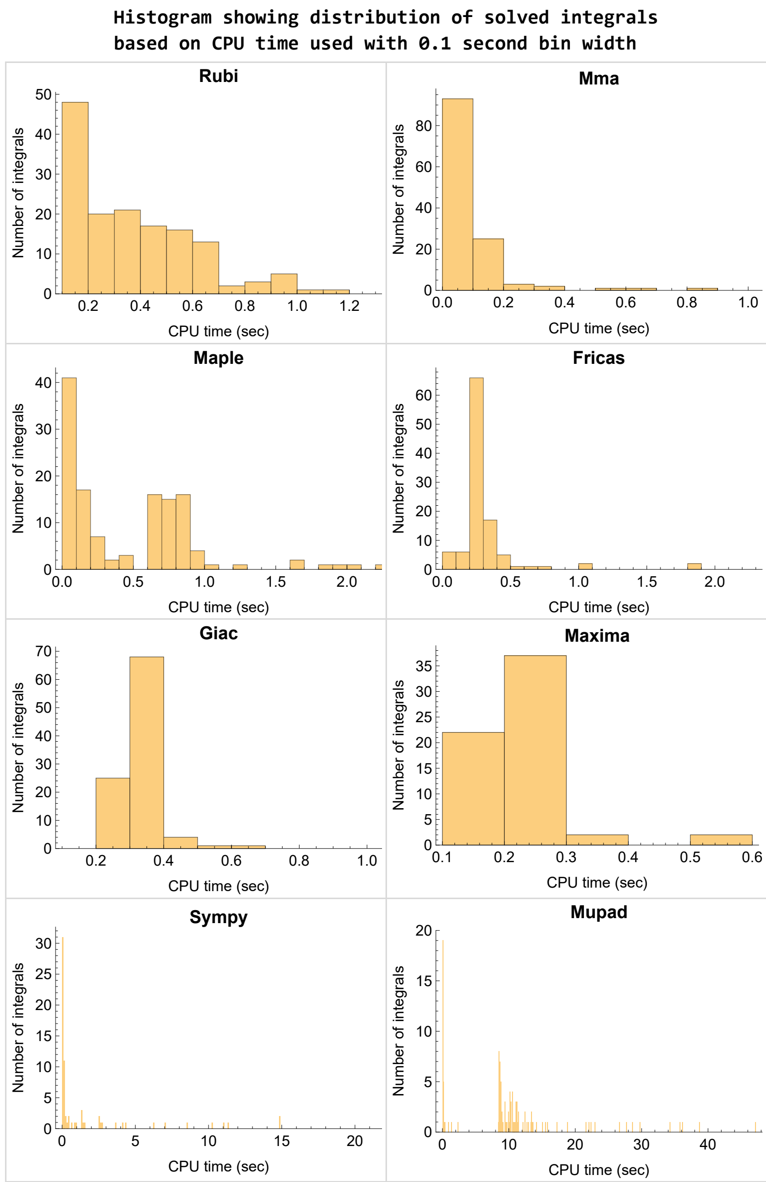


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

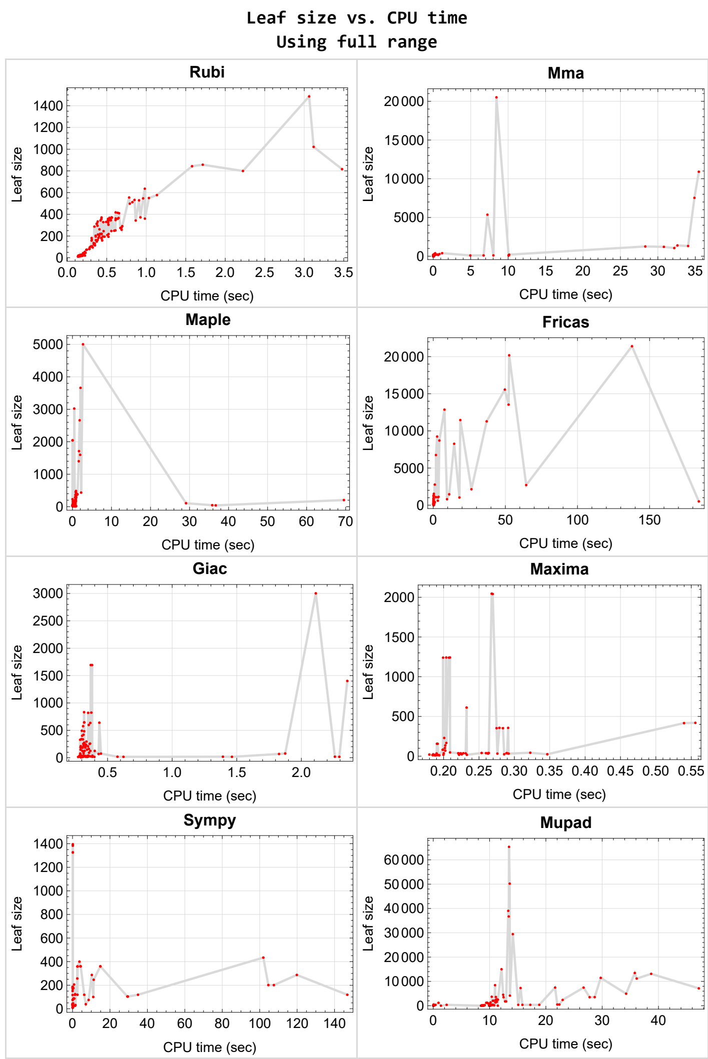


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{86, 155, 156\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {143}

Maple {140}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

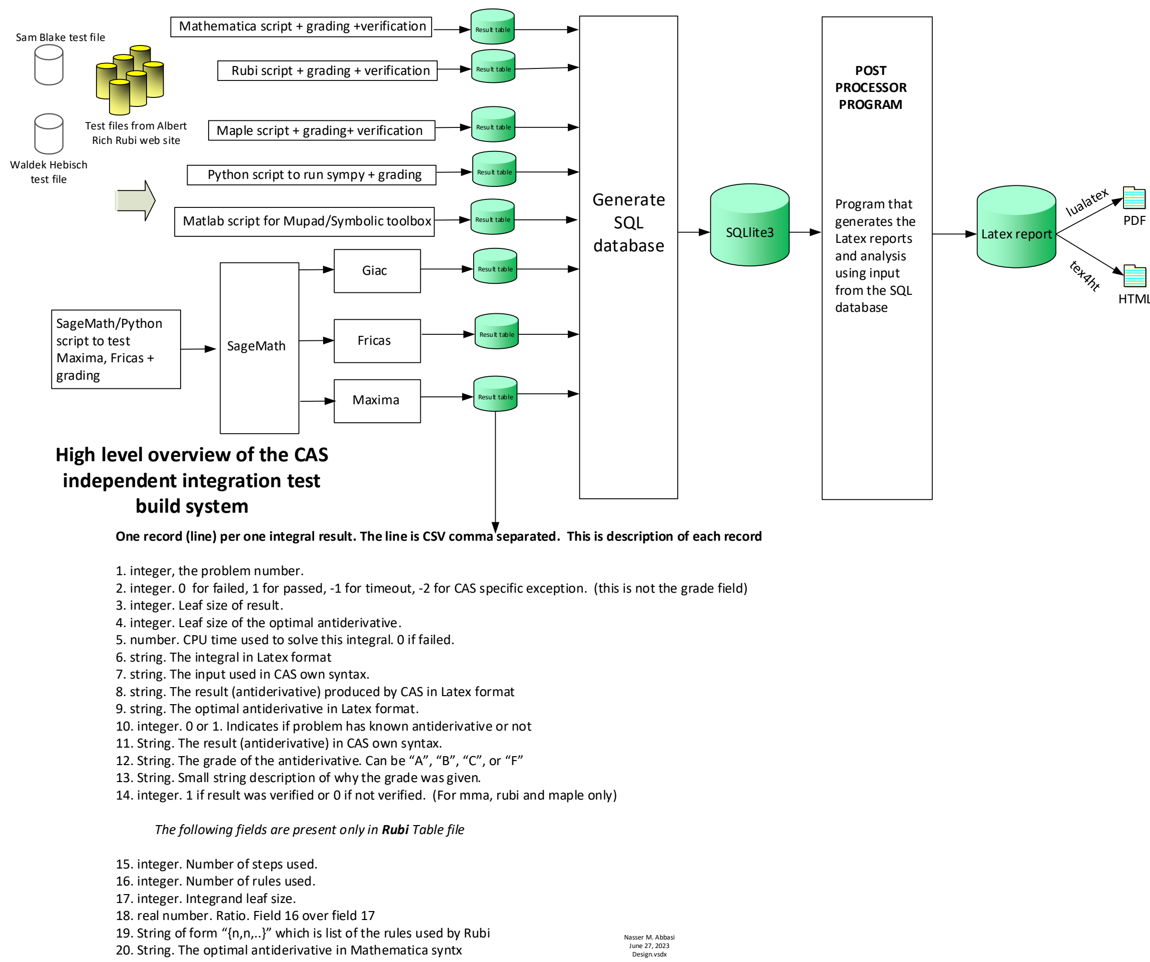
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 20, 21, 22, 32, 44, 46, 53, 55, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 87, 88, 89, 96, 100, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 141, 144, 149, 152, 153, 154 }

B grade { 93, 94, 95, 97, 98, 99, 101, 102, 103, 142, 143 }

C grade { 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 79, 80, 81, 82, 83, 84, 85, 138, 139, 140 }

F normal fail { 90, 91, 92, 145, 146, 147, 148, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 46, 48, 50, 53, 55, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 84, 85, 93, 94, 95, 97, 98, 99, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144 }

B grade { 79, 80, 81, 82, 83, 96, 100, 104, 128 }

C grade { 6, 7, 8, 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 43, 45, 47, 49, 51, 52, 54, 56, 58, 59, 60, 140 }

F normal fail { 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 44, 48, 53, 55, 57, 61, 62, 63, 64, 65, 66, 67, 105, 106, 107, 108, 113, 114, 115, 116, 121, 122, 123, 124, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144 }

B grade { 8, 14, 15, 16, 17, 18, 19, 43, 45, 46, 47, 49, 50, 51, 70, 71, 72, 73, 74, 75, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 109, 110, 111, 112, 117, 118, 119, 120, 125, 126, 127, 128 }

C grade { 35, 36, 37, 38, 39, 40, 41, 42, 52, 54, 56, 58, 59, 60, 79, 80, 81, 82 }

F normal fail { 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

F(-1) timedout fail { 68, 69, 76, 77, 78, 83, 84, 85 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 20, 21, 22, 23, 24, 32, 33, 53, 57, 93, 97, 101, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 121, 122, 123, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144 }

B grade { 94, 95, 96, 98, 99, 100, 102, 103, 104, 110, 111, 112, 118, 119, 120, 124, 126, 127, 128 }

C grade { }

F normal fail { 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

F(-1) timedout fail { }

F(-2) exception fail { 6, 7, 8, 9, 10, 11, 12, 13, 44, 48, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

B grade { 25, 26, 27, 28, 29, 30, 31, 46, 50, 93, 94, 95, 96, 97, 98, 99, 100, 104 }

C grade { }

F normal fail { 15, 16, 17, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 153, 154 }

F(-1) timedout fail { 14, 43, 45, 47, 49, 51 }

F(-2) exception fail { 87, 88, 144, 152 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144 }

C grade { }

F normal fail { }

F(-1) timedout fail { 35, 36, 37, 38, 39, 40, 41, 42, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 60, 105, 106, 107, 113, 114, 115, 121, 122, 123 }

B grade { 10, 11, 44, 93, 94, 95, 97, 98, 99, 101, 102, 103, 109, 110, 111, 117, 118, 119, 124, 125, 126, 127, 129, 130, 133, 134, 137, 138, 140, 144 }

C grade { }

F normal fail { 79, 80, 81, 82, 83, 84, 85, 141, 149 }

F(-1) timedout fail { 9, 12, 13, 14, 15, 16, 17, 18, 19, 42, 43, 45, 46, 47, 48, 49, 50, 51, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 92, 96, 100, 104, 108, 112, 116, 120, 128, 131, 132, 135, 136, 139, 142, 143, 148, 151, 152, 153, 154 }

F(-2) exception fail { 145, 146, 147, 150, 155, 156 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	164	165	166	166	187	182	158
N.S.	1	1.00	1.01	1.01	1.02	1.02	1.15	1.12	0.97
time (sec)	N/A	0.365	0.046	0.875	0.204	0.247	0.035	0.306	0.052

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	135	134	135	135	151	147	130
N.S.	1	1.00	1.00	0.99	1.00	1.00	1.12	1.09	0.96
time (sec)	N/A	0.328	0.028	0.676	0.202	0.260	0.029	0.291	0.034

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	104	103	102	102	117	112	102
N.S.	1	1.00	1.01	1.00	0.99	0.99	1.14	1.09	0.99
time (sec)	N/A	0.287	0.021	0.644	0.202	0.267	0.028	0.312	0.027

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	69	75	76	70
N.S.	1	1.00	1.00	0.96	0.95	0.95	1.03	1.04	0.96
time (sec)	N/A	0.239	0.016	0.674	0.202	0.260	0.027	0.288	0.023

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	36	39	40	38
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.93	0.95	0.90
time (sec)	N/A	0.196	0.008	0.101	0.200	0.255	0.020	0.295	0.026

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	162	176	67	0	465	175	191	165
N.S.	1	0.86	0.94	0.36	0.00	2.47	0.93	1.02	0.88
time (sec)	N/A	0.361	0.112	0.709	0.000	0.276	0.416	0.297	0.172

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	183	199	88	0	697	206	211	187
N.S.	1	0.86	0.93	0.41	0.00	3.27	0.97	0.99	0.88
time (sec)	N/A	0.391	0.131	0.651	0.000	0.299	0.818	0.292	10.843

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	218	209	114	0	941	246	236	221
N.S.	1	0.90	0.86	0.47	0.00	3.89	1.02	0.98	0.91
time (sec)	N/A	0.424	0.189	0.645	0.000	0.325	11.306	0.333	0.174

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	131	126	136	0	430	0	125	3586
N.S.	1	0.99	0.95	1.03	0.00	3.26	0.00	0.95	27.17
time (sec)	N/A	0.348	0.049	0.270	0.000	0.522	0.000	0.366	11.115

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	95	93	98	0	305	434	91	2624
N.S.	1	0.98	0.96	1.01	0.00	3.14	4.47	0.94	27.05
time (sec)	N/A	0.280	0.076	0.160	0.000	0.364	101.949	0.357	11.310

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	71	66	0	216	287	68	1632
N.S.	1	1.03	0.99	0.92	0.00	3.00	3.99	0.94	22.67
time (sec)	N/A	0.235	0.040	0.122	0.000	0.326	10.297	0.430	11.273

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	81	80	75	0	240	0	75	4149
N.S.	1	1.04	1.03	0.96	0.00	3.08	0.00	0.96	53.19
time (sec)	N/A	0.277	0.028	0.097	0.000	0.414	0.000	0.449	13.612

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	114	130	126	0	385	0	124	7282
N.S.	1	1.02	1.16	1.12	0.00	3.44	0.00	1.11	65.02
time (sec)	N/A	0.340	0.036	0.125	0.000	0.677	0.000	0.401	15.505

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	723	577	88	70	0	13535	0	0	13112
N.S.	1	0.80	0.12	0.10	0.00	18.72	0.00	0.00	18.14
time (sec)	N/A	1.081	0.037	0.079	0.000	52.228	0.000	0.000	38.703

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	550	88	67	0	8705	0	0	11453
N.S.	1	0.77	0.12	0.09	0.00	12.12	0.00	0.00	15.95
time (sec)	N/A	0.981	0.033	0.062	0.000	4.249	0.000	0.000	29.746

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	634	532	59	49	0	8268	0	0	7457
N.S.	1	0.84	0.09	0.08	0.00	13.04	0.00	0.00	11.76
time (sec)	N/A	0.842	0.033	0.061	0.000	14.503	0.000	0.000	26.698

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	634	512	61	47	0	6748	0	0	7469
N.S.	1	0.81	0.10	0.07	0.00	10.64	0.00	0.00	11.78
time (sec)	N/A	0.786	0.023	0.056	0.000	1.872	0.000	0.000	21.637

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	653	547	85	71	0	11285	0	0	11174
N.S.	1	0.84	0.13	0.11	0.00	17.28	0.00	0.00	17.11
time (sec)	N/A	0.918	0.033	0.102	0.000	37.035	0.000	0.000	36.131

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	655	529	89	68	0	11459	0	0	13466
N.S.	1	0.81	0.14	0.10	0.00	17.49	0.00	0.00	20.56
time (sec)	N/A	0.896	0.032	0.099	0.000	18.760	0.000	0.000	35.790

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	48	46	38	37	37	42	37	39
N.S.	1	1.04	1.00	0.83	0.80	0.80	0.91	0.80	0.85
time (sec)	N/A	0.217	0.028	0.048	0.290	0.279	0.063	0.383	10.137

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	24	24	32	24	26
N.S.	1	1.00	1.00	0.81	0.77	0.77	1.03	0.77	0.84
time (sec)	N/A	0.196	0.008	0.045	0.286	0.284	0.057	0.367	0.027

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	40	39	33	32	32	37	32	34
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.207	0.009	0.042	0.292	0.296	0.058	0.325	0.035

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	44	44	33	38	34	41	35	36
N.S.	1	1.07	1.07	0.80	0.93	0.83	1.00	0.85	0.88
time (sec)	N/A	0.214	0.017	0.054	0.289	0.326	0.069	0.315	10.153

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	45	25	24	28	36	24	26
N.S.	1	1.00	1.45	0.81	0.77	0.90	1.16	0.77	0.84
time (sec)	N/A	0.202	0.012	0.059	0.347	0.302	0.062	0.301	10.004

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	356	47	46	0	266	31	645	332
N.S.	1	0.85	0.11	0.11	0.00	0.64	0.07	1.54	0.79
time (sec)	N/A	0.610	0.012	0.046	0.000	0.317	0.082	0.319	0.396

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	362	48	44	0	300	32	820	309
N.S.	1	0.95	0.13	0.12	0.00	0.79	0.08	2.15	0.81
time (sec)	N/A	0.594	0.012	0.046	0.000	0.327	0.084	0.350	10.566

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	340	46	41	0	288	24	635	330
N.S.	1	0.90	0.12	0.11	0.00	0.76	0.06	1.68	0.87
time (sec)	N/A	0.518	0.011	0.040	0.000	0.316	0.079	0.366	10.547

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	367	55	44	0	237	22	824	281
N.S.	1	0.89	0.13	0.11	0.00	0.58	0.05	2.00	0.68
time (sec)	N/A	0.530	0.010	0.043	0.000	0.293	0.086	0.374	10.351

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	349	57	44	0	299	26	640	319
N.S.	1	0.85	0.14	0.11	0.00	0.73	0.06	1.56	0.78
time (sec)	N/A	0.501	0.011	0.045	0.000	0.299	0.084	0.436	10.351

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	372	47	40	0	313	31	832	313
N.S.	1	0.89	0.11	0.10	0.00	0.75	0.07	2.00	0.75
time (sec)	N/A	0.551	0.013	0.055	0.000	0.299	0.098	0.318	0.255

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	356	47	38	0	299	32	645	332
N.S.	1	0.85	0.11	0.09	0.00	0.72	0.08	1.54	0.79
time (sec)	N/A	0.528	0.012	0.057	0.000	0.298	0.090	0.319	10.538

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	41	37	33	32	32	37	32	34
N.S.	1	1.14	1.03	0.92	0.89	0.89	1.03	0.89	0.94
time (sec)	N/A	0.208	0.008	0.046	0.276	0.294	0.063	0.304	0.029

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	45	55	33	38	34	41	35	36
N.S.	1	1.15	1.41	0.85	0.97	0.87	1.05	0.90	0.92
time (sec)	N/A	0.212	0.011	0.052	0.260	0.286	0.067	0.298	8.860

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	45	55	33	0	34	41	35	36
N.S.	1	1.15	1.41	0.85	0.00	0.87	1.05	0.90	0.92
time (sec)	N/A	0.224	0.008	0.053	0.000	0.292	0.066	0.283	0.024

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	356	103	434	0	169	400	0	0
N.S.	1	0.90	0.26	1.10	0.00	0.43	1.01	0.00	0.00
time (sec)	N/A	0.416	8.031	2.217	0.000	0.092	3.679	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	334	101	398	0	137	257	0	0
N.S.	1	0.94	0.28	1.12	0.00	0.38	0.72	0.00	0.00
time (sec)	N/A	0.382	6.722	0.894	0.000	0.099	2.534	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	312	98	362	0	104	124	0	0
N.S.	1	0.99	0.31	1.15	0.00	0.33	0.39	0.00	0.00
time (sec)	N/A	0.365	4.947	0.893	0.000	0.092	1.411	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	287	98	333	0	72	119	0	0
N.S.	1	1.03	0.35	1.20	0.00	0.26	0.43	0.00	0.00
time (sec)	N/A	0.335	10.065	0.855	0.000	0.095	1.300	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	295	102	382	0	125	119	0	0
N.S.	1	1.02	0.35	1.32	0.00	0.43	0.41	0.00	0.00
time (sec)	N/A	0.371	10.072	1.293	0.000	0.096	6.245	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	317	129	401	0	190	119	0	0
N.S.	1	1.03	0.42	1.30	0.00	0.61	0.39	0.00	0.00
time (sec)	N/A	0.367	10.086	1.000	0.000	0.105	34.982	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	344	166	437	0	268	119	0	0
N.S.	1	0.99	0.48	1.25	0.00	0.77	0.34	0.00	0.00
time (sec)	N/A	0.426	10.121	0.898	0.000	0.101	146.823	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	371	200	484	0	346	0	0	0
N.S.	1	0.95	0.51	1.24	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.431	10.144	0.906	0.000	0.109	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	366	88	67	0	12866	0	0	50213
N.S.	1	0.85	0.20	0.15	0.00	29.71	0.00	0.00	115.97
time (sec)	N/A	0.632	0.054	0.073	0.000	7.770	0.000	0.000	13.574

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	71	66	0	216	287	68	3704
N.S.	1	1.03	0.99	0.92	0.00	3.00	3.99	0.94	51.44
time (sec)	N/A	0.238	0.040	0.154	0.000	0.421	119.858	1.827	10.386

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	330	59	51	0	15561	0	0	29445
N.S.	1	0.88	0.16	0.14	0.00	41.50	0.00	0.00	78.52
time (sec)	N/A	0.470	0.034	0.066	0.000	49.644	0.000	0.000	14.129

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	182	179	168	0	1535	0	1402	4501
N.S.	1	0.99	0.97	0.91	0.00	8.34	0.00	7.62	24.46
time (sec)	N/A	0.316	0.106	0.119	0.000	0.408	0.000	2.356	12.415

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	328	61	47	0	9245	0	0	36707
N.S.	1	0.87	0.16	0.13	0.00	24.65	0.00	0.00	97.89
time (sec)	N/A	0.452	0.033	0.062	0.000	2.697	0.000	0.000	13.406

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	81	80	74	0	240	0	77	8454
N.S.	1	1.04	1.03	0.95	0.00	3.08	0.00	0.99	108.38
time (sec)	N/A	0.273	0.026	0.123	0.000	0.755	0.000	1.875	10.971

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	345	85	73	0	21400	0	0	39028
N.S.	1	0.88	0.22	0.19	0.00	54.59	0.00	0.00	99.56
time (sec)	N/A	0.549	0.091	0.112	0.000	137.794	0.000	0.000	13.320

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	195	89	177	0	2772	0	3003	15013
N.S.	1	0.98	0.45	0.89	0.00	13.93	0.00	15.09	75.44
time (sec)	N/A	0.375	0.030	0.151	0.000	1.058	0.000	2.112	12.123

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	345	86	68	0	20184	0	0	65350
N.S.	1	0.88	0.22	0.17	0.00	51.23	0.00	0.00	165.86
time (sec)	N/A	0.524	0.049	0.119	0.000	52.641	0.000	0.000	13.454

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	414	46	34	0	104	170	208	56
N.S.	1	1.49	0.17	0.12	0.00	0.37	0.61	0.75	0.20
time (sec)	N/A	0.634	0.022	0.066	0.000	0.277	0.105	0.348	0.061

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	40	39	33	32	32	37	32	34
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.200	0.010	0.070	0.263	0.285	0.063	0.346	0.027

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	360	55	46	0	545	27	253	248
N.S.	1	1.01	0.15	0.13	0.00	1.54	0.08	0.71	0.70
time (sec)	N/A	0.635	0.015	0.078	0.000	0.296	1.354	0.361	0.094

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	54	44	39	0	41	42	31	20
N.S.	1	1.08	0.88	0.78	0.00	0.82	0.84	0.62	0.40
time (sec)	N/A	0.218	0.016	0.076	0.000	0.262	0.063	0.344	8.478

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	347	57	44	0	417	26	253	208
N.S.	1	0.98	0.16	0.12	0.00	1.17	0.07	0.71	0.59
time (sec)	N/A	0.616	0.012	0.065	0.000	0.305	1.350	0.312	0.001

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	44	44	33	38	34	41	38	36
N.S.	1	1.07	1.07	0.80	0.93	0.83	1.00	0.93	0.88
time (sec)	N/A	0.214	0.012	0.081	0.263	0.262	0.074	0.352	8.504

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	418	47	38	0	111	168	210	58
N.S.	1	1.49	0.17	0.14	0.00	0.40	0.60	0.75	0.21
time (sec)	N/A	0.599	0.015	0.091	0.000	0.260	0.105	0.326	8.549

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	95	49	40	0	169	76	81	56
N.S.	1	1.07	0.55	0.45	0.00	1.90	0.85	0.91	0.63
time (sec)	N/A	0.322	0.013	0.105	0.000	0.257	0.111	0.314	0.058

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	47	38	0	419	32	258	479
N.S.	1	1.00	0.13	0.10	0.00	1.13	0.09	0.70	1.29
time (sec)	N/A	0.559	0.014	0.102	0.000	0.294	1.543	0.365	0.041

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	283	286	0	1027	0	295	2490
N.S.	1	1.00	1.01	1.02	0.00	3.67	0.00	1.05	8.89
time (sec)	N/A	0.684	0.151	0.671	0.000	18.156	0.000	0.336	11.488

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	218	208	0	798	0	221	2051
N.S.	1	1.00	1.00	0.95	0.00	3.66	0.00	1.01	9.41
time (sec)	N/A	0.527	0.109	0.709	0.000	9.547	0.000	0.331	10.700

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	178	164	0	596	0	185	1367
N.S.	1	1.00	1.01	0.93	0.00	3.39	0.00	1.05	7.77
time (sec)	N/A	0.428	0.113	0.822	0.000	3.159	0.000	0.327	10.179

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	132	130	0	405	0	147	966
N.S.	1	1.00	0.89	0.87	0.00	2.72	0.00	0.99	6.48
time (sec)	N/A	0.366	0.077	0.743	0.000	1.066	0.000	0.341	9.718

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	122	107	105	0	305	0	125	801
N.S.	1	0.98	0.86	0.85	0.00	2.46	0.00	1.01	6.46
time (sec)	N/A	0.331	0.045	0.690	0.000	0.431	0.000	0.322	9.902

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	106	105	104	0	305	0	125	521
N.S.	1	0.86	0.85	0.85	0.00	2.48	0.00	1.02	4.24
time (sec)	N/A	0.308	0.047	0.681	0.000	0.453	0.000	0.305	10.160

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	159	152	160	0	504	0	162	2399
N.S.	1	1.01	0.96	1.01	0.00	3.19	0.00	1.03	15.18
time (sec)	N/A	0.444	0.111	0.769	0.000	184.148	0.000	0.355	11.089

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	194	207	0	0	0	206	2388
N.S.	1	1.00	1.01	1.07	0.00	0.00	0.00	1.07	12.37
time (sec)	N/A	0.512	0.109	0.871	0.000	0.000	0.000	0.325	22.980

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	252	277	0	0	0	274	3530
N.S.	1	1.00	1.00	1.10	0.00	0.00	0.00	1.09	14.01
time (sec)	N/A	0.597	0.138	0.742	0.000	0.000	0.000	0.319	28.669

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	338	360	0	2703	0	576	3503
N.S.	1	1.00	0.99	1.05	0.00	7.88	0.00	1.68	10.21
time (sec)	N/A	0.862	0.231	0.749	0.000	64.455	0.000	0.311	12.481

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	269	290	0	2139	0	483	2495
N.S.	1	1.00	0.98	1.06	0.00	7.81	0.00	1.76	9.11
time (sec)	N/A	0.659	0.186	0.742	0.000	26.472	0.000	0.319	11.200

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	207	228	0	1465	0	420	2037
N.S.	1	1.00	0.84	0.93	0.00	5.96	0.00	1.71	8.28
time (sec)	N/A	0.550	0.152	0.846	0.000	11.045	0.000	0.314	10.578

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	159	188	0	1120	0	339	1585
N.S.	1	1.00	0.82	0.97	0.00	5.77	0.00	1.75	8.17
time (sec)	N/A	0.460	0.143	0.858	0.000	3.638	0.000	0.297	11.246

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	148	187	0	1059	0	328	1768
N.S.	1	1.00	0.81	1.02	0.00	5.79	0.00	1.79	9.66
time (sec)	N/A	0.434	0.141	0.713	0.000	3.324	0.000	0.288	12.946

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	209	151	197	0	1079	0	336	1782
N.S.	1	1.11	0.80	1.04	0.00	5.71	0.00	1.78	9.43
time (sec)	N/A	0.499	0.129	0.741	0.000	1.860	0.000	0.309	12.751

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	249	246	281	0	0	0	401	3510
N.S.	1	1.00	0.99	1.13	0.00	0.00	0.00	1.62	14.15
time (sec)	N/A	0.592	0.162	0.955	0.000	0.000	0.000	0.303	27.771

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	287	346	0	0	0	493	4948
N.S.	1	1.00	0.99	1.19	0.00	0.00	0.00	1.69	17.00
time (sec)	N/A	0.692	0.218	0.998	0.000	0.000	0.000	0.300	34.253

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	370	455	0	0	0	598	7144
N.S.	1	1.00	0.99	1.22	0.00	0.00	0.00	1.61	19.20
time (sec)	N/A	0.920	0.265	0.818	0.000	0.000	0.000	0.354	47.165

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	981	1021	10904	5004	0	920	0	0	0
N.S.	1	1.04	11.12	5.10	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	3.101	35.469	2.698	0.000	0.116	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	778	799	7531	3661	0	734	0	0	0
N.S.	1	1.03	9.68	4.71	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	2.198	34.884	2.061	0.000	0.127	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	636	636	1314	2662	0	598	0	0	0
N.S.	1	1.00	2.07	4.19	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.991	34.034	1.835	0.000	0.107	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	555	1051	1711	0	490	0	0	0
N.S.	1	1.01	1.91	3.11	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.784	32.189	1.669	0.000	0.099	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	955	857	1258	3023	0	0	0	0	0
N.S.	1	0.90	1.32	3.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.680	28.320	0.451	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	929	843	1207	1400	0	0	0	0	0
N.S.	1	0.91	1.30	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.542	30.801	1.605	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1287	1486	1392	1597	0	0	0	0	0
N.S.	1	1.15	1.08	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.931	32.614	1.972	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	0	28	28
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.183	0.604	0.349	0.258	0.323	0.000	0.348	8.786

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	358	358	249	0	0	0	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.548	0.316	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	262	189	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.430	0.197	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	136	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	0.096	0.000	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	194	200	0	0	0	0	0	0	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.414	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	302	304	0	0	0	0	0	0	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.522	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	412	410	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.651	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	201	15	14	1234	1326	216	1203
N.S.	1	1.00	12.56	0.94	0.88	77.12	82.88	13.50	75.19
time (sec)	N/A	0.179	0.119	0.789	0.192	0.273	0.156	0.323	9.422

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	233	17	1240	1240	1384	246	1210
N.S.	1	1.00	12.94	0.94	68.89	68.89	76.89	13.67	67.22
time (sec)	N/A	0.188	0.117	0.171	0.208	0.261	0.144	0.323	9.433

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	233	17	1240	1240	1394	246	1210
N.S.	1	1.00	12.94	0.94	68.89	68.89	77.44	13.67	67.22
time (sec)	N/A	0.187	0.110	0.217	0.199	0.270	0.154	0.321	9.605

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	2042	2041	1297	0	1693	1395
N.S.	1	1.00	0.96	88.78	88.74	56.39	0.00	73.61	60.65
time (sec)	N/A	0.180	0.077	0.025	0.269	0.293	0.000	0.380	11.021

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	201	17	16	1238	1326	218	1208
N.S.	1	1.00	11.17	0.94	0.89	68.78	73.67	12.11	67.11
time (sec)	N/A	0.184	0.108	0.817	0.189	0.258	0.146	0.314	0.931

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	233	19	1242	1242	1384	246	1214
N.S.	1	1.00	11.65	0.95	62.10	62.10	69.20	12.30	60.70
time (sec)	N/A	0.226	0.120	0.172	0.204	0.259	0.141	0.324	9.465

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	233	19	1242	1242	1394	246	1214
N.S.	1	1.00	11.65	0.95	62.10	62.10	69.70	12.30	60.70
time (sec)	N/A	0.194	0.113	0.220	0.209	0.281	0.140	0.322	9.572

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	2046	2045	1299	0	1693	1401
N.S.	1	1.00	0.96	81.84	81.80	51.96	0.00	67.72	56.04
time (sec)	N/A	0.187	0.063	0.025	0.268	0.332	0.000	0.370	11.073

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	172	13	13	154	175	13	154
N.S.	1	1.00	11.47	0.87	0.87	10.27	11.67	0.87	10.27
time (sec)	N/A	0.151	0.008	0.612	0.187	0.252	0.052	0.291	0.105

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	182	15	156	156	182	15	156
N.S.	1	1.00	11.38	0.94	9.75	9.75	11.38	0.94	9.75
time (sec)	N/A	0.149	0.006	0.655	0.192	0.252	0.046	0.308	0.111

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	186	15	156	156	185	15	156
N.S.	1	1.00	11.62	0.94	9.75	9.75	11.56	0.94	9.75
time (sec)	N/A	0.157	0.004	0.703	0.191	0.253	0.048	0.294	0.108

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	189	0	189	229
N.S.	1	1.00	1.00	10.95	10.90	9.00	0.00	9.00	10.90
time (sec)	N/A	0.160	0.012	0.015	0.201	0.269	0.000	0.291	9.130

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	12	11	11	10	12	11
N.S.	1	1.00	0.91	1.09	1.00	1.00	0.91	1.09	1.00
time (sec)	N/A	0.138	0.004	0.898	0.194	0.287	0.077	0.293	8.633

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	16	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.159	0.006	0.033	0.187	0.266	0.162	0.623	8.592

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	16	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.168	0.006	0.039	0.189	0.264	0.215	0.382	0.032

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	33	24	23	19	0	19	121
N.S.	1	1.00	1.74	1.26	1.21	1.00	0.00	1.00	6.37
time (sec)	N/A	0.174	0.055	0.393	0.226	0.286	0.000	0.304	8.893

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	15	15	14	350	359	14	358
N.S.	1	1.00	0.94	0.94	0.88	21.88	22.44	0.88	22.38
time (sec)	N/A	0.138	0.010	0.886	0.185	0.279	2.546	0.341	2.374

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	352	352	360	16	360
N.S.	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00
time (sec)	N/A	0.160	0.012	0.431	0.284	0.302	4.194	1.463	15.833

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	352	352	360	16	360
N.S.	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00
time (sec)	N/A	0.165	0.010	0.222	0.275	0.285	14.853	2.294	18.831

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	416	394	0	21	496
N.S.	1	1.00	0.96	0.96	18.09	17.13	0.00	0.91	21.57
time (sec)	N/A	0.172	0.062	0.016	0.540	0.345	0.000	0.305	22.399

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	13	13	10	14	13
N.S.	1	1.00	0.92	1.08	1.00	1.00	0.77	1.08	1.00
time (sec)	N/A	0.136	0.004	0.889	0.191	0.275	0.091	0.272	0.026

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	14	18	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89
time (sec)	N/A	0.166	0.006	0.034	0.200	0.279	0.152	0.576	8.565

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	14	18	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89
time (sec)	N/A	0.169	0.006	0.041	0.186	0.270	0.218	0.350	0.034

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	34	26	25	21	0	21	199
N.S.	1	1.00	1.62	1.24	1.19	1.00	0.00	1.00	9.48
time (sec)	N/A	0.175	0.054	0.391	0.221	0.291	0.000	0.313	8.928

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	16	17	16	354	359	16	358
N.S.	1	1.00	0.89	0.94	0.89	19.67	19.94	0.89	19.89
time (sec)	N/A	0.141	0.011	0.856	0.190	0.290	2.746	0.331	10.652

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	356	356	360	18	360
N.S.	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00
time (sec)	N/A	0.163	0.014	0.424	0.291	0.296	4.361	1.393	15.096

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	356	356	360	18	360
N.S.	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00
time (sec)	N/A	0.172	0.013	0.211	0.279	0.296	14.848	2.260	17.239

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	419	397	0	23	496
N.S.	1	1.00	0.92	0.96	16.76	15.88	0.00	0.92	19.84
time (sec)	N/A	0.178	0.056	0.016	0.556	0.307	0.000	0.307	22.029

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	8
N.S.	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.134	0.006	0.632	0.188	0.294	0.060	0.296	8.581

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	17	15	14	17	13	12	15	13
N.S.	1	1.06	0.94	0.88	1.06	0.81	0.75	0.94	0.81
time (sec)	N/A	0.161	0.005	0.600	0.222	0.268	0.084	0.308	0.037

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	17	15	14	17	13	12	15	13
N.S.	1	1.06	0.94	0.88	1.06	0.81	0.75	0.94	0.81
time (sec)	N/A	0.163	0.007	0.638	0.233	0.258	0.094	0.295	8.653

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	17	19	18	47	17	39	17	28
N.S.	1	1.13	1.27	1.20	3.13	1.13	2.60	1.13	1.87
time (sec)	N/A	0.171	0.009	0.770	0.209	0.258	7.081	0.295	8.678

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	14	13	13	81	87	13	12
N.S.	1	1.00	0.93	0.87	0.87	5.40	5.80	0.87	0.80
time (sec)	N/A	0.139	0.014	0.676	0.184	0.287	0.451	0.292	9.900

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.143	0.020	0.679	0.199	0.286	0.661	0.397	1.366

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.155	0.025	0.710	0.199	0.272	0.928	0.300	11.460

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	105	0	20	107
N.S.	1	1.00	1.00	9.67	29.14	5.00	0.00	0.95	5.10
time (sec)	N/A	0.158	0.009	69.556	0.232	0.335	0.000	0.313	8.772

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	28	104	20	39
N.S.	1	1.00	0.95	1.05	1.00	1.40	5.20	1.00	1.95
time (sec)	N/A	0.147	0.006	0.880	0.180	0.293	29.582	0.311	8.813

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	33	33	201	23	49
N.S.	1	1.00	1.00	0.96	1.32	1.32	8.04	0.92	1.96
time (sec)	N/A	0.167	0.046	0.132	0.222	0.325	104.634	0.284	8.763

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	33	33	0	23	49
N.S.	1	1.00	1.00	0.96	1.32	1.32	0.00	0.92	1.96
time (sec)	N/A	0.175	0.071	0.259	0.224	0.322	0.000	0.300	8.737

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	40	39	38	0	27	56
N.S.	1	1.00	0.96	1.48	1.44	1.41	0.00	1.00	2.07
time (sec)	N/A	0.180	0.112	36.713	0.264	0.301	0.000	0.329	8.978

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	22	32	104	22	42
N.S.	1	1.00	0.95	1.05	1.00	1.45	4.73	1.00	1.91
time (sec)	N/A	0.144	0.052	0.901	0.185	0.271	29.298	0.336	8.698

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	37	37	201	25	52
N.S.	1	1.00	1.00	0.96	1.37	1.37	7.44	0.93	1.93
time (sec)	N/A	0.167	0.038	0.140	0.221	0.278	107.588	0.299	8.611

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	37	37	0	25	52
N.S.	1	1.00	1.00	0.96	1.37	1.37	0.00	0.93	1.93
time (sec)	N/A	0.172	0.068	0.261	0.228	0.262	0.000	0.281	8.584

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	45	43	42	0	29	59
N.S.	1	1.00	0.97	1.55	1.48	1.45	0.00	1.00	2.03
time (sec)	N/A	0.184	0.092	35.813	0.323	0.294	0.000	0.342	8.888

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	19	26	46	19	23
N.S.	1	1.00	0.89	1.05	1.00	1.37	2.42	1.00	1.21
time (sec)	N/A	0.151	0.011	0.674	0.185	0.249	0.306	0.295	8.562

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	97	31	35	31	75	22	31
N.S.	1	1.00	4.04	1.29	1.46	1.29	3.12	0.92	1.29
time (sec)	N/A	0.170	0.064	0.714	0.224	0.250	8.591	0.285	8.609

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	97	31	35	31	0	22	31
N.S.	1	1.00	4.04	1.29	1.46	1.29	0.00	0.92	1.29
time (sec)	N/A	0.174	0.060	0.761	0.231	0.262	0.000	0.283	8.582

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	B	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	26	26	111	106	40	36	100	26	34
N.S.	1	1.00	4.27	4.08	1.54	1.38	3.85	1.00	1.31
time (sec)	N/A	0.185	0.110	29.093	0.254	0.280	11.068	0.328	8.643

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	158	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.442	0.651	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	374	360	5363	0	0	0	0	0	0
N.S.	1	0.96	14.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.985	7.233	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	816	816	20515	0	0	0	0	0	0
N.S.	1	1.00	25.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.483	8.441	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	34	33	119	0	31
N.S.	1	1.00	1.00	0.77	0.72	0.70	2.53	0.00	0.66
time (sec)	N/A	0.233	0.090	0.030	0.190	0.258	2.605	0.000	8.818

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	245	245	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	210	219	0	0	0	0	0	0	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.428	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	206	214	0	0	0	0	0	0	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	194	203	0	0	0	0	0	0	0
N.S.	1	1.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.366	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	258	218	0	0	0	0	0	0
N.S.	1	0.98	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.677	0.576	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	212	221	0	0	0	0	0	0	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.438	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	210	220	0	0	0	0	0	0	0
N.S.	1	1.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.428	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	498	498	391	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.806	1.188	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	323	323	273	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.519	0.825	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	0.339	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	33	0	33	33
N.S.	1	1.00	1.06	1.00	1.06	1.06	0.00	1.06	1.06
time (sec)	N/A	0.176	1.119	0.124	0.233	0.280	0.000	0.350	8.701

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	46	0	33	33
N.S.	1	1.00	1.06	1.00	1.06	1.48	0.00	1.06	1.06
time (sec)	N/A	0.176	0.927	0.142	0.239	0.311	0.000	0.489	9.018

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [85] had the largest ratio of [.6206899999999999964]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	22	0.091
2	A	2	2	1.00	22	0.091
3	A	2	2	1.00	22	0.091
4	A	2	2	1.00	22	0.091
5	A	2	2	1.00	20	0.100
6	A	12	11	0.86	22	0.500
7	A	11	10	0.86	22	0.455
8	A	11	10	0.90	22	0.455
9	A	4	3	0.99	25	0.120
10	A	4	3	0.98	25	0.120
11	A	6	5	1.03	25	0.200
12	A	4	3	1.04	25	0.120
13	A	4	3	1.02	25	0.120
14	A	13	12	0.80	25	0.480
15	A	12	11	0.77	25	0.440
16	A	11	10	0.84	23	0.435
17	A	11	10	0.81	22	0.455
18	A	12	11	0.84	25	0.440
19	A	13	12	0.81	25	0.480
20	A	4	3	1.04	23	0.130
21	A	4	3	1.00	23	0.130
22	A	7	6	1.03	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	3	1.07	23	0.130
24	A	4	3	1.00	23	0.130
25	A	11	10	0.85	23	0.435
26	A	11	10	0.95	23	0.435
27	A	11	10	0.90	23	0.435
28	A	9	8	0.89	21	0.381
29	A	9	8	0.85	20	0.400
30	A	10	9	0.89	23	0.391
31	A	11	10	0.85	23	0.435
32	A	8	7	1.14	21	0.333
33	A	4	3	1.15	21	0.143
34	A	5	4	1.15	18	0.222
35	A	7	7	0.90	24	0.292
36	A	6	6	0.94	24	0.250
37	A	5	5	0.99	24	0.208
38	A	4	4	1.03	24	0.167
39	A	4	4	1.02	24	0.167
40	A	4	4	1.03	24	0.167
41	A	5	5	0.99	24	0.208
42	A	6	6	0.95	24	0.250
43	A	5	5	0.85	25	0.200
44	A	6	5	1.03	25	0.200
45	A	5	5	0.88	25	0.200
46	A	4	3	0.99	23	0.130
47	A	4	4	0.87	22	0.182
48	A	4	3	1.04	25	0.120
49	A	6	6	0.88	25	0.240
50	A	5	4	0.98	25	0.160
51	A	6	6	0.88	25	0.240
52	A	10	9	1.49	23	0.391
53	A	7	6	1.03	23	0.261
54	A	11	10	1.01	23	0.435
55	A	5	4	1.08	21	0.190
56	A	9	8	0.98	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	4	3	1.07	23	0.130
58	A	9	8	1.49	23	0.348
59	A	10	9	1.07	23	0.391
60	A	11	10	1.00	23	0.435
61	A	3	3	1.00	25	0.120
62	A	3	3	1.00	25	0.120
63	A	3	3	1.00	23	0.130
64	A	3	3	1.00	22	0.136
65	A	3	3	0.98	25	0.120
66	A	7	6	0.86	25	0.240
67	A	3	3	1.01	25	0.120
68	A	3	3	1.00	25	0.120
69	A	3	3	1.00	25	0.120
70	A	3	3	1.00	25	0.120
71	A	3	3	1.00	25	0.120
72	A	3	3	1.00	23	0.130
73	A	3	3	1.00	22	0.136
74	A	3	3	1.00	25	0.120
75	A	4	4	1.11	25	0.160
76	A	3	3	1.00	25	0.120
77	A	3	3	1.00	25	0.120
78	A	3	3	1.00	25	0.120
79	A	16	15	1.04	29	0.517
80	A	14	13	1.03	29	0.448
81	A	10	9	1.00	29	0.310
82	A	9	8	1.01	27	0.296
83	A	14	13	0.90	26	0.500
84	A	13	12	0.91	29	0.414
85	A	19	18	1.15	29	0.621
86	N/A	1	0	1.00	26	0.000
87	A	2	2	1.00	26	0.077
88	A	2	2	1.00	26	0.077
89	A	2	2	1.00	24	0.083

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
90	A	2	2	1.03	26	0.077
91	A	2	2	1.01	26	0.077
92	A	2	2	1.00	26	0.077
93	A	1	1	1.00	19	0.053
94	A	3	2	1.00	24	0.083
95	A	3	2	1.00	26	0.077
96	A	3	2	1.00	30	0.067
97	A	1	1	1.00	21	0.048
98	A	4	3	1.00	26	0.115
99	A	4	3	1.00	28	0.107
100	A	4	3	1.00	32	0.094
101	A	1	1	1.00	18	0.056
102	A	4	3	1.00	23	0.130
103	A	4	3	1.00	25	0.120
104	A	4	3	1.00	29	0.103
105	A	1	1	1.00	19	0.053
106	A	3	2	1.00	24	0.083
107	A	3	2	1.00	26	0.077
108	A	3	2	1.00	30	0.067
109	A	1	1	1.00	19	0.053
110	A	3	2	1.00	24	0.083
111	A	3	2	1.00	26	0.077
112	A	3	2	1.00	30	0.067
113	A	1	1	1.00	21	0.048
114	A	4	3	1.00	26	0.115
115	A	4	3	1.00	28	0.107
116	A	4	3	1.00	32	0.094
117	A	1	1	1.00	21	0.048
118	A	3	2	1.00	26	0.077
119	A	3	2	1.00	28	0.071
120	A	3	2	1.00	32	0.062
121	A	1	1	1.00	18	0.056
122	A	5	4	1.06	23	0.174
123	A	5	4	1.06	25	0.160

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	A	5	4	1.13	29	0.138
125	A	1	1	1.00	18	0.056
126	A	4	3	1.00	23	0.130
127	A	4	3	1.00	25	0.120
128	A	4	3	1.00	29	0.103
129	A	1	1	1.00	19	0.053
130	A	3	2	1.00	24	0.083
131	A	3	2	1.00	26	0.077
132	A	3	2	1.00	30	0.067
133	A	1	1	1.00	21	0.048
134	A	3	2	1.00	26	0.077
135	A	3	2	1.00	28	0.071
136	A	3	2	1.00	32	0.062
137	A	1	1	1.00	18	0.056
138	A	3	2	1.00	23	0.087
139	A	3	2	1.00	25	0.080
140	A	3	2	1.00	29	0.069
141	A	2	2	1.00	29	0.069
142	A	4	4	0.96	29	0.138
143	A	6	6	1.00	29	0.207
144	A	4	3	1.00	59	0.051
145	A	3	3	1.00	31	0.097
146	A	3	3	1.04	29	0.103
147	A	3	3	1.04	27	0.111
148	A	3	3	1.05	26	0.115
149	A	4	3	0.98	29	0.103
150	A	3	3	1.04	29	0.103
151	A	3	3	1.05	29	0.103
152	A	2	2	1.00	31	0.065
153	A	2	2	1.00	29	0.069
154	A	2	2	1.00	22	0.091
155	N/A	1	0	1.00	31	0.000
156	N/A	1	0	1.00	31	0.000

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int (d+ex^3)^5 (a+bx^3+cx^6)\,dx$

3.1.1 Optimal result

Integrand size = 22, antiderivative size = 163

$$\begin{aligned}\int (d+ex^3)^5 (a+bx^3+cx^6)\,dx = {} & ad^5x+\frac{1}{4}d^4(bd+5ae)x^4+\frac{1}{7}d^3\big(cd^2+5e(bd+2ae)\big)x^7 \\ & +\frac{1}{2}d^2e\big(cd^2+2e(bd+ae)\big)x^{10} \\ & +\frac{5}{13}de^2\big(2cd^2+e(2bd+ae)\big)x^{13} \\ & +\frac{1}{16}e^3\big(10cd^2+e(5bd+ae)\big)x^{16} \\ & +\frac{1}{19}e^4(5cd+be)x^{19}+\frac{1}{22}ce^5x^{22}\end{aligned}$$

output
```
a*d^5*x+1/4*d^4*(5*a*e+b*d)*x^4+1/7*d^3*(c*d^2+5*e*(2*a*e+b*d))*x^7+1/2*d^
2*e*(c*d^2+2*e*(a*e+b*d))*x^10+5/13*d*e^2*(2*c*d^2+e*(a*e+2*b*d))*x^13+1/1
6*e^3*(10*c*d^2+e*(a*e+5*b*d))*x^16+1/19*e^4*(b*e+5*c*d)*x^19+1/22*c*e^5*x
^22
```

3.1.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\int (d+ex^3)^5 (a+bx^3+cx^6)\, dx = ad^5x+\frac{1}{4}d^4(bd+5ae)x^4+\frac{1}{7}d^3(cd^2+5bde+10ae^2)x^7 +\frac{1}{2}d^2e(cd^2+2bde+2ae^2)x^{10} +\frac{5}{13}de^2(2cd^2+2bde+ae^2)x^{13} +\frac{1}{16}e^3(10cd^2+5bde+ae^2)x^{16} +\frac{1}{19}e^4(5cd+be)x^{19}+\frac{1}{22}ce^5x^{22}$$

input
```
Integrate[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

output
```
a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*b*d*e + 10*a*e^2)*x^
7)/7 + (d^2*e*(c*d^2 + 2*b*d*e + 2*a*e^2)*x^10)/2 + (5*d*e^2*(2*c*d^2 + 2*
b*d*e + a*e^2)*x^13)/13 + (e^3*(10*c*d^2 + 5*b*d*e + a*e^2)*x^16)/16 + (e^
4*(5*c*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

3.1.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^3)^5 (a+bx^3+cx^6)\, dx$$

$$\Big\downarrow \; 1737$$

$$\int \left(e^3x^{15}(e(ae+5bd)+10cd^2)+5de^2x^{12}(e(ae+2bd)+2cd^2)+5d^2ex^9(2e(ae+bd)+cd^2)+d^3x^6(5e(2ae+bd)\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{16}e^3x^{16}(e(ae+5bd)+10cd^2)+\frac{5}{13}de^2x^{13}(e(ae+2bd)+2cd^2)+\frac{1}{2}d^2ex^{10}(2e(ae+bd)+cd^2)+ \frac{1}{7}d^3x^7(5e(2ae+bd)+cd^2)+\frac{1}{4}d^4x^4(5ae+bd)+ad^5x+\frac{1}{19}e^4x^{19}(be+5cd)+\frac{1}{22}ce^5x^{22}$$

input
```
Int[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

output
```
a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*e*(b*d + 2*a*e))*x^7
)/7 + (d^2*e*(c*d^2 + 2*e*(b*d + a*e))*x^10)/2 + (5*d*e^2*(2*c*d^2 + e*(2*
b*d + a*e))*x^13)/13 + (e^3*(10*c*d^2 + e*(5*b*d + a*e))*x^16)/16 + (e^4*(
5*c*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

3.1.3.1 Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01

method	result
norman	$a\,d^5x+\left(\frac{5}{4}d^4ea+\frac{1}{4}d^5b\right)x^4+\left(\frac{10}{7}a\,d^3e^2+\frac{5}{7}b\,d^4e+\frac{1}{7}d^5c\right)x^7+\left(a\,d^2e^3+b\,d^3e^2+\frac{1}{2}c\,d^4e\right)x^{10}$
default	$\frac{c\,e^5x^{22}}{22}+\frac{(b\,e^5+5d\,e^4c)x^{19}}{19}+\frac{(a\,e^5+5bd\,e^4+10c\,d^2e^3)x^{16}}{16}+\frac{(5ad\,e^4+10b\,d^2e^3+10c\,d^3e^2)x^{13}}{13}+$
gosper	$a\,d^5x+\frac{5}{4}x^4d^4ea+\frac{1}{4}x^4d^5b+\frac{10}{7}x^7a\,d^3e^2+\frac{5}{7}x^7b\,d^4e+\frac{1}{7}x^7d^5c+x^{10}a\,d^2e^3+x^{10}b\,d^3e^2+\frac{1}{2}x^{10}$
risch	$a\,d^5x+\frac{5}{4}x^4d^4ea+\frac{1}{4}x^4d^5b+\frac{10}{7}x^7a\,d^3e^2+\frac{5}{7}x^7b\,d^4e+\frac{1}{7}x^7d^5c+x^{10}a\,d^2e^3+x^{10}b\,d^3e^2+\frac{1}{2}x^{10}$
parallelrisch	$a\,d^5x+\frac{5}{4}x^4d^4ea+\frac{1}{4}x^4d^5b+\frac{10}{7}x^7a\,d^3e^2+\frac{5}{7}x^7b\,d^4e+\frac{1}{7}x^7d^5c+x^{10}a\,d^2e^3+x^{10}b\,d^3e^2+\frac{1}{2}x^{10}$

input
```
int((e*x^3+d)^5*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
a*d^5*x+(5/4*d^4*e*a+1/4*d^5*b)*x^4+(10/7*a*d^3*e^2+5/7*b*d^4*e+1/7*d^5*c)
*x^7+(a*d^2*e^3+b*d^3*e^2+1/2*c*d^4*e)*x^10+(5/13*a*d*e^4+10/13*b*d^2*e^3+
10/13*c*d^3*e^2)*x^13+(1/16*a*e^5+5/16*b*d*e^4+5/8*c*d^2*e^3)*x^16+(1/19*b
*e^5+5/19*d*e^4*c)*x^19+1/22*c*e^5*x^22
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\int (d+ex^3)^5 (a+bx^3+cx^6)\, dx = \frac{1}{22}\,ce^5x^{22} + \frac{1}{19}\left(5\,cde^4 + be^5\right)x^{19} + \frac{1}{16}\left(10\,cd^2e^3 + 5\,bde^4 + ae^5\right)x^{16} + \frac{5}{13}\left(2\,cd^3e^2 + 2\,bd^2e^3 + ade^4\right)x^{13} + \frac{1}{2}\left(cd^4e + 2\,bd^3e^2 + 2\,ad^2e^3\right)x^{10} + \frac{1}{7}\left(cd^5 + 5\,bd^4e + 10\,ad^3e^2\right)x^7 + ad^5x + \frac{1}{4}\left(bd^5 + 5\,ad^4e\right)x^4$$

input
```
integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/22*c*e^5*x^22 + 1/19*(5*c*d*e^4 + b*e^5)*x^19 + 1/16*(10*c*d^2*e^3 + 5*b
*d*e^4 + a*e^5)*x^16 + 5/13*(2*c*d^3*e^2 + 2*b*d^2*e^3 + a*d*e^4)*x^13 + 1
/2*(c*d^4*e + 2*b*d^3*e^2 + 2*a*d^2*e^3)*x^10 + 1/7*(c*d^5 + 5*b*d^4*e + 1
0*a*d^3*e^2)*x^7 + a*d^5*x + 1/4*(b*d^5 + 5*a*d^4*e)*x^4
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.15

$$\int (d+ex^3)^5 (a+bx^3+cx^6)\, dx = ad^5x + \frac{ce^5x^{22}}{22} + x^{19}\left(\frac{be^5}{19} + \frac{5cde^4}{19}\right) + x^{16}\left(\frac{ae^5}{16} + \frac{5bde^4}{16} + \frac{5cd^2e^3}{8}\right) + x^{13} \cdot \left(\frac{5ade^4}{13} + \frac{10bd^2e^3}{13} + \frac{10cd^3e^2}{13}\right) + x^{10}\left(ad^2e^3 + bd^3e^2 + \frac{cd^4e}{2}\right) + x^7 \cdot \left(\frac{10ad^3e^2}{7} + \frac{5bd^4e}{7} + \frac{cd^5}{7}\right) + x^4 \cdot \left(\frac{5ad^4e}{4} + \frac{bd^5}{4}\right)$$

input
```
integrate((e*x**3+d)**5*(c*x**6+b*x**3+a),x)
```

output
```
a*d**5*x + c*e**5*x**22/22 + x**19*(b*e**5/19 + 5*c*d*e**4/19) + x**16*(a*
e**5/16 + 5*b*d*e**4/16 + 5*c*d**2*e**3/8) + x**13*(5*a*d*e**4/13 + 10*b*d
**2*e**3/13 + 10*c*d**3*e**2/13) + x**10*(a*d**2*e**3 + b*d**3*e**2 + c*d*
*4*e/2) + x**7*(10*a*d**3*e**2/7 + 5*b*d**4*e/7 + c*d**5/7) + x**4*(5*a*d*
*4*e/4 + b*d**5/4)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \left(d+ex^3\right)^5\left(a+bx^3+cx^6\right)\,dx &= \frac{1}{22}\,ce^5x^{22}+\frac{1}{19}\left(5\,cde^4+be^5\right)x^{19}\\ &+\frac{1}{16}\left(10\,cd^2e^3+5\,bde^4+ae^5\right)x^{16}\\ &+\frac{5}{13}\left(2\,cd^3e^2+2\,bd^2e^3+ade^4\right)x^{13}\\ &+\frac{1}{2}\left(cd^4e+2\,bd^3e^2+2\,ad^2e^3\right)x^{10}\\ &+\frac{1}{7}\left(cd^5+5\,bd^4e+10\,ad^3e^2\right)x^7\\ &+ad^5x+\frac{1}{4}\left(bd^5+5\,ad^4e\right)x^4\end{aligned}$$

input
```
integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
1/22*c*e^5*x^22 + 1/19*(5*c*d*e^4 + b*e^5)*x^19 + 1/16*(10*c*d^2*e^3 + 5*b
*d*e^4 + a*e^5)*x^16 + 5/13*(2*c*d^3*e^2 + 2*b*d^2*e^3 + a*d*e^4)*x^13 + 1
/2*(c*d^4*e + 2*b*d^3*e^2 + 2*a*d^2*e^3)*x^10 + 1/7*(c*d^5 + 5*b*d^4*e + 1
0*a*d^3*e^2)*x^7 + a*d^5*x + 1/4*(b*d^5 + 5*a*d^4*e)*x^4
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

$$\int \left(d+ex^3\right)^5 \left(a+bx^3+cx^6\right) \, dx = \frac{1}{22} ce^5x^{22} + \frac{5}{19} cde^4x^{19} + \frac{1}{19} be^5x^{19} + \frac{5}{8} cd^2e^3x^{16} + \frac{5}{16} bde^4x^{16} + \frac{1}{16} ae^5x^{16} + \frac{10}{13} cd^3e^2x^{13} + \frac{10}{13} bd^2e^3x^{13} + \frac{5}{13} ade^4x^{13} + \frac{1}{2} cd^4ex^{10} + bd^3e^2x^{10} + ad^2e^3x^{10} + \frac{1}{7} cd^5x^7 + \frac{5}{7} bd^4ex^7 + \frac{10}{7} ad^3e^2x^7 + \frac{1}{4} bd^5x^4 + \frac{5}{4} ad^4ex^4 + ad^5x$$

input
```
integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/22*c*e^5*x^22 + 5/19*c*d*e^4*x^19 + 1/19*b*e^5*x^19 + 5/8*c*d^2*e^3*x^16
 + 5/16*b*d*e^4*x^16 + 1/16*a*e^5*x^16 + 10/13*c*d^3*e^2*x^13 + 10/13*b*d^
2*e^3*x^13 + 5/13*a*d*e^4*x^13 + 1/2*c*d^4*e*x^10 + b*d^3*e^2*x^10 + a*d^2
*e^3*x^10 + 1/7*c*d^5*x^7 + 5/7*b*d^4*e*x^7 + 10/7*a*d^3*e^2*x^7 + 1/4*b*d
^5*x^4 + 5/4*a*d^4*e*x^4 + a*d^5*x
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.97

$$\int \left(d+ex^3\right)^5 \left(a+bx^3+cx^6\right) \, dx = x^4 \left(\frac{b\,d^5}{4} + \frac{5\,a\,e\,d^4}{4}\right) + x^{19} \left(\frac{b\,e^5}{19} + \frac{5\,c\,d\,e^4}{19}\right) + x^7 \left(\frac{c\,d^5}{7} + \frac{5\,b\,d^4\,e}{7} + \frac{10\,a\,d^3\,e^2}{7}\right) + x^{16} \left(\frac{5\,c\,d^2\,e^3}{8} + \frac{5\,b\,d\,e^4}{16} + \frac{a\,e^5}{16}\right) + \frac{c\,e^5\,x^{22}}{22} + a\,d^5\,x + \frac{d^2\,e\,x^{10}\,\left(c\,d^2+2\,b\,d\,e+2\,a\,e^2\right)}{2} + \frac{5\,d\,e^2\,x^{13}\,\left(2\,c\,d^2+2\,b\,d\,e+a\,e^2\right)}{13}$$

input
```
int((d + e*x^3)^5*(a + b*x^3 + c*x^6),x)
```

output
```
x^4*((b*d^5)/4 + (5*a*d^4*e)/4) + x^19*((b*e^5)/19 + (5*c*d*e^4)/19) + x^7
*((c*d^5)/7 + (10*a*d^3*e^2)/7 + (5*b*d^4*e)/7) + x^16*((a*e^5)/16 + (5*c*
d^2*e^3)/8 + (5*b*d*e^4)/16) + (c*e^5*x^22)/22 + a*d^5*x + (d^2*e*x^10*(2*
a*e^2 + c*d^2 + 2*b*d*e))/2 + (5*d*e^2*x^13*(a*e^2 + 2*c*d^2 + 2*b*d*e))/1
3
```

3.2 $\int (d+ex^3)^4 (a+bx^3+cx^6)\,dx$

3.2.1 Optimal result

Integrand size = 22, antiderivative size = 135

$$\begin{aligned}\int (d+ex^3)^4 (a+bx^3+cx^6)\,dx = {} & ad^4x+\frac{1}{4}d^3(bd+4ae)x^4+\frac{1}{7}d^2\left(cd^2+4bde+6ae^2\right)x^7 \\ & +\frac{1}{5}de\left(2cd^2+e(3bd+2ae)\right)x^{10} \\ & +\frac{1}{13}e^2\left(6cd^2+e(4bd+ae)\right)x^{13} \\ & +\frac{1}{16}e^3(4cd+be)x^{16}+\frac{1}{19}ce^4x^{19}\end{aligned}$$

output
```
a*d^4*x+1/4*d^3*(4*a*e+b*d)*x^4+1/7*d^2*(6*a*e^2+4*b*d*e+c*d^2)*x^7+1/5*d*
e*(2*c*d^2+e*(2*a*e+3*b*d))*x^10+1/13*e^2*(6*c*d^2+e*(a*e+4*b*d))*x^13+1/1
6*e^3*(b*e+4*c*d)*x^16+1/19*c*e^4*x^19
```

3.2.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (d+ex^3)^4 (a+bx^3+cx^6)\,dx = {} & ad^4x+\frac{1}{4}d^3(bd+4ae)x^4+\frac{1}{7}d^2\left(cd^2+4bde+6ae^2\right)x^7 \\ & +\frac{1}{5}de\left(2cd^2+3bde+2ae^2\right)x^{10} \\ & +\frac{1}{13}e^2\left(6cd^2+4bde+ae^2\right)x^{13} \\ & +\frac{1}{16}e^3(4cd+be)x^{16}+\frac{1}{19}ce^4x^{19}\end{aligned}$$

input
```
Integrate[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

output
```
a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7
)/7 + (d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^10)/5 + (e^2*(6*c*d^2 + 4*b*d*e
 + a*e^2)*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

3.2.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right) \, dx$$

$$\Big\downarrow \text{1737}$$

$$\int \left(e^2x^{12}\left(e(ae+4bd)+6cd^2\right)+d^2x^6\left(6ae^2+4bde+cd^2\right)+2dex^9\left(e(2ae+3bd)+2cd^2\right)+d^3x^3(4ae+bd)+ad^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{13}e^2x^{13}\left(e(ae+4bd)+6cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{5}dex^{10}\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+\frac{1}{16}e^3x^{16}(be+4cd)+\frac{1}{19}ce^4x^{19}$$

input
```
Int[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

output
```
a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7
)/7 + (d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^10)/5 + (e^2*(6*c*d^2 + e*(4*b*
d + a*e))*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

3.2.3.1 Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

method	result
norman	$a\,d^4x+\left(a\,d^3e+\frac{1}{4}d^4b\right)x^4+\left(\frac{6}{7}e^2d^2a+\frac{4}{7}d^3eb+\frac{1}{7}d^4c\right)x^7+\left(\frac{2}{5}d\,e^3a+\frac{3}{5}e^2d^2b+\frac{2}{5}d^3ec\right)x^{10}+$
default	$\frac{c\,e^4x^{19}}{19}+\frac{(b\,e^4+4d\,e^3c)x^{16}}{16}+\frac{(e^4a+4bd\,e^3+6e^2d^2c)x^{13}}{13}+\frac{(4d\,e^3a+6e^2d^2b+4d^3ec)x^{10}}{10}+\frac{(6e^2d^2a+4d^3eb+d^4c)x^7}{7}+$
gosper	$a\,d^4x+x^4a\,d^3e+\frac{1}{4}b\,x^4d^4+\frac{6}{7}x^7e^2d^2a+\frac{4}{7}x^7d^3eb+\frac{1}{7}x^7d^4c+\frac{2}{5}x^{10}d\,e^3a+\frac{3}{5}x^{10}e^2d^2b+\frac{2}{5}x^{10}d$
risch	$a\,d^4x+x^4a\,d^3e+\frac{1}{4}b\,x^4d^4+\frac{6}{7}x^7e^2d^2a+\frac{4}{7}x^7d^3eb+\frac{1}{7}x^7d^4c+\frac{2}{5}x^{10}d\,e^3a+\frac{3}{5}x^{10}e^2d^2b+\frac{2}{5}x^{10}d$
parallelrisch	$a\,d^4x+x^4a\,d^3e+\frac{1}{4}b\,x^4d^4+\frac{6}{7}x^7e^2d^2a+\frac{4}{7}x^7d^3eb+\frac{1}{7}x^7d^4c+\frac{2}{5}x^{10}d\,e^3a+\frac{3}{5}x^{10}e^2d^2b+\frac{2}{5}x^{10}d$

input
```
int((e*x^3+d)^4*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
a*d^4*x+(a*d^3*e+1/4*d^4*b)*x^4+(6/7*e^2*d^2*a+4/7*d^3*e*b+1/7*d^4*c)*x^7+
(2/5*d*e^3*a+3/5*e^2*d^2*b+2/5*d^3*e*c)*x^10+(1/13*e^4*a+4/13*b*d*e^3+6/13
*e^2*d^2*c)*x^13+(1/16*b*e^4+1/4*d*e^3*c)*x^16+1/19*c*e^4*x^19
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right)\, dx = \frac{1}{19}\, ce^4x^{19} + \frac{1}{16}\left(4\, cde^3 + be^4\right)x^{16} + \frac{1}{13}\left(6\, cd^2e^2 + 4\, bde^3 + ae^4\right)x^{13} + \frac{1}{5}\left(2\, cd^3e + 3\, bd^2e^2 + 2\, ade^3\right)x^{10} + \frac{1}{7}\left(cd^4 + 4\, bd^3e + 6\, ad^2e^2\right)x^7 + ad^4x + \frac{1}{4}\left(bd^4 + 4\, ad^3e\right)x^4$$

input
```
integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/19*c*e^4*x^19 + 1/16*(4*c*d*e^3 + b*e^4)*x^16 + 1/13*(6*c*d^2*e^2 + 4*b*
d*e^3 + a*e^4)*x^13 + 1/5*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^10 + 1/7
*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^7 + a*d^4*x + 1/4*(b*d^4 + 4*a*d^3*e)
*x^4
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12

$$\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right)\, dx = ad^4x + \frac{ce^4x^{19}}{19} + x^{16}\left(\frac{be^4}{16} + \frac{cde^3}{4}\right) + x^{13}\left(\frac{ae^4}{13} + \frac{4bde^3}{13} + \frac{6cd^2e^2}{13}\right) + x^{10} \cdot \left(\frac{2ade^3}{5} + \frac{3bd^2e^2}{5} + \frac{2cd^3e}{5}\right) + x^7 \cdot \left(\frac{6ad^2e^2}{7} + \frac{4bd^3e}{7} + \frac{cd^4}{7}\right) + x^4\left(ad^3e + \frac{bd^4}{4}\right)$$

input
```
integrate((e*x**3+d)**4*(c*x**6+b*x**3+a),x)
```

output
```
a*d**4*x + c*e**4*x**19/19 + x**16*(b*e**4/16 + c*d*e**3/4) + x**13*(a*e**
4/13 + 4*b*d*e**3/13 + 6*c*d**2*e**2/13) + x**10*(2*a*d*e**3/5 + 3*b*d**2*
e**2/5 + 2*c*d**3*e/5) + x**7*(6*a*d**2*e**2/7 + 4*b*d**3*e/7 + c*d**4/7)
+ x**4*(a*d**3*e + b*d**4/4)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int (d+ex^3)^4 (a+bx^3+cx^6)\,dx = \frac{1}{19}\,ce^4x^{19} + \frac{1}{16}\left(4\,cde^3+be^4\right)x^{16} + \frac{1}{13}\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^{13} + \frac{1}{5}\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^{10} + \frac{1}{7}\left(cd^4+4\,bd^3e+6\,ad^2e^2\right)x^7 + ad^4x + \frac{1}{4}\left(bd^4+4\,ad^3e\right)x^4$$

input
```
integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
1/19*c*e^4*x^19 + 1/16*(4*c*d*e^3 + b*e^4)*x^16 + 1/13*(6*c*d^2*e^2 + 4*b*
d*e^3 + a*e^4)*x^13 + 1/5*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^10 + 1/7
*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^7 + a*d^4*x + 1/4*(b*d^4 + 4*a*d^3*e)
*x^4
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int (d+ex^3)^4 (a+bx^3+cx^6)\,dx = \frac{1}{19}\,ce^4x^{19} + \frac{1}{4}\,cde^3x^{16} + \frac{1}{16}\,be^4x^{16} + \frac{6}{13}\,cd^2e^2x^{13} + \frac{4}{13}\,bde^3x^{13} + \frac{1}{13}\,ae^4x^{13} + \frac{2}{5}\,cd^3ex^{10} + \frac{3}{5}\,bd^2e^2x^{10} + \frac{2}{5}\,ade^3x^{10} + \frac{1}{7}\,cd^4x^7 + \frac{4}{7}\,bd^3ex^7 + \frac{6}{7}\,ad^2e^2x^7 + \frac{1}{4}\,bd^4x^4 + ad^3ex^4 + ad^4x$$

input
```
integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/19*c*e^4*x^19 + 1/4*c*d*e^3*x^16 + 1/16*b*e^4*x^16 + 6/13*c*d^2*e^2*x^13
 + 4/13*b*d*e^3*x^13 + 1/13*a*e^4*x^13 + 2/5*c*d^3*e*x^10 + 3/5*b*d^2*e^2*
x^10 + 2/5*a*d*e^3*x^10 + 1/7*c*d^4*x^7 + 4/7*b*d^3*e*x^7 + 6/7*a*d^2*e^2*
x^7 + 1/4*b*d^4*x^4 + a*d^3*e*x^4 + a*d^4*x
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int (d+ex^3)^4 (a+bx^3+cx^6)\,dx = x^4\left(\frac{b\,d^4}{4}+a\,e\,d^3\right)+x^{16}\left(\frac{b\,e^4}{16}+\frac{c\,d\,e^3}{4}\right) + x^7\left(\frac{c\,d^4}{7}+\frac{4\,b\,d^3\,e}{7}+\frac{6\,a\,d^2\,e^2}{7}\right) + x^{13}\left(\frac{6\,c\,d^2\,e^2}{13}+\frac{4\,b\,d\,e^3}{13}+\frac{a\,e^4}{13}\right)+\frac{c\,e^4\,x^{19}}{19} + a\,d^4\,x+\frac{d\,e\,x^{10}\,(2\,c\,d^2+3\,b\,d\,e+2\,a\,e^2)}{5}$$

input
```
int((d + e*x^3)^4*(a + b*x^3 + c*x^6),x)
```

output
```
x^4*((b*d^4)/4 + a*d^3*e) + x^16*((b*e^4)/16 + (c*d*e^3)/4) + x^7*((c*d^4)
/7 + (6*a*d^2*e^2)/7 + (4*b*d^3*e)/7) + x^13*((a*e^4)/13 + (6*c*d^2*e^2)/1
3 + (4*b*d*e^3)/13) + (c*e^4*x^19)/19 + a*d^4*x + (d*e*x^10*(2*a*e^2 + 2*c
*d^2 + 3*b*d*e))/5
```

3.3 $\int (d+ex^3)^3 (a+bx^3+cx^6)\,dx$

3.3.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int (d+ex^3)^3 (a+bx^3+cx^6)\,dx = ad^3x+\frac{1}{4}d^2(bd+3ae)x^4+\frac{1}{7}d\left(cd^2+3e(bd+ae)\right)x^7 +\frac{1}{10}e\left(3cd^2+e(3bd+ae)\right)x^{10} +\frac{1}{13}e^2(3cd+be)x^{13}+\frac{1}{16}ce^3x^{16}$$

output
```
a*d^3*x+1/4*d^2*(3*a*e+b*d)*x^4+1/7*d*(c*d^2+3*e*(a*e+b*d))*x^7+1/10*e*(3*
c*d^2+e*(a*e+3*b*d))*x^10+1/13*e^2*(b*e+3*c*d)*x^13+1/16*c*e^3*x^16
```

3.3.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int (d+ex^3)^3 (a+bx^3+cx^6)\,dx = ad^3x+\frac{1}{4}d^2(bd+3ae)x^4+\frac{1}{7}d\left(cd^2+3bde+3ae^2\right)x^7 +\frac{1}{10}e\left(3cd^2+3bde+ae^2\right)x^{10} +\frac{1}{13}e^2(3cd+be)x^{13}+\frac{1}{16}ce^3x^{16}$$

input
```
Integrate[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

output
```
a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^7)/
7 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13
+ (c*e^3*x^16)/16
```

3.3.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^3\right)^3 \left(a+bx^3+cx^6\right) \, dx$$

$$\Big\downarrow \ 1737$$

$$\int \left(ex^9\left(e(ae+3bd)+3cd^2\right)+dx^6\left(3e(ae+bd)+cd^2\right)+d^2x^3(3ae+bd)+ad^3+e^2x^{12}(be+3cd)+ce^3x^{15}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{10}ex^{10}\left(e(ae+3bd)+3cd^2\right)+\frac{1}{7}dx^7\left(3e(ae+bd)+cd^2\right)+\frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

input
```
Int[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

output
```
a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^7)/7
+ (e*(3*c*d^2 + e*(3*b*d + a*e))*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 +
(c*e^3*x^16)/16
```

3.3.3.1 Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

method	result
default	$\frac{c e^3 x^{16}}{16}+\frac{(b e^3+3c d e^2) x^{13}}{13}+\frac{(a e^3+3d e^2 b+3c d^2 e) x^{10}}{10}+\frac{(3d e^2 a+3b d^2 e+d^3 c) x^7}{7}+\frac{(3a d^2 e+b d^3) x^4}{4}+a d^3 x$
norman	$a d^3 x+\left(\frac{3}{4}a d^2 e+\frac{1}{4}b d^3\right) x^4+\left(\frac{3}{7}d e^2 a+\frac{3}{7}b d^2 e+\frac{1}{7}d^3 c\right) x^7+\left(\frac{1}{10}a e^3+\frac{3}{10}d e^2 b+\frac{3}{10}c d^2 e\right) x^{10}$
gosper	$a d^3 x+\frac{3}{4}x^4 a d^2 e+\frac{1}{4}x^4 b d^3+\frac{3}{7}x^7 d e^2 a+\frac{3}{7}x^7 b d^2 e+\frac{1}{7}x^7 d^3 c+\frac{1}{10}x^{10} a e^3+\frac{3}{10}x^{10} d e^2 b+\frac{3}{10}x^{1}$
risch	$a d^3 x+\frac{3}{4}x^4 a d^2 e+\frac{1}{4}x^4 b d^3+\frac{3}{7}x^7 d e^2 a+\frac{3}{7}x^7 b d^2 e+\frac{1}{7}x^7 d^3 c+\frac{1}{10}x^{10} a e^3+\frac{3}{10}x^{10} d e^2 b+\frac{3}{10}x^{1}$
parallelrisch	$a d^3 x+\frac{3}{4}x^4 a d^2 e+\frac{1}{4}x^4 b d^3+\frac{3}{7}x^7 d e^2 a+\frac{3}{7}x^7 b d^2 e+\frac{1}{7}x^7 d^3 c+\frac{1}{10}x^{10} a e^3+\frac{3}{10}x^{10} d e^2 b+\frac{3}{10}x^{1}$

input
```
int((e*x^3+d)^3*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/16*c*e^3*x^16+1/13*(b*e^3+3*c*d*e^2)*x^13+1/10*(a*e^3+3*b*d*e^2+3*c*d^2*
e)*x^10+1/7*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^7+1/4*(3*a*d^2*e+b*d^3)*x^4+a*d^
3*x
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int \left(d+ex^3\right)^3\left(a+bx^3+cx^6\right)\,dx=\frac{1}{16}\,ce^3x^{16}+\frac{1}{13}\,(3\,cde^2+be^3)x^{13}+\frac{1}{10}\,(3\,cd^2e+3\,bde^2+ae^3)x^{10}+\frac{1}{7}\,(cd^3+3\,bd^2e+3\,ade^2)x^7+ad^3x+\frac{1}{4}\,(bd^3+3\,ad^2e)x^4$$

input
```
integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/16*c*e^3*x^16 + 1/13*(3*c*d*e^2 + b*e^3)*x^13 + 1/10*(3*c*d^2*e + 3*b*d*
e^2 + a*e^3)*x^10 + 1/7*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^7 + a*d^3*x + 1/
4*(b*d^3 + 3*a*d^2*e)*x^4
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14

$$\int (d+ex^3)^3 (a+bx^3+cx^6)\,dx = ad^3x + \frac{ce^3x^{16}}{16} + x^{13}\left(\frac{be^3}{13} + \frac{3cde^2}{13}\right) + x^{10}\left(\frac{ae^3}{10} + \frac{3bde^2}{10} + \frac{3cd^2e}{10}\right) + x^7 \cdot \left(\frac{3ade^2}{7} + \frac{3bd^2e}{7} + \frac{cd^3}{7}\right) + x^4 \cdot \left(\frac{3ad^2e}{4} + \frac{bd^3}{4}\right)$$

input
```
integrate((e*x**3+d)**3*(c*x**6+b*x**3+a),x)
```

output
```
a*d**3*x + c*e**3*x**16/16 + x**13*(b*e**3/13 + 3*c*d*e**2/13) + x**10*(a*
e**3/10 + 3*b*d*e**2/10 + 3*c*d**2*e/10) + x**7*(3*a*d*e**2/7 + 3*b*d**2*e
/7 + c*d**3/7) + x**4*(3*a*d**2*e/4 + b*d**3/4)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int (d+ex^3)^3 (a+bx^3+cx^6)\,dx = \frac{1}{16}ce^3x^{16} + \frac{1}{13}(3\,cde^2 + be^3)x^{13} + \frac{1}{10}(3\,cd^2e + 3\,bde^2 + ae^3)x^{10} + \frac{1}{7}(cd^3 + 3\,bd^2e + 3\,ade^2)x^7 + ad^3x + \frac{1}{4}(bd^3 + 3\,ad^2e)x^4$$

input
```
integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
1/16*c*e^3*x^16 + 1/13*(3*c*d*e^2 + b*e^3)*x^13 + 1/10*(3*c*d^2*e + 3*b*d*
e^2 + a*e^3)*x^10 + 1/7*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^7 + a*d^3*x + 1/
4*(b*d^3 + 3*a*d^2*e)*x^4
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

$$\int (d+ex^3)^3 (a+bx^3+cx^6)\, dx = \frac{1}{16}ce^3x^{16} + \frac{3}{13}cde^2x^{13} + \frac{1}{13}be^3x^{13} + \frac{3}{10}cd^2ex^{10} + \frac{3}{10}bde^2x^{10} + \frac{1}{10}ae^3x^{10} + \frac{1}{7}cd^3x^7 + \frac{3}{7}bd^2ex^7 + \frac{3}{7}ade^2x^7 + \frac{1}{4}bd^3x^4 + \frac{3}{4}ad^2ex^4 + ad^3x$$

input
```
integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/16*c*e^3*x^16 + 3/13*c*d*e^2*x^13 + 1/13*b*e^3*x^13 + 3/10*c*d^2*e*x^10
+ 3/10*b*d*e^2*x^10 + 1/10*a*e^3*x^10 + 1/7*c*d^3*x^7 + 3/7*b*d^2*e*x^7 +
3/7*a*d*e^2*x^7 + 1/4*b*d^3*x^4 + 3/4*a*d^2*e*x^4 + a*d^3*x
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int (d+ex^3)^3 (a+bx^3+cx^6)\, dx = x^4\left(\frac{b\,d^3}{4}+\frac{3\,a\,e\,d^2}{4}\right)+x^{13}\left(\frac{b\,e^3}{13}+\frac{3\,c\,d\,e^2}{13}\right) + x^7\left(\frac{c\,d^3}{7}+\frac{3\,b\,d^2\,e}{7}+\frac{3\,a\,d\,e^2}{7}\right) + x^{10}\left(\frac{3\,c\,d^2\,e}{10}+\frac{3\,b\,d\,e^2}{10}+\frac{a\,e^3}{10}\right)+\frac{c\,e^3\,x^{16}}{16}+a\,d^3\,x$$

input
```
int((d + e*x^3)^3*(a + b*x^3 + c*x^6),x)
```

output
```
x^4*((b*d^3)/4 + (3*a*d^2*e)/4) + x^13*((b*e^3)/13 + (3*c*d*e^2)/13) + x^7
*((c*d^3)/7 + (3*a*d*e^2)/7 + (3*b*d^2*e)/7) + x^10*((a*e^3)/10 + (3*b*d*e
^2)/10 + (3*c*d^2*e)/10) + (c*e^3*x^16)/16 + a*d^3*x
```

3.4 $\int (d+ex^3)^2 (a+bx^3+cx^6)\, dx$

3.4.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int (d+ex^3)^2 (a+bx^3+cx^6)\, dx = ad^2x+\frac{1}{4}d(bd+2ae)x^4+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7 + \frac{1}{10}e(2cd+be)x^{10}+\frac{1}{13}ce^2x^{13}$$

output
```
a*d^2*x+1/4*d*(2*a*e+b*d)*x^4+1/7*(c*d^2+e*(a*e+2*b*d))*x^7+1/10*e*(b*e+2*
c*d)*x^10+1/13*c*e^2*x^13
```

3.4.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (d+ex^3)^2 (a+bx^3+cx^6)\, dx = ad^2x+\frac{1}{4}d(bd+2ae)x^4+\frac{1}{7}\left(cd^2+2bde+ae^2\right)x^7 + \frac{1}{10}e(2cd+be)x^{10}+\frac{1}{13}ce^2x^{13}$$

input
```
Integrate[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

output
```
a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7 + (e
*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

3.4.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^3\right)^2 \left(a+bx^3+cx^6\right) \, dx$$

$$\Big\downarrow \text{1737}$$

$$\int \left(x^6 \left(e(ae+2bd)+cd^2\right)+dx^3(2ae+bd)+ad^2+ex^9(be+2cd)+ce^2x^{12}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7 \left(e(ae+2bd)+cd^2\right)+\frac{1}{4}dx^4(2ae+bd)+ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

input
```
Int[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

output
```
a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7 + (e
*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

3.4.3.1 Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result
default	$\frac{c\,e^{2} x^{13}}{13}+\frac{\left(b\,e^{2}+2 d c e \right) x^{10}}{10}+\frac{\left(a\,e^{2}+2 b d e +c\,d^{2}\right) x^{7}}{7}+\frac{\left(2 e d a +b\,d^{2}\right) x^{4}}{4}+a\,d^{2} x$
norman	$\frac{c\,e^{2} x^{13}}{13}+\left(\frac{1}{10} b\,e^{2}+\frac{1}{5} d c e \right) x^{10}+\left(\frac{1}{7} a\,e^{2}+\frac{2}{7} b d e +\frac{1}{7} c\,d^{2}\right) x^{7}+\left(\frac{1}{2} e d a +\frac{1}{4} b\,d^{2}\right) x^{4}+a\,d^{2} x$
gosper	$\frac{1}{13} c\,e^{2} x^{13}+\frac{1}{10} x^{10} b\,e^{2}+\frac{1}{5} x^{10} d c e +\frac{1}{7} x^{7} a\,e^{2}+\frac{2}{7} x^{7} b d e +\frac{1}{7} x^{7} c\,d^{2}+\frac{1}{2} x^{4} e d a +\frac{1}{4} b\,x^{4} d^{2}+a\,d^{2} x$
risch	$\frac{1}{13} c\,e^{2} x^{13}+\frac{1}{10} x^{10} b\,e^{2}+\frac{1}{5} x^{10} d c e +\frac{1}{7} x^{7} a\,e^{2}+\frac{2}{7} x^{7} b d e +\frac{1}{7} x^{7} c\,d^{2}+\frac{1}{2} x^{4} e d a +\frac{1}{4} b\,x^{4} d^{2}+a\,d^{2} x$
parallelrisch	$\frac{1}{13} c\,e^{2} x^{13}+\frac{1}{10} x^{10} b\,e^{2}+\frac{1}{5} x^{10} d c e +\frac{1}{7} x^{7} a\,e^{2}+\frac{2}{7} x^{7} b d e +\frac{1}{7} x^{7} c\,d^{2}+\frac{1}{2} x^{4} e d a +\frac{1}{4} b\,x^{4} d^{2}+a\,d^{2} x$

input
```
int((e*x^3+d)^2*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/13*c*e^2*x^13+1/10*(b*e^2+2*c*d*e)*x^10+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/
4*(2*a*d*e+b*d^2)*x^4+a*d^2*x
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \left(d+ex^3\right)^2\left(a+bx^3+cx^6\right)\,dx=\frac{1}{13}\,ce^2x^{13}+\frac{1}{10}\,\left(2\,cde+be^2\right)x^{10}+\frac{1}{7}\,\left(cd^2+2\,bde+ae^2\right)x^7+\frac{1}{4}\,\left(bd^2+2\,ade\right)x^4+ad^2x$$

input
```
integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/13*c*e^2*x^13 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/7*(c*d^2 + 2*b*d*e + a*e
^2)*x^7 + 1/4*(b*d^2 + 2*a*d*e)*x^4 + a*d^2*x
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int (d+ex^3)^2 \left(a+bx^3+cx^6\right) dx = ad^2x + \frac{ce^2x^{13}}{13} + x^{10}\left(\frac{be^2}{10}+\frac{cde}{5}\right) + x^7\left(\frac{ae^2}{7}+\frac{2bde}{7}+\frac{cd^2}{7}\right) + x^4\left(\frac{ade}{2}+\frac{bd^2}{4}\right)$$

input
```
integrate((e*x**3+d)**2*(c*x**6+b*x**3+a),x)
```

output
```
a*d**2*x + c*e**2*x**13/13 + x**10*(b*e**2/10 + c*d*e/5) + x**7*(a*e**2/7
+ 2*b*d*e/7 + c*d**2/7) + x**4*(a*d*e/2 + b*d**2/4)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (d+ex^3)^2 \left(a+bx^3+cx^6\right) dx = \frac{1}{13}ce^2x^{13} + \frac{1}{10}\left(2cde+be^2\right)x^{10} + \frac{1}{7}\left(cd^2+2bde+ae^2\right)x^7 + \frac{1}{4}\left(bd^2+2ade\right)x^4 + ad^2x$$

input
```
integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
1/13*c*e^2*x^13 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/7*(c*d^2 + 2*b*d*e + a*e
^2)*x^7 + 1/4*(b*d^2 + 2*a*d*e)*x^4 + a*d^2*x
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int (d+ex^3)^2 \left(a+bx^3+cx^6\right) dx = \frac{1}{13}ce^2x^{13} + \frac{1}{5}cdex^{10} + \frac{1}{10}be^2x^{10} + \frac{1}{7}cd^2x^7 + \frac{2}{7}bdex^7 + \frac{1}{7}ae^2x^7 + \frac{1}{4}bd^2x^4 + \frac{1}{2}adex^4 + ad^2x$$

input
```
integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/13*c*e^2*x^13 + 1/5*c*d*e*x^10 + 1/10*b*e^2*x^10 + 1/7*c*d^2*x^7 + 2/7*b
*d*e*x^7 + 1/7*a*e^2*x^7 + 1/4*b*d^2*x^4 + 1/2*a*d*e*x^4 + a*d^2*x
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \left(d+ex^3\right)^2 \left(a+bx^3+cx^6\right)\,dx = x^7\left(\frac{c\,d^2}{7}+\frac{2\,b\,d\,e}{7}+\frac{a\,e^2}{7}\right)+x^4\left(\frac{b\,d^2}{4}+\frac{a\,e\,d}{2}\right) + x^{10}\left(\frac{b\,e^2}{10}+\frac{c\,d\,e}{5}\right)+\frac{c\,e^2\,x^{13}}{13}+a\,d^2\,x$$

input
```
int((d + e*x^3)^2*(a + b*x^3 + c*x^6),x)
```

output
```
x^7*((a*e^2)/7 + (c*d^2)/7 + (2*b*d*e)/7) + x^4*((b*d^2)/4 + (a*d*e)/2) +
x^10*((b*e^2)/10 + (c*d*e)/5) + (c*e^2*x^13)/13 + a*d^2*x
```

3.5 $\int (d + ex^3) (a + bx^3 + cx^6) \, dx$

3.5.1 Optimal result

Integrand size = 20, antiderivative size = 42

$$\int (d + ex^3) (a + bx^3 + cx^6) \, dx = adx + \frac{1}{4}(bd + ae)x^4 + \frac{1}{7}(cd + be)x^7 + \frac{1}{10}cex^{10}$$

output
```
a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (d + ex^3) (a + bx^3 + cx^6) \, dx = adx + \frac{1}{4}(bd + ae)x^4 + \frac{1}{7}(cd + be)x^7 + \frac{1}{10}cex^{10}$$

input
```
Integrate[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
```

output
```
a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

3.5.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^3)(a+bx^3+cx^6)\,dx$$

$$\Big\downarrow \ 1737$$

$$\int (x^3(ae+bd)+ad+x^6(be+cd)+cex^9)\,dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

input
```
Int[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
```

output
```
a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

3.5.3.1 Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.5.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
default	$adx+\frac{(ae+bd)x^4}{4}+\frac{(be+cd)x^7}{7}+\frac{ce\,x^{10}}{10}$	37
norman	$\frac{ce\,x^{10}}{10}+\left(\frac{be}{7}+\frac{cd}{7}\right)x^7+\left(\frac{ae}{4}+\frac{bd}{4}\right)x^4+adx$	39
gosper	$\frac{1}{10}ce\,x^{10}+\frac{1}{7}x^7be+\frac{1}{7}x^7cd+\frac{1}{4}x^4ae+\frac{1}{4}x^4bd+adx$	41
risch	$\frac{1}{10}ce\,x^{10}+\frac{1}{7}x^7be+\frac{1}{7}x^7cd+\frac{1}{4}x^4ae+\frac{1}{4}x^4bd+adx$	41
parallelrisch	$\frac{1}{10}ce\,x^{10}+\frac{1}{7}x^7be+\frac{1}{7}x^7cd+\frac{1}{4}x^4ae+\frac{1}{4}x^4bd+adx$	41

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (d+ex^3)(a+bx^3+cx^6)\,dx=\frac{1}{10}cex^{10}+\frac{1}{7}(cd+be)x^7+\frac{1}{4}(bd+ae)x^4+adx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/10*c*e*x^10 + 1/7*(c*d + b*e)*x^7 + 1/4*(b*d + a*e)*x^4 + a*d*x
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int (d+ex^3)(a+bx^3+cx^6)\,dx=adx+\frac{cex^{10}}{10}+x^7\left(\frac{be}{7}+\frac{cd}{7}\right)+x^4\left(\frac{ae}{4}+\frac{bd}{4}\right)$$

input
```
integrate((e*x**3+d)*(c*x**6+b*x**3+a),x)
```

output
```
a*d*x + c*e*x**10/10 + x**7*(b*e/7 + c*d/7) + x**4*(a*e/4 + b*d/4)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \left(d+ex^3\right)\left(a+bx^3+cx^6\right)\,dx=\frac{1}{10}cex^{10}+\frac{1}{7}(cd+be)x^7+\frac{1}{4}(bd+ae)x^4+adx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
1/10*c*e*x^10 + 1/7*(c*d + b*e)*x^7 + 1/4*(b*d + a*e)*x^4 + a*d*x
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \left(d+ex^3\right)\left(a+bx^3+cx^6\right)\,dx=\frac{1}{10}cex^{10}+\frac{1}{7}cdx^7+\frac{1}{7}bex^7+\frac{1}{4}bdx^4+\frac{1}{4}aex^4+adx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/10*c*e*x^10 + 1/7*c*d*x^7 + 1/7*b*e*x^7 + 1/4*b*d*x^4 + 1/4*a*e*x^4 + a*
d*x
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \left(d+ex^3\right)\left(a+bx^3+cx^6\right)\,dx=\frac{c\,e\,x^{10}}{10}+\left(\frac{b\,e}{7}+\frac{c\,d}{7}\right)\,x^7+\left(\frac{a\,e}{4}+\frac{b\,d}{4}\right)\,x^4+a\,d\,x$$

input
```
int((d + e*x^3)*(a + b*x^3 + c*x^6),x)
```

output
```
x^4*((a*e)/4 + (b*d)/4) + x^7*((b*e)/7 + (c*d)/7) + a*d*x + (c*e*x^10)/10
```

3.6 $\int \frac{a+bx^3+cx^6}{d+ex^3} dx$

3.6.1 Optimal result

Integrand size = 22, antiderivative size = 188

$$\int \frac{a+bx^3+cx^6}{d+ex^3} dx = -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} - \frac{(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{7/3}} + \frac{(cd^2-bde+ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}} - \frac{(cd^2-bde+ae^2)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{7/3}}$$

output
```
-(-b*e+c*d)*x/e^2+1/4*c*x^4/e+1/3*(a*e^2-b*d*e+c*d^2)*ln(d^(1/3)+e^(1/3)*x
)/d^(2/3)/e^(7/3)-1/6*(a*e^2-b*d*e+c*d^2)*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(
2/3)*x^2)/d^(2/3)/e^(7/3)-1/3*(a*e^2-b*d*e+c*d^2)*arctan(1/3*(d^(1/3)-2*e^
(1/3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(7/3)*3^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$$

$$= \frac{12\sqrt[3]{e}(-cd+be)x+3ce^{4/3}x^4-\frac{4\sqrt{3}(cd^2+e(-bd+ae))\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{2/3}}+\frac{4(cd^2+e(-bd+ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d^{2/3}}-\frac{2(cd^2+}{}}{12e^{7/3}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

output
```
(12*e^(1/3)*(-(c*d) + b*e)*x + 3*c*e^(4/3)*x^4 - (4*Sqrt[3]*(c*d^2 + e*(-(
b*d) + a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (4*(c*
d^2 + e*(-(b*d) + a*e))*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - (2*(c*d^2 + e*
(-(b*d) + a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3))/(
12*e^(7/3))
```

3.6.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.86, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1741, 27, 913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$$

$$\Big\downarrow \; 1741$$

$$\frac{\int \frac{4(ae-(cd-be)x^3)}{ex^3+d}dx}{4e}+\frac{cx^4}{4e}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{ae-(cd-be)x^3}{ex^3+d}dx}{e}+\frac{cx^4}{4e}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{(ae^2-bde+cd^2)\int \frac{1}{ex^3+d}dx}{e}-\frac{x(cd-be)}{e}}{e}+\frac{cx^4}{4e}$$

$$\Big\downarrow \text{750}$$

$$\frac{\frac{(ae^2-bde+cd^2)\left(\frac{\int \frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{e}x+\sqrt[3]{d}}dx}{3d^{2/3}}\right)}{e}-\frac{x(cd-be)}{e}}{e}+\frac{cx^4}{4e}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{(ae^2-bde+cd^2)\left(\frac{\int \frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{e}-\frac{x(cd-be)}{e}}{e}+\frac{cx^4}{4e}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{(ae^2-bde+cd^2)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{e}-\frac{x(cd-be)}{e}}{e}+\frac{cx^4}{4e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(ae^2-bde+cd^2)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{\int \frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{e}-\frac{x(cd-be)}{e}}{e}+\frac{cx^4}{4e}$$

$$\Big\downarrow \text{27}$$

3.6. $\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$

$$\frac{\frac{(ae^2-bde+cd^2)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{e}-\frac{x(cd-be)}{e}}{e}+\frac{cx^4}{4e}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{(ae^2-bde+cd^2)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{e}-\frac{x(cd-be)}{e}}{e}+\frac{cx^4}{4e}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{(ae^2-bde+cd^2)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{e}-\frac{x(cd-be)}{e}}{e}+\frac{cx^4}{4e}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{(ae^2-bde+cd^2)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{2\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{e}-\frac{x(cd-be)}{e}}{e}+\frac{cx^4}{4e}$$

3.6. $\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$

input
```
Int[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

output
```
(c*x^4)/(4*e) + (-((((c*d - b*e)*x)/e) + ((c*d^2 - b*d*e + a*e^2)*(Log[d^(1
/3) + e^(1/3)*x]/(3*d^(2/3)*e^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*e^(1/3)*
x)/d^(1/3))/Sqrt[3]])/e^(1/3)) - Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)
*x^2]/(2*e^(1/3)))/(3*d^(2/3))))/e)/e
```

3.6.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

3.6.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.71 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{c\,x^{4}}{4e}+\frac{bx}{e}-\frac{cdx}{e^{2}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(e\,\_Z^{3}+d\right)}\frac{\left(a\,e^{2}-bde+c\,d^{2}\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3e^{3}}$	67
default	$\frac{\frac{1}{4}c\,x^{4}e+bex-cdx}{e^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)\left(a\,e^{2}-bde+c\,d^{2}\right)}{e^{2}}$	133

input
```
int((c*x^6+b*x^3+a)/(e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*c*x^4/e+1/e*b*x-c*d*x/e^2+1/3/e^3*sum((a*e^2-b*d*e+c*d^2)/_R^2*ln(x-_R
),_R=RootOf(_Z^3*e+d))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.47

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$$
$$= \left[\frac{3\,cd^2e^2x^4+6\,\sqrt{\frac{1}{3}}(cd^3e-bd^2e^2+ade^3)\sqrt{-\frac{(d^2e)^{\frac{1}{3}}}{e}}\log\left(\frac{2\,dex^3-3\,(d^2e)^{\frac{1}{3}}dx-d^2+3\,\sqrt{\frac{1}{3}}\left(2\,dex^2+(d^2e)^{\frac{2}{3}}x-(d^2e)^{\frac{1}{3}}d\right)\sqrt{-}}{ex^3+d}\right.}{} \right.$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="fricas")
```

output
```
[1/12*(3*c*d^2*e^2*x^4 + 6*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt(
-(d^2*e)^(1/3)/e)*log((2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)
*(2*d*e*x^2 + (d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(
e*x^3 + d)) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e
)^(2/3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log
(d*e*x + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x)/(d^2*e^3), 1/12*(3*c
*d^2*e^2*x^4 + 12*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt((d^2*e)^(
1/3)/e)*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e
)^(1/3)/e)/d^2) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d
^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)
*log(d*e*x + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x)/(d^2*e^3)]
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = \frac{cx^4}{4e}+x\left(\frac{b}{e}-\frac{cd}{e^2}\right)$$
$$+\text{RootSum}\left(27t^3d^2e^7-a^3e^6+3a^2bde^5-3a^2cd^2e^4-3ab^2d^2e^4+6abcd^3e^3-3ac^2d^4e^2+b^3d^3e^3-3b^2cd^4e$$

input
```
integrate((c*x**6+b*x**3+a)/(e*x**3+d),x)
```

output
```
c*x**4/(4*e) + x*(b/e - c*d/e**2) + RootSum(27*_t**3*d**2*e**7 - a**3*e**6
 + 3*a**2*b*d*e**5 - 3*a**2*c*d**2*e**4 - 3*a*b**2*d**2*e**4 + 6*a*b*c*d**
3*e**3 - 3*a*c**2*d**4*e**2 + b**3*d**3*e**3 - 3*b**2*c*d**4*e**2 + 3*b*c*
*2*d**5*e - c**3*d**6, Lambda(_t, _t*log(3*_t*d*e**2/(a*e**2 - b*d*e + c*d
**2) + x)))
```

3.6.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.02

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = -\frac{\sqrt{3}(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\left(-de^2\right)^{\frac{2}{3}}e} - \frac{(cd^2-bde+ae^2)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{2}{3}}e} - \frac{(cd^2e^2-bde^3+ae^4)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,de^4} + \frac{ce^3x^4-4\,cde^2x+4\,be^3x}{4\,e^4}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(c*d^2 - b*d*e + a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3
))/(-d/e)^(1/3))/((-d*e^2)^(2/3)*e) - 1/6*(c*d^2 - b*d*e + a*e^2)*log(x^2
+ x*(-d/e)^(1/3) + (-d/e)^(2/3))/((-d*e^2)^(2/3)*e) - 1/3*(c*d^2*e^2 - b*d
*e^3 + a*e^4)*(-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/(d*e^4) + 1/4*(c*e^3
*x^4 - 4*c*d*e^2*x + 4*b*e^3*x)/e^4
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = x\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)+\frac{c\,x^4}{4\,e}+\frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}} + \frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}} - \frac{\ln\left(d^{1/3}-2\,e^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}}$$

input
```
int((a + b*x^3 + c*x^6)/(d + e*x^3),x)
```

output
```
x*(b/e - (c*d)/e^2) + (c*x^4)/(4*e) + (log(e^(1/3)*x + d^(1/3))*(a*e^2 + c
*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3)) + (log(3^(1/2)*d^(1/3)*1i + 2*e^(1/3)*x
 - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(
7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 +
 1/2)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3))
```

3.6. $\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$

3.7 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx$

3.7.1 Optimal result

Integrand size = 22, antiderivative size = 213

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx = \frac{cx}{e^2}+\frac{(cd^2-bde+ae^2)\,x}{3de^2(d+ex^3)}+\frac{(4cd^2-e(bd+2ae))\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{3\sqrt{3}d^{5/3}e^{7/3}} - \frac{(4cd^2-e(bd+2ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}e^{7/3}} + \frac{(4cd^2-e(bd+2ae))\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{18d^{5/3}e^{7/3}}$$

output
```
c*x/e^2+1/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)-1/9*(4*c*d^2-e*(2*a*e+b*
d))*ln(d^(1/3)+e^(1/3)*x)/d^(5/3)/e^(7/3)+1/18*(4*c*d^2-e*(2*a*e+b*d))*ln(
d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(5/3)/e^(7/3)+1/9*(4*c*d^2-e*(2*a
*e+b*d))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(5/3)/e^(7/3)
*3^(1/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx$$

$$=\frac{18c\sqrt[3]{e}x+\frac{6\sqrt[3]{e}(cd^2+e(-bd+ae))x}{d(d+ex^3)}+\frac{2\sqrt{3}(4cd^2-e(bd+2ae))\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{5/3}}-\frac{2(4cd^2-e(bd+2ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d^{5/3}}+\frac{(4cd^2}{}}{18e^{7/3}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

output
```
(18*c*e^(1/3)*x + (6*e^(1/3)*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^3))
 + (2*Sqrt[3]*(4*c*d^2 - e*(b*d + 2*a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3
))/Sqrt[3]])/d^(5/3) - (2*(4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3
)*x])/d^(5/3) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)
*x + e^(2/3)*x^2])/d^(5/3))/(18*e^(7/3))
```

3.7.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1739, 913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx$$

$$\Big\downarrow \ 1739$$

$$\frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^3)}-\frac{\int \frac{-3cdex^3+cd^2-e(bd+2ae)}{ex^3+d}dx}{3de^2}$$

$$\Big\downarrow \ 913$$

$$\frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^3)}-\frac{(4cd^2-e(2ae+bd))\int \frac{1}{ex^3+d}dx-3cdx}{3de^2}$$

$$\Big\downarrow \textbf{750}$$

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}-\frac{\left(4cd^2-e(2ae+bd)\right)\left(\frac{\int \frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{e}x+\sqrt[3]{d}}dx}{3d^{2/3}}\right)-3cdx}{3de^2}$$

$$\Big\downarrow \textbf{16}$$

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}-\frac{\left(4cd^2-e(2ae+bd)\right)\left(\frac{\int \frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-3cdx}{3de^2}$$

$$\Big\downarrow \textbf{1142}$$

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}-\frac{\left(4cd^2-e(2ae+bd)\right)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-3cdx}{3de^2}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}-\frac{\left(4cd^2-e(2ae+bd)\right)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{\int \frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-3cdx}{3de^2}$$

$$\Big\downarrow \textbf{27}$$

3.7. $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^2}\,dx$

$$\frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^3)}-\frac{(4cd^2-e(2ae+bd))\left(\frac{\frac{3}{2}\sqrt[3]{d}\int\frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-3cdx}{3de^2}$$

$\downarrow$ 1082

$$\frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^3)}-\frac{(4cd^2-e(2ae+bd))\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-3cdx}{3de^2}$$

$\downarrow$ 217

$$\frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^3)}-\frac{(4cd^2-e(2ae+bd))\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-3cdx}{3de^2}$$

$\downarrow$ 1103

3.7. $\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx$

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}-\frac{\left(4cd^2-e(2ae+bd)\right)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{2\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-3cdx}{3de^2}$$

input
```
Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^3)) - (-3*c*d*x + (4*c*d^2 -
 e*(b*d + 2*a*e))*(Log[d^(1/3) + e^(1/3)*x]/(3*d^(2/3)*e^(1/3)) + (-((Sqrt
[3]*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/e^(1/3)) - Log[d^(2/3) -
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]/(2*e^(1/3)))/(3*d^(2/3))))/(3*d*e^2)
```

3.7.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

3.7. $\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx$

3.7.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.41

method	result	size
risch	$\frac{cx}{e^2}+\frac{(a e^2-bde+c d^2)x}{3d e^2(e x^3+d)}+\frac{\sum_{\_R=\operatorname{RootOf}(e\_Z^3+d)}\frac{(2a e^2+bde-4c d^2)\ln(x-\_R)}{\_R^2}}{9e^3 d}$	88
default	$\frac{cx}{e^2}+\frac{\frac{(a e^2-bde+c d^2)x}{3d(e x^3+d)}+\frac{(2a e^2+bde-4c d^2)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)}{3d}}{e^2}$	156

input
```
int((c*x^6+b*x^3+a)/(e*x^3+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*x/e^2+1/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)+1/9/e^3/d*sum((2*a*e^2+b
*d*e-4*c*d^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 697, normalized size of antiderivative = 3.27

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx = \left[\frac{18\,cd^3e^2x^4-3\sqrt{\frac{1}{3}}(4\,cd^4e-bd^3e^2-2\,ad^2e^3+(4\,cd^3e^2-bd^2e^3-2\,ade^4)x^3)\sqrt{-\frac{(d^2e)^{\frac{1}{3}}}{e}}\log\left(\frac{2\,dex^3-3\,(d^2e)^{\frac{1}{3}}\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
[1/18*(18*c*d^3*e^2*x^4 - 3*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2*a*d^2*e^3
 + (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt(-(d^2*e)^(1/3)/e)*log((
2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*e)^(
2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d)) + (4*c*d^3
- b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)
*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) - 2*(4*c*d^3 - b*d^2*e -
 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x
+ (d^2*e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2*e^3)*x)/(d^3*e^4*x^3 +
 d^4*e^3), 1/18*(18*c*d^3*e^2*x^4 - 6*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2
*a*d^2*e^3 + (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt((d^2*e)^(1/3)
/e)*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1
/3)/e)/d^2) + (4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*
e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) -
 2*(4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(
d^2*e)^(2/3)*log(d*e*x + (d^2*e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2
*e^3)*x)/(d^3*e^4*x^3 + d^4*e^3)]
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx = \frac{cx}{e^2} + \frac{x(ae^2-bde+cd^2)}{3d^2e^2+3de^3x^3}$$
$$+\mathrm{RootSum}\Big(729t^3d^5e^7 - 8a^3e^6 - 12a^2bde^5 + 48a^2cd^2e^4 - 6ab^2d^2e^4 + 48abcd^3e^3 - 96ac^2d^4e^2 - b^3d^3e^3 +$$

input
```
integrate((c*x**6+b*x**3+a)/(e*x**3+d)**2,x)
```

output
```
c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(3*d**2*e**2 + 3*d*e**3*x**3) + Roo
tSum(729*_t**3*d**5*e**7 - 8*a**3*e**6 - 12*a**2*b*d*e**5 + 48*a**2*c*d**2
*e**4 - 6*a*b**2*d**2*e**4 + 48*a*b*c*d**3*e**3 - 96*a*c**2*d**4*e**2 - b*
*3*d**3*e**3 + 12*b**2*c*d**4*e**2 - 48*b*c**2*d**5*e + 64*c**3*d**6, Lamb
da(_t, _t*log(9*_t*d**2*e**2/(2*a*e**2 + b*d*e - 4*c*d**2) + x)))
```

3.7.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx = \frac{cx}{e^2} + \frac{\sqrt{3}(4\,cd^2-bde-2\,ae^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{9\,(-de^2)^{\frac{2}{3}}\,de} + \frac{(4\,cd^2-bde-2\,ae^2)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{18\,(-de^2)^{\frac{2}{3}}\,de} + \frac{(4\,cd^2-bde-2\,ae^2)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{9\,d^2e^2} + \frac{cd^2x-bdex+ae^2x}{3\,(ex^3+d)de^2}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
c*x/e^2 + 1/9*sqrt(3)*(4*c*d^2 - b*d*e - 2*a*e^2)*arctan(1/3*sqrt(3)*(2*x
+ (-d/e)^(1/3))/(-d/e)^(1/3))/((-d*e^2)^(2/3)*d*e) + 1/18*(4*c*d^2 - b*d*e
 - 2*a*e^2)*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/((-d*e^2)^(2/3)*d*e)
+ 1/9*(4*c*d^2 - b*d*e - 2*a*e^2)*(-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/
(d^2*e^2) + 1/3*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^3 + d)*d*e^2)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx$$
$$=\frac{c\,x}{e^2}+\frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\,\left(-4\,c\,d^2+b\,d\,e+2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}}+\frac{x\,(c\,d^2-b\,d\,e+a\,e^2)}{3\,d\,\left(e^3\,x^3+d\,e^2\right)}$$
$$+\frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-4\,c\,d^2+b\,d\,e+2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}}$$
$$-\frac{\ln\left(d^{1/3}-2\,e^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-4\,c\,d^2+b\,d\,e+2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}}$$

input
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^2,x)
```

output
```
(c*x)/e^2 + (log(e^(1/3)*x + d^(1/3))*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^(5
/3)*e^(7/3)) + (x*(a*e^2 + c*d^2 - b*d*e))/(3*d*(d*e^2 + e^3*x^3)) + (log(
3^(1/2)*d^(1/3)*1i + 2*e^(1/3)*x - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*a*e^
2 - 4*c*d^2 + b*d*e))/(9*d^(5/3)*e^(7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^
(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*
d^(5/3)*e^(7/3))
```

3.8 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$

3.8.1 Optimal result

Integrand size = 22, antiderivative size = 242

$$\begin{aligned}\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx &= \frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^3)^2}-\frac{(7cd^2-e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)} \\ &\quad-\frac{(2cd^2+e(bd+5ae))\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{9\sqrt{3}d^{8/3}e^{7/3}} \\ &\quad+\frac{(2cd^2+e(bd+5ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{27d^{8/3}e^{7/3}} \\ &\quad-\frac{(2cd^2+e(bd+5ae))\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{54d^{8/3}e^{7/3}}\end{aligned}$$

output
```
1/6*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^2-1/18*(7*c*d^2-e*(5*a*e+b*d))*x
/d^2/e^2/(e*x^3+d)+1/27*(2*c*d^2+e*(5*a*e+b*d))*ln(d^(1/3)+e^(1/3)*x)/d^(8
/3)/e^(7/3)-1/54*(2*c*d^2+e*(5*a*e+b*d))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2
/3)*x^2)/d^(8/3)/e^(7/3)-1/27*(2*c*d^2+e*(5*a*e+b*d))*arctan(1/3*(d^(1/3)-
2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(8/3)/e^(7/3)*3^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$$

$$= \frac{-\frac{3d^{2/3}\sqrt[3]{e}x(cd^2(4d+7ex^3)-e(bd(-2d+ex^3)+ae(8d+5ex^3)))}{(d+ex^3)^2}-2\sqrt{3}(2cd^2+e(bd+5ae))\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)+2(2cd^2}{54d^{8/3}e^{7/3}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

output
```
((-3*d^(2/3)*e^(1/3)*x*(c*d^2*(4*d + 7*e*x^3) - e*(b*d*(-2*d + e*x^3) + a*
e*(8*d + 5*e*x^3))))/(d + e*x^3)^2 - 2*Sqrt[3]*(2*c*d^2 + e*(b*d + 5*a*e))
*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] + 2*(2*c*d^2 + e*(b*d + 5*a*e
))*Log[d^(1/3) + e^(1/3)*x] - (2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^
(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(54*d^(8/3)*e^(7/3))
```

3.8.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1739, 910, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$$

$$\Big\downarrow \text{1739}$$

$$\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^3)^2}-\frac{\int \frac{-6cdex^3+cd^2-e(bd+5ae)}{(ex^3+d)^2}dx}{6de^2}$$

$$\Big\downarrow \text{910}$$

$$\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^3)^2}-\frac{\frac{x(7cd^2-e(5ae+bd))}{3d(d+ex^3)}-\frac{2(e(5ae+bd)+2cd^2)\int \frac{1}{ex^3+d}dx}{3d}}{6de^2}$$

$\downarrow$ 750

$$\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\frac{x\left(7cd^2-e(5ae+bd)\right)}{3d(d+ex^3)}-\frac{2\left(e(5ae+bd)+2cd^2\right)\left(\frac{\int\frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{e}x+\sqrt[3]{d}}dx}{3d^{2/3}}\right)}{3d}}{6de^2}$$

$\downarrow$ 16

$$\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\frac{x\left(7cd^2-e(5ae+bd)\right)}{3d(d+ex^3)}-\frac{2\left(e(5ae+bd)+2cd^2\right)\left(\frac{\int\frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{3d}}{6de^2}$$

$\downarrow$ 1142

$$\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\frac{x\left(7cd^2-e(5ae+bd)\right)}{3d(d+ex^3)}-\frac{2\left(e(5ae+bd)+2cd^2\right)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int\frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{3d}}{6de^2}$$

$\downarrow$ 25

$$\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\frac{x\left(7cd^2-e(5ae+bd)\right)}{3d(d+ex^3)}-\frac{2\left(e(5ae+bd)+2cd^2\right)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int\frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{\int\frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{3d}}{6de^2}$$

$\downarrow$ 27

3.8. $\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$

$$\frac{\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^3)^2}-\frac{\frac{x(7cd^2-e(5ae+bd))}{3d(d+ex^3)}-\frac{2(e(5ae+bd)+2cd^2)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int\frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{3d}}{6de^2}}{}$$

$$\Big\downarrow \; 1082$$

$$\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^3)^2}-\frac{\frac{x(7cd^2-e(5ae+bd))}{3d(d+ex^3)}-\frac{2(e(5ae+bd)+2cd^2)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{3d}}{6de^2}$$

$$\Big\downarrow \; 217$$

$$\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^3)^2}-\frac{\frac{x(7cd^2-e(5ae+bd))}{3d(d+ex^3)}-\frac{2(e(5ae+bd)+2cd^2)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{3d}}{6de^2}$$

$$\Big\downarrow \; 1103$$

3.8. $\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$

$$\frac{\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^3)^2}-\frac{\frac{x(7cd^2-e(5ae+bd))}{3d(d+ex^3)}-\frac{2(e(5ae+bd)+2cd^2)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{2\sqrt[3]{e}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{3d}}{6de^2}}{}$$

input
```
Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^3)^2) - (((7*c*d^2 - e*(b*d
+ 5*a*e))*x)/(3*d*(d + e*x^3)) - (2*(2*c*d^2 + e*(b*d + 5*a*e))*(Log[d^(1/
3) + e^(1/3)*x]/(3*d^(2/3)*e^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*e^(1/3)*x
)/d^(1/3))/Sqrt[3]])/e^(1/3)) - Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*
x^2]/(2*e^(1/3)))/(3*d^(2/3))))/(3*d))/(6*d*e^2)
```

3.8.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

3.8. $\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$

3.8.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.47

method	result
risch	$\frac{\frac{\left(5a\,e^{2}+bde-7c\,d^{2}\right)x^{4}}{18d^{2}e}+\frac{\left(4a\,e^{2}-bde-2c\,d^{2}\right)x}{9d\,e^{2}}}{\left(e\,x^{3}+d\right)^{2}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e\_Z^{3}+d\right)}\frac{\left(5a\,e^{2}+bde+2c\,d^{2}\right)\ln\left(x-\_R\right)}{\_R^{2}}}{27e^{3}d^{2}}$
default	$\frac{\frac{\left(5a\,e^{2}+bde-7c\,d^{2}\right)x^{4}}{18d^{2}e}+\frac{\left(4a\,e^{2}-bde-2c\,d^{2}\right)x}{9d\,e^{2}}}{\left(e\,x^{3}+d\right)^{2}}+\frac{\left(5a\,e^{2}+bde+2c\,d^{2}\right)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right.}{3}\right.}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right.}{9e^{2}d^{2}}$

input
```
int((c*x^6+b*x^3+a)/(e*x^3+d)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/18*(5*a*e^2+b*d*e-7*c*d^2)/d^2/e*x^4+1/9*(4*a*e^2-b*d*e-2*c*d^2)/d/e^2*
x)/(e*x^3+d)^2+1/27/e^3/d^2*sum((5*a*e^2+b*d*e+2*c*d^2)/_R^2*ln(x-_R),_R=R
ootOf(_Z^3*e+d))
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(201) = 402.

Time = 0.32 (sec) , antiderivative size = 941, normalized size of antiderivative = 3.89

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$$

$$= \left[-\frac{3\,(7\,cd^4e^2 - bd^3e^3 - 5\,ad^2e^4)x^4 - 3\,\sqrt{\tfrac{1}{3}}(2\,cd^5e + bd^4e^2 + 5\,ad^3e^3 + (2\,cd^3e^3 + bd^2e^4 + 5\,ade^5)x^6 + 2\,(2}{} \right.$$

$$-\frac{3\,(7\,cd^4e^2 - bd^3e^3 - 5\,ad^2e^4)x^4 - 6\,\sqrt{\tfrac{1}{3}}(2\,cd^5e + bd^4e^2 + 5\,ad^3e^3 + (2\,cd^3e^3 + bd^2e^4 + 5\,ade^5)x^6 + 2\,(2\,c}{}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="fricas")
```

3.8. $\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$

output
```
[-1/54*(3*(7*c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^4 - 3*sqrt(1/3)*(2*c*d
^5*e + b*d^4*e^2 + 5*a*d^3*e^3 + (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6
 + 2*(2*c*d^4*e^2 + b*d^3*e^3 + 5*a*d^2*e^4)*x^3)*sqrt(-(d^2*e)^(1/3)/e)*l
og((2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*
e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d)) + ((2*c
*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2
*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)
^(2/3)*x + (d^2*e)^(1/3)*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2
*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3
)*(d^2*e)^(2/3)*log(d*e*x + (d^2*e)^(2/3)) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*
a*d^3*e^3)*x)/(d^4*e^5*x^6 + 2*d^5*e^4*x^3 + d^6*e^3), -1/54*(3*(7*c*d^4*e
^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^4 - 6*sqrt(1/3)*(2*c*d^5*e + b*d^4*e^2 + 5
*a*d^3*e^3 + (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6 + 2*(2*c*d^4*e^2 +
b*d^3*e^3 + 5*a*d^2*e^4)*x^3)*sqrt((d^2*e)^(1/3)/e)*arctan(sqrt(1/3)*(2*(d
^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/e)/d^2) + ((2*c*d^2*e^
2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*
e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*
x + (d^2*e)^(1/3)*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4
+ b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*
e)^(2/3)*log(d*e*x + (d^2*e)^(2/3)) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*a*d^...
```

3.8.6 Sympy [A] (verification not implemented)

Time = 11.31 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx = \frac{x^4\cdot(5ae^3+bde^2-7cd^2e)+x(8ade^2-2bd^2e-4cd^3)}{18d^4e^2+36d^3e^3x^3+18d^2e^4x^6}$$
$$+\mathrm{RootSum}\Big(19683t^3d^8e^7-125a^3e^6-75a^2bde^5-150a^2cd^2e^4-15ab^2d^2e^4-60abcd^3e^3-60ac^2d^4e^2-b^3$$

input
```
integrate((c*x**6+b*x**3+a)/(e*x**3+d)**3,x)
```

output
```
(x**4*(5*a*e**3 + b*d*e**2 - 7*c*d**2*e) + x*(8*a*d*e**2 - 2*b*d**2*e - 4*
c*d**3))/(18*d**4*e**2 + 36*d**3*e**3*x**3 + 18*d**2*e**4*x**6) + RootSum(
19683*_t**3*d**8*e**7 - 125*a**3*e**6 - 75*a**2*b*d*e**5 - 150*a**2*c*d**2
*e**4 - 15*a*b**2*d**2*e**4 - 60*a*b*c*d**3*e**3 - 60*a*c**2*d**4*e**2 - b
**3*d**3*e**3 - 6*b**2*c*d**4*e**2 - 12*b*c**2*d**5*e - 8*c**3*d**6, Lambd
a(_t, _t*log(27*_t*d**3*e**2/(5*a*e**2 + b*d*e + 2*c*d**2) + x)))
```

3.8.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx = -\frac{\sqrt{3}(2\,cd^2+bde+5\,ae^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{27\,\left(-de^2\right)^{\frac{2}{3}}d^2e} - \frac{(2\,cd^2+bde+5\,ae^2)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{54\,\left(-de^2\right)^{\frac{2}{3}}d^2e} - \frac{(2\,cd^2+bde+5\,ae^2)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{27\,d^3e^2} - \frac{7\,cd^2ex^4-bde^2x^4-5\,ae^3x^4+4\,cd^3x+2\,bd^2ex-8\,ade^2x}{18\,(ex^3+d)^2d^2e^2}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(2*c*d^2 + b*d*e + 5*a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d/e)
^(1/3))/(-d/e)^(1/3))/((-d*e^2)^(2/3)*d^2*e) - 1/54*(2*c*d^2 + b*d*e + 5*a
*e^2)*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/((-d*e^2)^(2/3)*d^2*e) - 1/
27*(2*c*d^2 + b*d*e + 5*a*e^2)*(-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/(d^
3*e^2) - 1/18*(7*c*d^2*e*x^4 - b*d*e^2*x^4 - 5*a*e^3*x^4 + 4*c*d^3*x + 2*b
*d^2*e*x - 8*a*d*e^2*x)/((e*x^3 + d)^2*d^2*e^2)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx = \frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\,\left(2\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{27\,d^{8/3}\,e^{7/3}} - \frac{\frac{x\,\left(2\,c\,d^2+b\,d\,e-4\,a\,e^2\right)}{9\,d\,e^2}-\frac{x^4\,\left(-7\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{18\,d^2\,e}}{d^2+2\,d\,e\,x^3+e^2\,x^6} + \frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{27\,d^{8/3}\,e^{7/3}} - \frac{\ln\left(d^{1/3}-2\,e^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{27\,d^{8/3}\,e^{7/3}}$$

input
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^3,x)
```

output
```
(log(e^(1/3)*x + d^(1/3))*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^(8/3)*e^(7/3)
) - ((x*(2*c*d^2 - 4*a*e^2 + b*d*e))/(9*d*e^2) - (x^4*(5*a*e^2 - 7*c*d^2 +
 b*d*e))/(18*d^2*e))/(d^2 + e^2*x^6 + 2*d*e*x^3) + (log(3^(1/2)*d^(1/3)*1i
 + 2*e^(1/3)*x - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*
e))/(27*d^(8/3)*e^(7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3)*x + d^(1/3)
)*((3^(1/2)*1i)/2 + 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^(8/3)*e^(7/3))
```

3.9 $\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx$

3.9.1 Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{(b^2cd-2ac^2d-b^3e+3abce)\,\mathrm{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}} - \frac{(bcd-b^2e+ace)\log(a+bx^3+cx^6)}{6c^3}$$

output
```
1/3*(-b*e+c*d)*x^3/c^2+1/6*e*x^6/c-1/6*(a*c*e-b^2*e+b*c*d)*ln(c*x^6+b*x^3+
a)/c^3-1/3*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((2*c*x^3+b)/(-4*a*c
+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{2c(cd-be)x^3+c^2ex^6+\frac{2(b^2cd-2ac^2d-b^3e+3abce)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(-bcd+b^2e-ace)\log(a+bx^3+cx^6)}{6c^3}$$

input
```
Integrate[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*(c*d - b*e)*x^3 + c^2*e*x^6 + (2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b
*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*
c*d) + b^2*e - a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

3.9.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 \left(d+e x^3\right)}{a+b x^3+c x^6} \, dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3} \int \frac{x^6 \left(e x^3+d\right)}{c x^6+b x^3+a}dx^3$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3} \int \left(\frac{e x^3}{c}+\frac{c d-b e}{c^2}-\frac{\left(-e b^2+c d b+a c e\right) x^3+a (c d-b e)}{c^2 \left(c x^6+b x^3+a\right)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(-\frac{\text{arctanh}\left(\frac{b+2 c x^3}{\sqrt{b^2-4 a c}}\right) \left(3 a b c e-2 a c^2 d+b^3 (-e)+b^2 c d\right)}{c^3 \sqrt{b^2-4 a c}}-\frac{\left(a c e+b^2 (-e)+b c d\right) \log \left(a+b x^3+c x^6\right)}{2 c^3}+\frac{x^3 (c d-}{c^2}$$

input
```
Int[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(((c*d - b*e)*x^3)/c^2 + (e*x^6)/(2*c) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3
*a*b*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]
) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^3 + c*x^6])/(2*c^3))/3
```

3.9.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.9.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{-\frac{1}{2} c e \,x^{6}+e \,x^{3} b -c d \,x^{3}}{3 c^{2}}+\frac{\frac{\left(-a c e +b^{2} e -b c d \right) \ln \left(c \,x^{6}+b \,x^{3}+a \right)}{2 c}+\frac{2 \left(a b e -a c d -\frac{\left(-a c e +b^{2} e -b c d \right) b}{2 c}\right) \arctan \left(\frac{2 c \,x^{3}+b}{\sqrt{4 a c -b^{2}}}\right)}{\sqrt{4 a c -b^{2}}}}{3 c^{2}}$	136
risch	Expression too large to display	2131

input
```
int(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^2*(-1/2*c*e*x^6+e*x^3*b-c*d*x^3)+1/3/c^2*(1/2*(-a*c*e+b^2*e-b*c*d)/
c*ln(c*x^6+b*x^3+a)+2*(a*b*e-a*c*d-1/2*(-a*c*e+b^2*e-b*c*d)*b/c)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.26

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)ex^6+2\,((b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e)x^3+\sqrt{b^2-4\,ac}((b^2c-2\,ac^2)d-(b^3-3\,abc)e)}{6\,(b^2c^3}\right.$$

input
```
integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*((b^2*c^2 - 4*a*c^3)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*
b*c^2)*e)*x^3 + sqrt(b^2 - 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e
)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*a*
c))/(c*x^6 + b*x^3 + a)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4), 1/6*((b^2*c^2 - 4*
a*c^3)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*x^3 - 2*s
qrt(-b^2 + 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*arctan(-(2*c*x
^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3*c - 4*a*b*c^2)*d - (b^4
- 5*a*b^2*c + 4*a^2*c^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4)]
```

3.9.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.9.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{cex^6+2\,cdx^3-2\,bex^3}{6\,c^2} - \frac{(bcd-b^2e+ace)\log\left(cx^6+bx^3+a\right)}{6\,c^3} + \frac{(b^2cd-2\,ac^2d-b^3e+3\,abce)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/6*(c*e*x^6 + 2*c*d*x^3 - 2*b*e*x^3)/c^2 - 1/6*(b*c*d - b^2*e + a*c*e)*lo
g(c*x^6 + b*x^3 + a)/c^3 + 1/3*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*a
rctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 11.12 (sec) , antiderivative size = 3586, normalized size of antiderivative = 27.17

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
x^3*(d/(3*c) - (b*e)/(3*c^2)) + (e*x^6)/(6*c) - (log(a + b*x^3 + c*x^6)*(3
*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a
*c^4 - 9*b^2*c^3)) - (atan((4*c^6*(4*a*c - b^2)^(3/2)*(x^3*((b*((b^5*c^3*d
^3 - b^8*e^3 - 2*a*b^3*c^4*d^3 + a^2*b*c^5*d^3 + a^3*c^5*d^2*e - 3*b^6*c^2
*d^2*e - 8*a^2*b^4*c^2*e^3 + 4*a^3*b^2*c^3*e^3 + 5*a*b^6*c*e^3 + 3*b^7*c*d
*e^2 + 9*a*b^4*c^3*d^2*e - 12*a*b^5*c^2*d*e^2 - 4*a^3*b*c^4*d*e^2 - 7*a^2*
b^2*c^4*d^2*e + 14*a^2*b^3*c^3*d*e^2)/c^6 - (((6*a^2*c^7*d^2 + 12*b^4*c^5*
d^2 + 12*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^
5*e^2 - 24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*
b^3*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^
3*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*
a*c^4 - 9*b^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 1
5*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*
c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((45*b
^3*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3
*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a
*c^4 - 9*b^2*c^3))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*
c - b^2)^(1/2)) - (9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*
e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b
^2)^(1/2)*(36*a*c^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b...
```

3.9. $\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx$

3.10 $\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6} dx$

3.10.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6} dx = \frac{ex^3}{3c} + \frac{(bcd-b^2e+2ace)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} + \frac{(cd-be)\log(a+bx^3+cx^6)}{6c^2}$$

output
```
1/3*e*x^3/c+1/6*(-b*e+c*d)*ln(c*x^6+b*x^3+a)/c^2+1/3*(2*a*c*e-b^2*e+b*c*d)
*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6} dx = \frac{2cex^3 + \frac{2(-bcd+b^2e-2ace)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (cd-be)\log(a+bx^3+cx^6)}{6c^2}$$

input
```
Integrate[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*e*x^3 + (2*(-(b*c*d) + b^2*e - 2*a*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^
2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*d - b*e)*Log[a + b*x^3 + c*x^6])/(6*c
^2)
```

3.10.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (d+ex^3)}{a+bx^3+cx^6} \, dx$$

$$\Big\downarrow \; 1802$$

$$\frac{1}{3}\int \frac{x^3 (ex^3+d)}{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{3}\int \left(\frac{e}{c}-\frac{ae-(cd-be)x^3}{c(cx^6+bx^3+a)}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\frac{\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)(2ace+b^2(-e)+bcd)}{c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log(a+bx^3+cx^6)}{2c^2}+\frac{ex^3}{c}\right)$$

input
```
Int[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
((e*x^3)/c + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4
*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[a + b*x^3 + c*x^6])/(2*
c^2))/3
```

3.10.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.10.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{e\,x^3}{3c}+\frac{\frac{(-be+cd)\ln\left(c\,x^6+b\,x^3+a\right)}{2c}+\frac{2\left(-ae-\frac{(-be+cd)b}{2c}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3c}$	98
risch	Expression too large to display	1400

input
```
int(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*e*x^3/c+1/3/c*(1/2*(-b*e+c*d)/c*ln(c*x^6+b*x^3+a)+2*(-a*e-1/2*(-b*e+c*
d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.14

$$\int\frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx$$
$$=\left[\frac{2\,(b^2c-4\,ac^2)ex^3+(bcd-(b^2-2\,ac)e)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+((b^2c-4}{6\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + (b*c*d - (b^2 - 2*a*c)*e)*sqrt(b^2 - 4*a
*c)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*
a*c))/(c*x^6 + b*x^3 + a)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log
(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 +
 2*(b*c*d - (b^2 - 2*a*c)*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^3 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*
log(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

3.10.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(94) = 188$.

Time = 101.95 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.47

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)} - \frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)+2acd+3b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2}{6c^2\cdot(4ac-b^2)}\right.}{2ace-b^2e+bcd}\right. + \left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)} - \frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)+2acd+3b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bc}{6c^2\cdot(4ac-b^2)}\right.}{2ace-b^2e+bcd}\right. + \frac{ex^3}{3c}$$

input
```
integrate(x**5*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) -
 (b*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(-sqrt(-4*a*c + b**2
)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2
)) + 2*a*c*d + 3*b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(
6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)
) + (sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)
) - (b*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(sqrt(-4*a*c + b*
*2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c*
*2)) + 2*a*c*d + 3*b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/
(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d
)) + e*x**3/(3*c)
```

3.10.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex^3}{3\,c} + \frac{(cd-be)\log\left(cx^6+bx^3+a\right)}{6\,c^2} - \frac{(bcd-b^2e+2\,ace)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/3*e*x^3/c + 1/6*(c*d - b*e)*log(c*x^6 + b*x^3 + a)/c^2 - 1/3*(b*c*d - b^
2*e + 2*a*c*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c
)*c^2)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 11.31 (sec) , antiderivative size = 2624, normalized size of antiderivative = 27.05

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
(e*x^3)/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d
- 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) + (atan((4*c^3*(4*a*c - b^2)^(3/
2)*(x^3*((b*((b^2*c^3*d^3 - b^5*e^3 - a^2*b*c^2*e^3 + a^2*c^3*d*e^2 - 3*b^
3*c^2*d^2*e + 2*a*b^3*c*e^3 + 3*b^4*c*d*e^2 + 2*a*b*c^3*d^2*e - 4*a*b^2*c^
2*d*e^2)/c^3 - (((6*a^2*c^4*e^2 + 12*b^2*c^4*d^2 + 12*b^4*c^2*e^2 - 18*a*b
^2*c^3*e^2 - 24*b^3*c^3*d*e + 18*a*b*c^4*d*e)/c^3 - (((45*b^2*c^5*d - 45*b
^3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c
*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*
d - 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2
*c*d - 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((((45*b^2*c^5*d - 45*b^
3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*
d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2
*(4*a*c - b^2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e + 12*a
*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2
*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) + (3*b^2*(2
*a*c*e - b^2*e + b*c*d)^2*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))
/(4*c*(4*a*c - b^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*(
(((((45*b^2*c^5*d - 45*b^3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*
e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e
 - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^...
```

3.11 $\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6} dx$

3.11.1 Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6} dx = -\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{e\log\left(a+bx^3+cx^6\right)}{6c}$$

output
```
1/6*e*ln(c*x^6+b*x^3+a)/c-1/3*(-b*e+2*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2
)^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6} dx = \frac{-\frac{2(-2cd+be)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+e\log\left(a+bx^3+cx^6\right)}{6c}$$

input
```
Integrate[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] + e*Log[a + b*x^3 + c*x^6])/(6*c)
```

3.11.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1798, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(d+ex^3\right)}{a+bx^3+cx^6}dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{3}\int \frac{ex^3+d}{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{(2cd-be)\int \frac{1}{cx^6+bx^3+a}dx^3}{2c}+\frac{e\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3}{2c}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{e\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3}{2c}-\frac{(2cd-be)\int \frac{1}{-x^6+b^2-4ac}d\left(2cx^3+b\right)}{c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{e\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3}{2c}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\frac{e\log\left(a+bx^3+cx^6\right)}{2c}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

input
```
Int[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(-(((2*c*d - b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 -
4*a*c])) + (e*Log[a + b*x^3 + c*x^6])/(2*c))/3
```

3.11.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.11.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

method	result
default	$\frac{e\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{6c}+\frac{2\left(d-\frac{be}{2c}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3\sqrt{4ac-b^{2}}}$
risch	$\frac{2\ln\left(\left(-4abce+8a\,c^{2}d+b^{3}e-2b^{2}cd+\sqrt{-\left(be-2cd\right)^{2}\left(4ac-b^{2}\right)}\,b\right)x^{3}+2\sqrt{-\left(be-2cd\right)^{2}\left(4ac-b^{2}\right)}\,a\right)ae}{3\left(4ac-b^{2}\right)}-\frac{\ln\left(\left(-4abce+8a\,c^{2}d+b^{3}e-2\right.\right.}{}$

input
```
int(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*e*ln(c*x^6+b*x^3+a)/c+2/3*(d-1/2/c*b*e)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x^3+b)/(4*a*c-b^2)^(1/2))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.00

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \left[\frac{(b^2-4\,ac)e\log\left(cx^6+bx^3+a\right)-\sqrt{b^2-4\,ac}(2\,cd-be)\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{6\left(b^2c-4\,ac^2\right)}, \frac{(b^2-\cdots}{\cdots}\right.$$

input
```
integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*((b^2 - 4*a*c)*e*log(c*x^6 + b*x^3 + a) - sqrt(b^2 - 4*a*c)*(2*c*d -
b*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4
*a*c))/(c*x^6 + b*x^3 + a)))/(b^2*c - 4*a*c^2), 1/6*((b^2 - 4*a*c)*e*log(c
*x^6 + b*x^3 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^3 +
b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

3.11.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(65) = 130$.

Time = 10.30 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.99

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \left(\frac{e}{6c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{6c\left(4ac-b^2\right)}\right)\log\left(x^3+\frac{-12ac\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)+2ae+3b^2\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)\cdots}{be-2cd}\right.$$
$$+\left(\frac{e}{6c} + \frac{\sqrt{-4ac+b^2}(be-2cd)}{6c\left(4ac-b^2\right)}\right)\log\left(x^3+\frac{-12ac\left(\frac{e}{6c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)+2ae+3b^2\left(\frac{e}{6c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)\cdots}{be-2cd}\right.$$

input
```
integrate(x**2*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2)))*log(x**
3 + (-12*a*c*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b*
*2))) + 2*a*e + 3*b**2*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(
4*a*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(6*c) + sqrt(-4*a*c + b**2)*(b*
e - 2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3 + (-12*a*c*(e/(6*c) + sqrt(-4*a*
c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) + 2*a*e + 3*b**2*(e/(6*c) +
sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*
d))
```

3.11.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{e\log\left(cx^6+bx^3+a\right)}{6\,c} + \frac{(2\,cd-be)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c}$$

input
```
integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/6*e*log(c*x^6 + b*x^3 + a)/c + 1/3*(2*c*d - b*e)*arctan((2*c*x^3 + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 11.27 (sec) , antiderivative size = 1632, normalized size of antiderivative = 22.67

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
- (log(a + b*x^3 + c*x^6)*(3*b^2*e - 12*a*c*e))/(2*(36*a*c^2 - 9*b^2*c)) -
 (atan((b*(4*a*c - b^2)^(3/2)*(a*c*d*e^2 - a*b*e^3 - ((3*b^2*e - 12*a*c*e)
*(((3*b^2*e - 12*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e
- 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^
2 - 12*a*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) + ((((b*e - 2*c*d)*(72*a*b*c^2
*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))
/(6*c*(4*a*c - b^2)^(1/2)) + (9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)
)/((36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(6*c*(4*a*c -
 b^2)^(1/2)) + (3*a*b*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(2*(36*a*c^2
 - 9*b^2*c)*(4*a*c - b^2))))/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e
- 6*b^2*c*d*e^2)) - (4*x^3*((b*(b^2*e^3 + c^2*d^2*e + ((3*b^2*e - 12*a*c*e
)*(6*c^3*d^2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*
c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c))
+ 12*b^2*c*e^2 - 18*b*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) - 2*b*c*d*e^2 - (
(((b*e - 2*c*d)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c
*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^2*c^2*(3*b^2*
e - 12*a*c*e)*(b*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))
*(b*e - 2*c*d))/(6*c*(4*a*c - b^2)^(1/2)) - (3*b^2*c*(3*b^2*e - 12*a*c*e)*
(b*e - 2*c*d)^2)/(4*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2))))/(4*a^2*c) - ((2*
a*c - b^2)*(((3*b^2*e - 12*a*c*e)*(((b*e - 2*c*d)*(45*b^2*c^2*e - 36*b*...
```

3.12 $\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx$

3.12.1 Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \frac{(bd-2ae)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}$$

output
```
d*ln(x)/a-1/6*d*ln(c*x^6+b*x^3+a)/a+1/3*(-2*a*e+b*d)*arctanh((2*c*x^3+b)/(
-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

3.12.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \frac{d\log(x)}{a}-\frac{\operatorname{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a}$$

input
```
Integrate[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

output
```
(d*Log[x])/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[
x - #1] + c*d*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

3.12.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}dx$$

$$\Big\downarrow \ 1802$$

$$\frac{1}{3}\int \frac{ex^3+d}{x^3\left(cx^6+bx^3+a\right)}dx^3$$

$$\Big\downarrow \ 1200$$

$$\frac{1}{3}\int \left(\frac{d}{ax^3}+\frac{-cdx^3-bd+ae}{a\left(cx^6+bx^3+a\right)}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^3+cx^6\right)}{2a}+\frac{d\log\left(x^3\right)}{a}\right)$$

input
```
Int[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

output
```
(((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x^3])/a - (d*Log[a + b*x^3 + c*x^6])/(2*a))/3
```

3.12.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.12.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

method	result
default	$\frac{d\ln(x)}{a}+\frac{-\frac{d\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2}+\frac{2\left(ae-\frac{bd}{2}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3a}$
risch	$\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^{2}-b^{2}a\right)\_Z^{2}+\left(4acd-b^{2}d\right)\_Z+a\,e^{2}-bde+c\,d^{2}\right)}\_R\ln\left(\left(\left(-14ac+4b^{2}\right)\_R^{2}+\left(be-7cd\right)\_R-3e^{2}\right)x^{3}+b\right.\right.}{3}$

input
```
int((e*x^3+d)/x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
d*ln(x)/a+1/3/a*(-1/2*d*ln(c*x^6+b*x^3+a)+2*(a*e-1/2*b*d)/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.08

$$\int\frac{d+ex^{3}}{x\left(a+bx^{3}+cx^{6}\right)}\,dx$$
$$=\left[-\frac{(b^{2}-4\,ac)d\log\left(cx^{6}+bx^{3}+a\right)-6\,(b^{2}-4\,ac)d\log\left(x\right)+\sqrt{b^{2}-4\,ac}(bd-2\,ae)\log\left(\frac{2\,c^{2}x^{6}+2\,bcx^{3}+b^{2}-2\,ac}{cx^{6}+b}\right.}{6\,(ab^{2}-4\,a^{2}c)}\right.$$
$$-\frac{(b^{2}-4\,ac)d\log\left(cx^{6}+bx^{3}+a\right)-6\,(b^{2}-4\,ac)d\log\left(x\right)-2\,\sqrt{-b^{2}+4\,ac}(bd-2\,ae)\arctan\left(-\frac{(2\,cx^{3}+b)\sqrt{}}{b^{2}-4}\right.}{6\,(ab^{2}-4\,a^{2}c)}$$

input
```
integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) +
 sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c
- (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)))/(a*b^2 - 4*a^2*c)
, -1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x)
- 2*sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a
*c)/(b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

3.12.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)/x/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.12.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx$$
$$= -\frac{d\log(cx^6+bx^3+a)}{6\,a} + \frac{d\log(|x|)}{a} - \frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a}$$

input
```
integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
-1/6*d*log(c*x^6 + b*x^3 + a)/a + d*log(abs(x))/a - 1/3*(b*d - 2*a*e)*arct
an((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 4149, normalized size of antiderivative = 53.19

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x)
```

output
```
(d*log(x))/a - (log(a + b*x^3 + c*x^6)*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 -
 36*a^2*c)) - (atan((48*a^4*x^3*(4*a*c - b^2)^2*(((((((3*b^2*d - 12*a*c*d)
*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*
(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a
*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4
)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a
*b^2 - 36*a^2*c)) - ((2*a*e - b*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2
*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9
*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*
a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(6*a*(4*a*c - b^2)^(1/2)))*(3*b^2*d -
12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c)) + ((2*a*e - b*d)*(5*b*c^3*e^3 - ((3*b^
2*d - 12*a*c*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3
*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c))
 + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c))
+ 42*b*c^4*d*e))/(2*(9*a*b^2 - 36*a^2*c)) + 7*c^4*d*e^2))/(6*a*(4*a*c - b^
2)^(1/2)) - ((((2*a*e - b*d)*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b
^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*
c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*
(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4
*a*c - b^2)^(1/2))))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*...
```

3.12. $\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx$

3.13 $\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)} \, dx$

3.13.1 Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)} \, dx = -\frac{d}{3ax^3} - \frac{\left(b^2d-2acd-abe\right)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} - \frac{(bd-ae)\log(x)}{a^2} + \frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}$$

output
```
-1/3*d/a/x^3-(-a*e+b*d)*ln(x)/a^2+1/6*(-a*e+b*d)*ln(c*x^6+b*x^3+a)/a^2-1/3
*(-a*b*e-2*a*c*d+b^2*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*
c+b^2)^(1/2)
```

3.13.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.16

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)} \, dx = -\frac{d}{3ax^3} + \frac{(-bd+ae)\log(x)}{a^2} + \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&, \frac{b^2d\log(x-\#1)-acd\log(x-\#1)-abe\log(x-\#1)+bcd\log(x-\#1)\#1^3-ace\log(x-\#1)\#1^3}{b+2c\#1^3}\right.}{3a^2}$$

input
```
Integrate[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/3*d/(a*x^3) + ((-(b*d) + a*e)*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6
 & , (b^2*d*Log[x - #1] - a*c*d*Log[x - #1] - a*b*e*Log[x - #1] + b*c*d*Lo
g[x - #1]*#1^3 - a*c*e*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2)
```

3.13.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}dx$$

$$\Big\downarrow \; 1802$$

$$\frac{1}{3}\int \frac{ex^3+d}{x^6\left(cx^6+bx^3+a\right)}dx^3$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{3}\int \left(\frac{d}{ax^6}+\frac{c(bd-ae)x^3+b^2d-acd-abe}{a^2\left(cx^6+bx^3+a\right)}+\frac{ae-bd}{a^2x^3}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)\left(-abe-2acd+b^2d\right)}{a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{2a^2}-\frac{\log\left(x^3\right)(bd-ae)}{a^2}-\frac{d}{ax^3}\right)$$

input
```
Int[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

output
```
(-(d/(a*x^3)) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2
- 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x^3])/a^2 + ((b*d -
a*e)*Log[a + b*x^3 + c*x^6])/(2*a^2))/3
```

3.13.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.13.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result
default	$-\frac{d}{3a\,x^{3}}+\frac{(ae-bd)\ln(x)}{a^{2}}-\frac{\frac{(ace-bcd)\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2c}+\frac{2\left(abe+acd-b^{2}d-\frac{(ace-bcd)b}{2c}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3a^{2}}$
risch	$-\frac{d}{3a\,x^{3}}+\frac{\ln(x)e}{a}-\frac{\ln(x)bd}{a^{2}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(4a^{2}ce-a\,b^{2}e-4abcd+b^{3}d\right)\_Z+e^{2}ac-bcde+c^{2}d^{2}\right)}\_R\ln\left(\left(\left(-14a\right.\right.\right.\right.}{}$

input
```
int((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/a/x^3+(a*e-b*d)/a^2*ln(x)-1/3/a^2*(1/2*(a*c*e-b*c*d)/c*ln(c*x^6+b*x
^3+a)+2*(a*b*e+a*c*d-b^2*d-1/2*(a*c*e-b*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.44

$$\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx$$
$$= \left[\frac{(abe-(b^2-2ac)d)\sqrt{b^2-4ac}x^3\log\left(\frac{2c^2x^6+2bcx^3+b^2-2ac+(2cx^3+b)\sqrt{b^2-4ac}}{cx^6+bx^3+a}\right)+((b^3-4abc)d-(ab^2-4a^2\right.}{6(a^2b^2-4a^3c)}$$

input
```
integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*((a*b*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b
*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a
)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a)
- 6*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^
2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^3), 1/6*(2*(a*b*e - (b^2 - 2*a*c)*d)*sqrt(-
b^2 + 4*a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a) - 6*
((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^2*c)
*d)/((a^2*b^2 - 4*a^3*c)*x^3)]
```

3.13.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)/x**4/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.13.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx = \frac{(bd-ae)\log(cx^6+bx^3+a)}{6\,a^2} - \frac{(bd-ae)\log(|x|)}{a^2} + \frac{(b^2d-2\,acd-abe)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a^2} + \frac{bdx^3-aex^3-ad}{3\,a^2x^3}$$

input
```
integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/6*(b*d - a*e)*log(c*x^6 + b*x^3 + a)/a^2 - (b*d - a*e)*log(abs(x))/a^2 +
 1/3*(b^2*d - 2*a*c*d - a*b*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(s
qrt(-b^2 + 4*a*c)*a^2) + 1/3*(b*d*x^3 - a*e*x^3 - a*d)/(a^2*x^3)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 7282, normalized size of antiderivative = 65.02

$$\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x)
```

output
```
(log(x)*(a*e - b*d))/a^2 - (log(((((((((a*e - b*d + a^2*(-(a*b*e - b^2*d +
2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((27*b^2*c^3*(a*b*e - b^2*d + a*c*d
))/a + (9*b*c^4*x^3*(2*b^2*d + 7*a*b*e - 28*a*c*d))/a + (9*b^2*c^3*(a*b +
4*b^2*x^3 - 14*a*c*x^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^
4*(4*a*c - b^2)))^(1/2)))/(2*a^2)))/(6*a^2) - (3*c^5*d*x^3*(11*b^2*d - 14*
a*b*e + 14*a*c*d))/a^2 + (9*b*c^4*d*(3*a*b*e - 3*b^2*d + a*c*d))/a^2)*(a*e
 - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6
*a^2) + (c^5*d^2*(9*a*b*e - 9*b^2*d + a*c*d))/a^3 + (c^6*d^2*x^3*(7*a*e -
12*b*d))/a^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c -
 b^2)))^(1/2)))/(6*a^2) + (c^6*d^3*(a*e - b*d))/a^4 - (c^7*d^4*x^3)/a^4)*(
((((((b*d - a*e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(
1/2))*((27*b^2*c^3*(a*b*e - b^2*d + a*c*d))/a + (9*b*c^4*x^3*(2*b^2*d + 7*
a*b*e - 28*a*c*d))/a - (9*b^2*c^3*(a*b + 4*b^2*x^3 - 14*a*c*x^3)*(b*d - a*
e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(2*a^2)
))/(6*a^2) + (3*c^5*d*x^3*(11*b^2*d - 14*a*b*e + 14*a*c*d))/a^2 - (9*b*c^4
*d*(3*a*b*e - 3*b^2*d + a*c*d))/a^2)*(b*d - a*e + a^2*(-(a*b*e - b^2*d + 2
*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) + (c^5*d^2*(9*a*b*e - 9*b^2
*d + a*c*d))/a^3 + (c^6*d^2*x^3*(7*a*e - 12*b*d))/a^3)*(b*d - a*e + a^2*(-
(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) - (c^6*d^
3*(a*e - b*d))/a^4 + (c^7*d^4*x^3)/a^4))*(3*b^3*d - 3*a*b^2*e + 12*a^2*...
```

3.13. $\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx$

3.14 $\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx$

3.14.1 Optimal result

Integrand size = 25, antiderivative size = 723

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*e*x^2/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(c*d-b*
e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^
2)^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b
^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(-2*a*c*e+b^2*e-b*
c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*
arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(
c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)*3^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/
2))^(1/3))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5
/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/
3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(
2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b+(-4*a*c+b^2)^(
1/2))^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(
1/3))*3^(1/2))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/
c^(5/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

3.14.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.12

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{3ex^2-2\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{ae\log(x-\#1)-cd\log(x-\#1)\#1^3+be\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{6c}$$

input
```
Integrate[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(3*e*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x
- #1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

3.14.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.80, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {1826, 27, 1834, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (d+e x^3)}{a+b x^3+c x^6} \, dx$$

$$\Big\downarrow \; 1826$$

$$\frac{e x^2}{2 c}-\frac{\int \frac{2 x \left(a e-(c d-b e) x^3\right)}{c x^6+b x^3+a}dx}{2 c}$$

$$\Big\downarrow \; 27$$

$$\frac{e x^2}{2 c}-\frac{\int \frac{x \left(a e-(c d-b e) x^3\right)}{c x^6+b x^3+a}dx}{c}$$

$$\Big\downarrow \; 1834$$

$$\frac{e x^2}{2 c}-\frac{-\frac{1}{2} \left(-\frac{2 a c e+b^2 (-e)+b c d}{\sqrt{b^2-4 a c}}-b e+c d\right) \int \frac{2 x}{2 c x^3+b-\sqrt{b^2-4 a c}}dx-\frac{1}{2} \left(\frac{2 a c e+b^2 (-e)+b c d}{\sqrt{b^2-4 a c}}-b e+c d\right) \int \frac{2 x}{2 c x^3+b+\sqrt{b^2-4 a c}}dx}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{e x^2}{2 c}-\frac{-\left(\left(-\frac{2 a c e+b^2 (-e)+b c d}{\sqrt{b^2-4 a c}}-b e+c d\right) \int \frac{x}{2 c x^3+b-\sqrt{b^2-4 a c}}dx\right)-\left(\frac{2 a c e+b^2 (-e)+b c d}{\sqrt{b^2-4 a c}}-b e+c d\right) \int \frac{x}{2 c x^3+b+\sqrt{b^2-4 a c}}dx}{c}$$

$$\Big\downarrow \; 821$$

$$\frac{e x^2}{2 c}-\frac{-\left(\left(-\frac{2 a c e+b^2 (-e)+b c d}{\sqrt{b^2-4 a c}}-b e+c d\right) \left(\frac{\int \frac{\sqrt[3]{2} \sqrt[3]{c} x+\sqrt[3]{b-\sqrt{b^2-4 a c}}}{2^{2/3} c^{2/3} x^2-\sqrt[3]{2} \sqrt[3]{c} \sqrt[3]{b-\sqrt{b^2-4 a c}} x+\left(b-\sqrt{b^2-4 a c}\right)^{2/3}}dx}{3 \sqrt[3]{2} \sqrt[3]{c} \sqrt[3]{b-\sqrt{b^2-4 a c}}}-\frac{\int \frac{1}{\sqrt[3]{2} \sqrt[3]{c} x+\sqrt[3]{b-\sqrt{b^2 \cdots}}}}{3 \sqrt[3]{2} \sqrt[3]{c} \sqrt[3]{b-\sqrt{b^2 \cdots}}} \cdots\right)\right)}{\cdots}$$

$$\Big\downarrow \; 16$$

$$\frac{ex^2}{2c}-$$

$$-\left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}\ldots}\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b\ldots}}}\right.\right.$$

$\downarrow$ 1142

$$\frac{ex^2}{2c}-$$

$$-\left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int -\ldots}{2^{2}\ldots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.\right.$$

$\downarrow$ 25

$$\frac{ex^2}{2c}-$$

$$-\left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int \ldots}{2^{2/3}\ldots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.\right.$$

$\downarrow$ 27

3.14. $\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx$

$$\frac{ex^2}{2c}-$$

$$-\left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int\cdots}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots\right.\right.$$

$\downarrow$ 1082

$$\frac{ex^2}{2c}-$$

$$-\left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{\cdots}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\cdots}}\cdots}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots\right.\right.$$

$\downarrow$ 217

$$\frac{ex^2}{2c}-$$

$$-\left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\cdots}{\sqrt[3]{b-\sqrt{\cdots}}}}{\sqrt{3}}\cdots\right)}{\sqrt[3]{2}\sqrt[3]{c}}\cdots}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots\right.\right.$$

$\downarrow$ 1103

3.14. $\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx$

$$\frac{ex^2}{2c} - \left(- \left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd \right) \left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}} - \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{\cdots}}{\sqrt[3]{b-\sqrt{b^{\cdots}}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}} \cdots \right.\right.$$

input
```
Int[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(e*x^2)/(2*c) - (-((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c
])*(-1/3*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c
^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)
*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) +
Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c]
)^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(
b - Sqrt[b^2 - 4*a*c])^(1/3)))) - (c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/S
qrt[b^2 - 4*a*c])*(-1/3*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3
)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[
(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*c^(1/3))) + Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sq
rt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^
(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))))/c
```

3.14.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

3.14. $\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx$

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.14.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{e\,x^{2}}{2c}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{4}(be-cd)+ae\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	70
risch	$\frac{e\,x^{2}}{2c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left((-be+cd)\_R^{4}-ae\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	71

input
```
int(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/c*e*x^2-1/3/c*sum((_R^4*(b*e-c*d)+a*e*_R)/(2*_R^5*c+_R^2*b)*ln(x-_R),_
R=RootOf(_Z^6*c+_Z^3*b+a))
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13535 vs. 2(583) = 1166.

Time = 52.23 (sec) , antiderivative size = 13535, normalized size of antiderivative = 18.72

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.14.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.14.7 Maxima [F]

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x^4}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
1/2*e*x^2/c - integrate(-((c*d - b*e)*x^4 - a*e*x)/(c*x^6 + b*x^3 + a), x)
/c
```

3.14.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
Timed out
```

3.14.9 Mupad [B] (verification not implemented)

Time = 38.70 (sec) , antiderivative size = 13112, normalized size of antiderivative = 18.14

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*
a*c*e^2 - 2*b*c*d*e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8*e^3 + 16
*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^
4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c
^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2
*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b
^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^
2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*
c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d
*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3
*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8*e
^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a
*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4
8*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^
3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2
- 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c
^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5
*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^
3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*...
```

3.15 $\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6} dx$

3.15.1 Optimal result

Integrand size = 25, antiderivative size = 718

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex}{c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
e*x/c+1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(c*d-b*e+(-2*
a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/
2))^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1
/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(
-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*arctan
(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(c*d-b*
e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(2/3)+1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1
/3))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b
+(-4*a*c+b^2)^(1/2))^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(
b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(2*a*c*
e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^
(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*
3^(1/2))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3
)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

3.15.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.12

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{ae\log(x-\#1)-cd\log(x-\#1)\#1^3+be\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

input
```
Integrate[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(e*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x - #
1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

3.15.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 550, normalized size of antiderivative = 0.77, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1826, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (d+ex^3)}{a+bx^3+cx^6} \, dx$$

$$\Big\downarrow \text{ 1826}$$

$$\frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^3}{cx^6+bx^3+a}dx}{c}$$

$$\Big\downarrow \text{ 1752}$$

$$\frac{ex}{c} - \frac{-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{ex}{c} - \frac{-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\sqrt[3]{b-\cdots}}\cdots}{3\left(b-\sqrt{b^2-4\cdots}\right.}\cdots\right.}{\cdots}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{ex}{c} - \frac{-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b\cdots}}\right.\cdots}{3\sqrt[3]{c}\left(b-\sqrt{\cdots}\right.}\cdots\right.}{\cdots}$$

$\Big\downarrow$ 27

$$\frac{ex}{c}-$$

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{2\ 2^{2/3}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{\ldots}}\right.}{3\sqrt[3]{c}\left(b-\ldots\right.}\right.$$

$\Big\downarrow$ 1142

$$\frac{ex}{c}-$$

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$\Big\downarrow$ 25

3.15. $\int\frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx$

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{ex}{c}-\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}}{}\right.$$

$\downarrow$ 27

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{ex}{c}-\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}}{}\right.$$

$\downarrow$ 1082

3.15. $\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx$

$$\frac{ex}{c}-$$

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int}{-\left(1-\frac{2\sqrt[3]{}}{\sqrt[3]{b-}}\right.}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$\Big\downarrow$ 217

$$\frac{ex}{c}-$$

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{}{\sqrt[3]{b}}}{}\right.}{2\sqrt[3]{}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$\Big\downarrow$ 1103

3.15. $\int \frac{x^3\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$

$$\frac{ex}{c} - \frac{-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-\ldots}\right)\ldots\right)}{4\sqrt[3]{c}}\ldots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\ldots\right)}{\ldots}$$

input
```
Int[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(e*x)/c - (-1/2*((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])
*((2^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1
/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) -
Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c]
)^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^
(2/3)))) - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*((2^
(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(
b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2
^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(
b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/
3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b + Sqrt[b^2 - 4*a*c])^(2/3)
)))/2)/c
```

3.15. $\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx$

3.15.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

3.15.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{ex}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-be+cd\right)\_R^{3}-ae\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67
risch	$\frac{ex}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-be+cd\right)\_R^{3}-ae\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67

input
```
int(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
e*x/c+1/3/c*sum(((-b*e+c*d)*_R^3-a*e)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf
(_Z^6*c+_Z^3*b+a))
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8705 vs. $2(580) = 1160$.

Time = 4.25 (sec) , antiderivative size = 8705, normalized size of antiderivative = 12.12

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.15.7 Maxima [F]

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x^3}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
e*x/c - integrate(-((c*d - b*e)*x^3 - a*e)/(c*x^6 + b*x^3 + a), x)/c
```

3.15.8 Giac [F]

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x^3}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*x^3/(c*x^6 + b*x^3 + a), x)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 29.75 (sec) , antiderivative size = 11453, normalized size of antiderivative = 15.95

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
log((3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*
d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*
e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/
3)*((2^(1/3)*(81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c -
b^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2
*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b
^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^
2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4
*a*c - b^2)^3))^(1/3))/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d
^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*
b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*
b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e
+ 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 +
 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - ...
```

3.16 $\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$

3.16.1 Optimal result

Integrand size = 23, antiderivative size = 634

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$- \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(e+(-b*e+2*c*d)/(-
4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(
2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c
+b^2)^(1/2))^(2/3))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b
-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*
c+b^2)^(1/2))^(1/3))*3^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/
c^(2/3)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(
-4*a*c+b^2)^(1/2))^(1/3))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/
3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3
)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(e+(b*e-2*c
*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/6*a
rctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(e
+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)*3^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/3)
```

3.16.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.09

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]$$

input
```
Integrate[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1
 + 2*c*#1^4) & ]/3
```

3.16.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 532, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1834, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \; 1834$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \; 27$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \; 821$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \; 16$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \; 1142$$

3.16. $\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{2}\sqrt[3]{\cdots}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{\cdots}}}{\cdots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{2}\sqrt[3]{\cdots}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{\cdots}}}{\cdots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\downarrow 25$$

3.16. $\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int\frac{\sqrt[3]{2}\sqrt[3]{c}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}}}{}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int\frac{\sqrt[3]{2}\sqrt[3]{c}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}}}{}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$\downarrow$ 27

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int\frac{}{2^{2/3}c^{2/3}x^2-\sqrt[3]{}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int\frac{}{2^{2/3}c^{2/3}x^2-\sqrt[3]{}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$\downarrow$ 1082

3.16. $\int\frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4}}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4}}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\downarrow \; 217$$

3.16. $\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$\downarrow$ 1103

3.16. $\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

input
```
Int[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b - Sqrt[b^2 - 4*a*c])^(1
/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/
3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(
1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(
1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3))) + (e - (
2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) +
2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (-((S
qrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sq
rt[3]])/(2^(1/3)*c^(1/3))) + Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c
^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c
^(1/3)))/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)))
```

3.16. $\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$

3.16.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.16.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{4}e+\_Rd\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	49
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{4}e+\_Rd\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	49

input
```
int(x*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((_R^4*e+_R*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a
))
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8268 vs. 2(496) = 992.

Time = 14.50 (sec) , antiderivative size = 8268, normalized size of antiderivative = 13.04

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.16.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate(x*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.16.7 Maxima [F]

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

3.16.8 Giac [F]

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 26.70 (sec) , antiderivative size = 7457, normalized size of antiderivative = 11.76

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3
+ a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3
- b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2
) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*
c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (2^(2/3)*(27*c^3*x*(4*a*c -
b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*b*c^3*(
4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^
3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^
3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(
a*c^2*(4*a*c - b^2)^3))^(2/3))/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d
^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^
3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/6 - 108*a^2*c^4*d^2*e -
45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e
^2 + 108*a^2*b*c^3*d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)...
```

3.17 $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

3.17.1 Optimal result

Integrand size = 22, antiderivative size = 634

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(e+(-b*e+2*c*d)/(-4
*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/12*ln(2^(2
/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+
b^2)^(1/2))^(2/3))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b-
(-4*a*c+b^2)^(1/2))^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c
+b^2)^(1/2))^(1/3))*3^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c
^(1/3)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-
4*a*c+b^2)^(1/2))^(1/3))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3
)/(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)
*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(e+(b*e-2*c*
d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ar
ctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(e+
(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(2/3)
```

3.17.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.10

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]$$

input
```
Integrate[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1
^2 + 2*c*#1^5) & ]/3
```

3.17.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 512, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{ 1752}$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$$\Big\downarrow \text{ 750}$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}\ldots}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\ldots\right.$$
$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b+\sqrt{b^2-\ldots}}}{\sqrt[3]{2}}}\ldots}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\ldots\right.$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}\ldots\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4a\ldots}\right.}\ldots\right.$$
$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}\ldots\right.}{3\sqrt[3]{c}\left(\sqrt{b^2-4ac}+\ldots\right.}\ldots\right.$$

3.17. $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4a}}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4}\right.}\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+}\right.}{3\sqrt[3]{c}\left(\sqrt{b^2-4ac}\right.}\right.$$

$$\Big\downarrow \text{1142}$$

3.17. $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int -\cdots}{2c^{2/3}\cdots}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int -\cdots}{2c^{2/3}\cdots}\cdots\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}\cdots}\right.$$

$\downarrow$ 25

3.17. $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int\frac{\quad}{2c^{2/3}x^2\cdots}}{\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int\frac{\quad}{2c^{2/3}x^2\cdots}}{\cdots}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}\cdots}\right.$$

$\downarrow$ 27

3.17. $\int\frac{d+ex^3}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{\cdots}{2c^{2/3}\cdots}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\cdots\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}\,x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{\cdots}{2c^{2/3}\cdots}\cdots\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}\cdots}\cdots\right.$$

$$\Big\downarrow\ 1082$$

3.17. $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-}}}\right.}}{}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-}}}\right.}}{}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

↓ 217

3.17. $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{\cdots}}{\sqrt[3]{b-\sqrt{b\cdots}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{\cdots}}{\sqrt[3]{\sqrt{b^2-\cdots}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\cdots\right)}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\cdots\right.$$

$\downarrow$ 1103

3.17. $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\right.}{4\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}\right.}{4\sqrt[3]{c}}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

input
```
Int[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

3.17. $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

output
```
((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c
])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) +
 (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2
 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) -
2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*
c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + ((e - (2*c*d - b*e)/Sqr
t[b^2 - 4*a*c])*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1
/3)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt
[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[
3]])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sq
rt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b + Sqrt
[b^2 - 4*a*c])^(2/3))))/2
```

3.17.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.17.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{3}e+d\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	47
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{3}e+d\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	47

input
```
int((e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((_R^3*e+d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6748 vs. 2(496) = 992.

Time = 1.87 (sec) , antiderivative size = 6748, normalized size of antiderivative = 10.64

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.17.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.17.7 Maxima [F]

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \int \frac{ex^3+d}{cx^6+bx^3+a}\,dx$$

input
```
integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

3.17.8 Giac [F]

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx=\int \frac{ex^3+d}{cx^6+bx^3+a}\,dx$$

input
```
integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 21.64 (sec) , antiderivative size = 7469, normalized size of antiderivative = 11.78

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx=\text{Too large to display}$$

input
```
int((d + e*x^3)/(a + b*x^3 + c*x^6),x)
```

output
```
log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2
*e^2 - 4*b*c^2*d^3*e) - (2^(2/3)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e
^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(
1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-
(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*((2^(1/3)*(81*c^3*x*(4*a*
c - b^2)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^5*c*d^3
+ a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*
c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*
a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*
a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^
2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/
3))/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*
a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4
8*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(
4*a*c - b^2)^3))^(2/3))/18 - 36*a*c^5*d^3 + 9*b^2*c^4*d^3 + 9*a*b^3*c^2*e^
3 - 36*a^2*b*c^3*e^3 + 108*a^2*c^4*d*e^2 - 27*a*b^2*c^3*d*e^2))/6)*(-(b...
```

3.18 $\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx$

3.18.1 Optimal result

Integrand size = 25, antiderivative size = 653

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+ \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
-d/a/x+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(d+(
-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/1
2*c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^
(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^
(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c
^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(d+(-2*a*e+b*d)/(-4*a*c+b^
2)^(1/2))*2^(1/3)/a*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*ln(2^
(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^
(1/2))*2^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*c^(1/3)*ln(2^(2/3)*c^(2
/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/
2))^(2/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b+(-4*a*c+b^2)^(1
/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(
1/2))^(1/3))*3^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a*3^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

3.18.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.13

$$\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx$$
$$= -\frac{d}{ax} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{3a}$$

input
```
Integrate[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

output
```
-(d/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x
- #1] + c*d*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

3.18.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 547, normalized size of antiderivative = 0.84, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1828, 1834, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx$$

$$\Big\downarrow \ 1828$$

$$-\frac{\int \frac{x(cdx^3+bd-ae)}{cx^6+bx^3+a}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 1834$$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 27$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx+c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 821$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+c(}{a}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \ 16$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+}{a}$$

$$\frac{d}{ax}$$

$\downarrow$ 1142

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\int -\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b\ldots}\right)}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\ldots}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\ldots\right.}{\ldots}$$

$$\frac{d}{ax}$$

$\downarrow$ 25

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\int \frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b\ldots}\right)}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\ldots}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\ldots\right.}{\ldots}$$

$$\frac{d}{ax}$$

$\downarrow$ 27

3.18. $\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{b-\dots}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\dots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\dots\right.}{\dots}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \ 1082$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\dots}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\dots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\dots\right.}{\dots}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \ 217$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\dots}{\dots}\right.}{\dots}$$

$$\frac{d}{ax}$$

3.18. $\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx$

$$\Big\downarrow \text{1103}$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\right.}{}\right.}{}$$

$$\frac{d}{ax}$$

input
```
Int[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

output
```
-(d/(a*x)) - (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b - Sqrt[
b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b - Sqrt[b^2 -
4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b
^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b - Sqrt[b^2 - 4*a
*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(
2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^
(1/3))) + c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b + Sqrt[b^2
- 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*
c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 -
 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b + Sqrt[b^2 - 4*a*c])
^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)
*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3
))))/a
```

3.18. $\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx$

3.18.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.18.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.11

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-cd\_R^{4}+(ae-bd)\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3a}-\frac{d}{ax}$
risch	$-\frac{d}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^{7}c^{3}-48b^{2}c^{2}a^{6}+12a^{5}b^{4}c-b^{6}a^{4}\right)\_Z^{6}+\left(-16a^{5}c^{2}e^{3}+8a^{4}b^{2}ce^{3}+48a^{4}bc^{2}de^{2}+48a^{4}c^{3}d^{2}e-a^{3}b^{4}e^{3}-24a^{3}b^{3}cde^{2}-\right.\right.}\right.}{}$

input
```
int((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*sum((-c*d*_R^4+(a*e-b*d)*_R)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z
^6*c+_Z^3*b+a))-d/a/x
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11285 vs. 2(517) = 1034.

Time = 37.04 (sec) , antiderivative size = 11285, normalized size of antiderivative = 17.28

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)/x**2/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.18.7 Maxima [F]

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx = \int \frac{ex^3+d}{(cx^6+bx^3+a)x^2}\,dx$$

input
```
integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((c*d*x^4 + (b*d - a*e)*x)/(c*x^6 + b*x^3 + a), x)/a - d/(a*x)
```

3.18.8 Giac [F]

$$\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx = \int \frac{ex^3+d}{(cx^6+bx^3+a)x^2}\,dx$$

input
```
integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^2), x)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 36.13 (sec) , antiderivative size = 11174, normalized size of antiderivative = 17.11

$$\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x)
```

output
```
log((2^(1/3)*(-(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) +
8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3
+ 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e
- 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6
*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(a^4*(4*a*c - b^2)^3))^(2/3)*((2^(2/3)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^
2 - 2*a^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^
2 + 6*a^2*b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*(-(b^7*d^3 - a
^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*
c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5
*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*
e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2
*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3
))/2)*(-(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*...
```

3.19 $\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx$

3.19.1 Optimal result

Integrand size = 25, antiderivative size = 655

$$\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx$$

$$= -\frac{d}{2ax^2} + \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
-1/2*d/a/x^2-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3)
)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/
3)+1/12*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(
1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/
2))*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(
1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(d+(-2*a*e+b*d)/(-4*
a*c+b^2)^(1/2))*2^(2/3)/a*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*c^(2/3)
*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(d+(2*a*e-b*d)/(-4*a*c
+b^2)^(1/2))*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/12*c^(2/3)*ln(2^(2/3
)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^
2)^(1/2))^(2/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b+(-4*a*c+b
^2)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+
b^2)^(1/2))^(1/3))*3^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a*3
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

3.19.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.14

$$\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx = -\frac{d}{2ax^2} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3a}$$

input
```
Integrate[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/2*d/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Lo
g[x - #1] + c*d*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

3.19.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {1828, 27, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}dx$$

$$\Big\downarrow \text{ 1828}$$

$$-\frac{\int \frac{2(cdx^3+bd-ae)}{cx^6+bx^3+a}dx}{2a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{cdx^3+bd-ae}{cx^6+bx^3+a}dx}{a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \text{ 1752}$$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \text{ 750}$$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

$$\frac{d}{2ax^2}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{\cdots}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\big\downarrow$ 27

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\cdots\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\big\downarrow$ 1142

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int -\cdots}{2c^{2/3}x^2-2^{2}\cdots}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\big\downarrow$ 25

3.19. $\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\;2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{\cdots}}{2c^{2/3}x^2-2^{2/3}\cdots}}{\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\big\downarrow$ 27

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\;2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{\cdots}{2c^{2/3}x^2-2^{2\cdots}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\big\downarrow$ 1082

3.19. $\int\frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\ 2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-\cdots}\cdots}{2\sqrt[3]{c}\cdots}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 217

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\ 2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}\cdots}}{\sqrt{3}}\cdots\right)}{2\sqrt[3]{c}}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}$$

$$\frac{d}{2ax^2}$$

3.19. $\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx$

$\big\downarrow$ 1103

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}\cdots\right.}{4\sqrt[3]{c}}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{d}{2ax^2}$$

input
```
Int[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/2*d/(a*x^2) - ((c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(
b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^
2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1
/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^
2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2
/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + (c*
(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c]
)^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) +
(2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2
- 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2
^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c
^(1/3))))/(3*(b + Sqrt[b^2 - 4*a*c])^(2/3))))/2)/a
```

3.19. $\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx$

3.19.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.19.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.10

method	result
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}cd+ae-bd\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3a}-\frac{d}{2a\,x^{2}}$
risch	$-\frac{d}{2a\,x^{2}}+\frac{\left(\_R=\operatorname{RootOf}\left(\left(64c^{3}a^{8}-48b^{2}c^{2}a^{7}+12b^{4}c\,a^{6}-b^{6}a^{5}\right)\_Z^{6}+\left(16a^{5}b\,c^{2}e^{3}+48a^{5}c^{3}d\,e^{2}-8a^{4}b^{3}c\,e^{3}-72a^{4}b^{2}c^{2}d\,e^{2}-96a^{4}b\,c^{3}d^{2}e-16a^{4}\right.\right.\right.}{}$

input
```
int((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*sum((-_R^3*c*d+a*e-b*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+
_Z^3*b+a))-1/2*d/a/x^2
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11459 vs. 2(517) = 1034.

Time = 18.76 (sec) , antiderivative size = 11459, normalized size of antiderivative = 17.49

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)/x**3/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

3.19.7 Maxima [F]

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx = \int \frac{ex^3+d}{(cx^6+bx^3+a)x^3}\,dx$$

input
```
integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((c*d*x^3 + b*d - a*e)/(c*x^6 + b*x^3 + a), x)/a - 1/2*d/(a*x^2)
```

3.19.8 Giac [F]

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx=\int \frac{ex^3+d}{(cx^6+bx^3+a)x^3}\,dx$$

input
```
integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^3), x)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 35.79 (sec) , antiderivative size = 13466, normalized size of antiderivative = 20.56

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

input
```
int((d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x)
```

output
```
log(- (2^(2/3)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d
^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*
c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 +
96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e
^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c
- b^2)^3))^(1/3)*((2^(1/3)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d +
a*c*d) + (81*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 +
16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a
^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48
*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^
5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4
*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b
^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3...
```

3.20 $\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx$

3.20.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx=-\frac{x^6}{6}-\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
-1/6*x^6+1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx=-\frac{x^6}{6}+\frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

input
```
Integrate[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/6*x^6 + ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

3.20.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(1-x^3)}{x^6-x^3+1} dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3}\int \frac{x^6(1-x^3)}{x^6-x^3+1}dx^3$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3}\int \left(\frac{x^3}{x^6-x^3+1}-x^3\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{x^6}{2}+\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
(-1/2*x^6 - ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/2)/3
```

3.20.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.20.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x^6}{6}+\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	38
risch	$-\frac{x^6}{6}+\frac{\ln(4x^6-4x^3+4)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	40

input
```
int(x^8*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^6+1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx=-\frac{1}{6}\,x^6+\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
 + 1)
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^6}{6} + \frac{\log\left(x^6-x^3+1\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**8*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-x**6/6 + log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)
/3)/9
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{6}\,x^6 + \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right) + \frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
 + 1)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{6}\,x^6 + \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right) + \frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
 + 1)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = \frac{\ln(x^6-x^3+1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9} - \frac{x^6}{6}$$

input
```
int(-(x^8*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x
^6/6
```

3.21 $\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx$

3.21.1 Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^3}{3} - \frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
-1/3*x^3-2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^3}{3} + \frac{2\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Integrate[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/3*x^3 + (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

3.21.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(1-x^3)}{x^6-x^3+1}\,dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3}\int \frac{x^3(1-x^3)}{x^6-x^3+1}dx^3$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3}\int \left(\frac{1}{x^6-x^3+1}-1\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-x^3\right)$$

input
```
Int[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
(-x^3 - (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/Sqrt[3])/3
```

3.21.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.21.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	25
risch	$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	25

input
```
int(x^5*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3+2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx=-\frac{1}{3}\,x^3+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

input
```
integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx=-\frac{x^3}{3}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**5*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-x**3/3 + 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\,x^3 + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

input
```
integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\,x^3 + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

input
```
integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9} - \frac{x^3}{3}$$

input
```
int(-(x^5*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
- (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x^3/3
```

3.22 $\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx$

3.22.1 Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx=-\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
-1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx=\frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

input
```
Integrate[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

3.22.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1798, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-x^3)}{x^6-x^3+1}\,dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{3}\int \frac{1-x^3}{x^6-x^3+1}dx^3$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^6-x^3+1}dx^3-\frac{1}{2}\int -\frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^6-x^3+1}dx^3+\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3-\int \frac{1}{-x^6-3}d(2x^3-1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3+\frac{\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\frac{\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3] - Log[1 - x^3 + x^6]/2)/3
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.22.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}$	33
risch	$-\frac{\ln\left(4x^{6}-4x^{3}+4\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}$	35

input
```
int(x^2*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = -\frac{\log\left(x^6-x^3+1\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**2*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right) - \frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = -\frac{\ln\left(x^6-x^3+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

input
```
int(-(x^2*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
- log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

3.23 $\int \frac{1-x^3}{x(1-x^3+x^6)}\,dx$

3.23.1 Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx=\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
ln(x)-1/6*ln(x^6-x^3+1)+1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

3.23.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx=\log(x)-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

output
```
Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^3) & ]/
3
```

3.23.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^3}{x\left(x^6-x^3+1\right)}dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{3}\int \frac{1-x^3}{x^3\left(x^6-x^3+1\right)}dx^3$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3}\int \left(\frac{1}{x^3}-\frac{x^3}{x^6-x^3+1}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^3\right)-\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

output
```
(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[x^3] - Log[1 - x^3 + x^6]/2)/3
```

3.23.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.23.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x)-\frac{\ln(x^6-x^3+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$	33
default	$-\frac{\ln(x^6-x^3+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}+\ln(x)$	35

input
```
int((-x^3+1)/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1-x^3}{x(1-x^3+x^6)}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

input
```
integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + lo
g(x)
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx = \log\left(x\right) - \frac{\log\left(x^6-x^3+1\right)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((-x**3+1)/x/(x**6-x**3+1),x)
```

output
```
log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \frac{1}{3}\log\left(x^3\right)$$

input
```
integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/
3*log(x^3)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log\left(|x|\right)$$

input
```
integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + lo
g(abs(x))
```

3.23.9 Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx=\ln\left(x\right)-\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

input
```
int(-(x^3 - 1)/(x*(x^6 - x^3 + 1)),x)
```

output
```
log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/
3))/9
```

3.24 $\int \frac{1-x^3}{x^4(1-x^3+x^6)}\,dx$

3.24.1 Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{1}{3x^3}+\frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
-1/3/x^3+2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

3.24.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{1}{3x^3}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

output
```
-1/3*1/x^3 - RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-1 + 2*#1^3) & ]/3
```

3.24.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^3}{x^4 (x^6-x^3+1)} \, dx$$
$$\Big\downarrow \; 1802$$
$$\frac{1}{3} \int \frac{1-x^3}{x^6 (x^6-x^3+1)}dx^3$$
$$\Big\downarrow \; 1200$$
$$\frac{1}{3} \int \left(\frac{1}{x^6}+\frac{1}{-x^6+x^3-1}\right)dx^3$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{3} \left(\frac{2 \arctan \left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^3}\right)$$

input
```
Int[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

output
```
(-x^(-3) + (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/Sqrt[3])/3
```

3.24.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.24.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$	25
risch	$-\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$	25

input
```
int((-x^3+1)/x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))-1/3/x^3
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{2\,\sqrt{3}x^3\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)+3}{9\,x^3}$$

input
```
integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/9*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3)/x^3
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

input
```
integrate((-x**3+1)/x**4/(x**6-x**3+1),x)
```

output
```
-2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9 - 1/(3*x**3)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^3-1\right)\right)-\frac{1}{3\,x^3}$$

input
```
integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^3-1\right)\right)-\frac{1}{3\,x^3}$$

input
```
integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

3.24.9 Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}-\frac{1}{3\,x^3}$$

input
```
int(-(x^3 - 1)/(x^4*(x^6 - x^3 + 1)),x)
```

output
```
(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - 1/(3*x^3)
```

3.25 $\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx$

3.25.1 Optimal result

Integrand size = 23, antiderivative size = 418

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^4}{4} - \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output
```
-1/4*x^4+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^
(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))
*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(
1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2)
)^(2/3)+1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I
*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(
1/2))^(2/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2
*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))
^(2/3)
```

3.25.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.11

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^4}{4}+\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/4*x^4 + RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/
3
```

3.25.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1826, 27, 1710, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(1-x^3)}{x^6-x^3+1}\,dx$$

$$\Big\downarrow \text{1826}$$

$$-\frac{1}{4}\int -\frac{4x^3}{x^6-x^3+1}dx-\frac{x^4}{4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^3}{x^6-x^3+1}dx-\frac{x^4}{4}$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx-\frac{x^4}{4}$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{x^4}{4}$$

$$\downarrow 16$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{x^4}{4}$$

$$\downarrow 25$$

3.25. $\int \frac{x^6\left(1-x^3\right)}{1-x^3+x^6}\,dx$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{x^4}{4}$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\cdots}{x^2+\cdots}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\cdots}{x^2+\cdots}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{x^4}{4}$$

$$\Big\downarrow \; 1082$$

3.25. $\int \frac{x^6\left(1-x^3\right)}{1-x^3+x^6}\,dx$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{x^4}{4}$$

$\downarrow$ 217

3.25. $\int \frac{x^6\left(1-x^3\right)}{1-x^3+x^6}\,dx$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-\cdots\right)}}}{\sqrt{3}}\cdots\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\cdots\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+\cdots\right)}}}{\sqrt{3}}\cdots\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\cdots\right.$$

$$\frac{x^4}{4}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x\cdots\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\cdots\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x\cdots\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\cdots\right.$$

$$\frac{x^4}{4}$$

3.25. $\int \frac{x^6\left(1-x^3\right)}{1-x^3+x^6}\,dx$

input
```
Int[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/4*x^4 + ((3 + I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1
 - I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(
1/3))/Sqrt[3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x +
 2^(2/3)*x^2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3))))/6 + ((3 - I*Sqrt[3])*(Log
[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[
3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt
[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt
[3])/2)^(2/3))))/6
```

3.25.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

3.25.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.11

method	result	size
default	$-\frac{x^4}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	46
risch	$-\frac{x^4}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	46

input
```
int(x^6*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^4+1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.64

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{4}\,x^4 - \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(i\sqrt{-3}+i\right)+36\,x\right) + \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(i\sqrt{-3}-i\right)+36\,x\right) + \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(-i\sqrt{-3}+i\right)+36\,x\right) - \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(-i\sqrt{-3}-i\right)+36\,x\right) + \frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(-i\cdot 18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}+18\,x\right) + \frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(i\cdot 18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}+18\,x\right)$$

input
```
integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/4*x^4 - 1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3
)*sqrt(3)*(I*sqrt(3) - 3)^(1/3)*(I*sqrt(-3) + I) + 36*x) + 1/108*18^(2/3)*
(-I*sqrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*sqrt(3)*(-I*sqrt(3) - 3
)^(1/3)*(I*sqrt(-3) - I) + 36*x) + 1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(s
qrt(-3) - 1)*log(18^(2/3)*sqrt(3)*(I*sqrt(3) - 3)^(1/3)*(-I*sqrt(-3) + I)
+ 36*x) - 1/108*18^(2/3)*(-I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3
)*sqrt(3)*(-I*sqrt(3) - 3)^(1/3)*(-I*sqrt(-3) - I) + 36*x) + 1/54*18^(2/3)
*(I*sqrt(3) - 3)^(1/3)*log(-I*18^(2/3)*sqrt(3)*(I*sqrt(3) - 3)^(1/3) + 18*
x) + 1/54*18^(2/3)*(-I*sqrt(3) - 3)^(1/3)*log(I*18^(2/3)*sqrt(3)*(-I*sqrt(
3) - 3)^(1/3) + 18*x)
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^4}{4} - \operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(-1458t^4 + 9t + x\right)\right)\right)$$

input
```
integrate(x**6*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-x**4/4 - RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t*
*4 + 9*_t + x)))
```

3.25.7 Maxima [F]

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x^6}{x^6-x^3+1}\,dx$$

input
```
integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/4*x^4 + integrate(x^3/(x^6 - x^3 + 1), x)
```

3.25.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(272) = 544$.

Time = 0.32 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.54

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6} \, dx = \text{Too large to display}$$

input
```
integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/4*x^4 - 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9
*pi)^2 + 2*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9
*pi)*sin(4/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sq
rt(3) - 1)*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(
2*sqrt(3)*cos(2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(
3)*sin(2/9*pi)^4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)
^3 + sqrt(3)*cos(2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2
/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/
9*pi)^4 - 12*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4
 - 8*cos(1/9*pi)^3*sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos
(1/9*pi) + sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/(
(1/2*I*sqrt(3) + 1/2)*sin(1/9*pi))) - 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/
9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9
*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi)
)*log((-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3
)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(
2/9*pi)^4 + 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin
(2/9*pi) - cos(2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2
 + 1) + 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*
sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*si...
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.79

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{x^4}{4} - \frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{1/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{1/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{1/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{12}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{1/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{12}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(-(x^6*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
 36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(
3^(1/2)*12i - 36)^(1/3))/18 - x^4/4 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(-
 3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)
*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2
^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i -
3)^(4/3))/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)
*log(x - (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(
- 3^(1/2)*1i - 3)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6
)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 +
(2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^
(1/3) + 3^(5/6)*1i))/36
```

3.26 $\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$

3.26.1 Optimal result

Integrand size = 23, antiderivative size = 382

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^2}{2}+\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} - \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}} + \frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
-1/2*x^2+1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1
/2))/(1-I*3^(1/2))^(1/3)-1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1
/2))^(1/3))*3^(1/2))/(1+I*3^(1/2))^(1/3)+1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1-I*
3^(1/2))^(1/3))/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*
x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(1/3)/(1-I*3^(1/2))^(1/3)*3^(
1/2)-1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(1/3)*
3^(1/2)+1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^
(2/3))*2^(1/3)/(1+I*3^(1/2))^(1/3)*3^(1/2)
```

3.26.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.13

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^2}{2}+\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1+2\#1^4}\&\right]$$

input
```
Integrate[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/2*x^2 + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

3.26.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1826, 27, 1711, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(1-x^3)}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1826}$$

$$-\frac{1}{2}\int -\frac{2x}{x^6-x^3+1}dx-\frac{x^2}{2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x}{x^6-x^3+1}dx-\frac{x^2}{2}$$

$$\Big\downarrow \text{1711}$$

$$\frac{i\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{i\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{x^2}{2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2i\int \frac{x}{-2x^3-i\sqrt{3}+1}dx}{\sqrt{3}}+\frac{2i\int \frac{x}{-2x^3+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{x^2}{2}$$

$$\Big\downarrow \text{821}$$

$$-\frac{2i\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{3}}+\frac{2i\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{3}}-\frac{x^2}{2}$$

$$\Big\downarrow \; 16$$

$$-\frac{2i\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}+\frac{2i\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-\frac{x^2}{2}$$

$$\Big\downarrow \; 1142$$

3.26. $\int \frac{x^4\left(1-x^3\right)}{1-x^3+x^6}\,dx$

$$-\frac{2i\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\cdots\right.}{3\,2^{2/3}\sqrt[3]{1-\cdots}}\right.}{\sqrt{3}}$$

$$\frac{2i\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+\cdots\right.}{3\,2^{2/3}\sqrt[3]{1+i\cdots}}\right.}{\sqrt{3}}$$

$$\frac{x^2}{2}$$

$$\Big\downarrow\ 1082$$

3.26. $\int \frac{x^4\left(1-x^3\right)}{1-x^3+x^6}\,dx$

$$-\frac{2i\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1\cdots}}\cdots\right.}{\sqrt{3}}$$

$$\frac{2i\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1+\cdots}}\cdots\right.}{\sqrt{3}}$$

$$\frac{x^2}{2}$$

$\downarrow$ 217

3.26. $\int \frac{x^4\left(1-x^3\right)}{1-x^3+x^6}\,dx$

$$-\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}+$$

$$\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-$$

$$\frac{x^2}{2}$$

$$\Big\downarrow \text{1103}$$

3.26. $\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$

$$-\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}+\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-\frac{x^2}{2}$$

input
```
Int[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/2*x^2 - ((2*I)*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1
 - I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/
3))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1
/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3))))/Sqrt[3]
+ ((2*I)*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1 + I*Sqrt
[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/Sqrt[3]
```

3.26. $\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$

3.26.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

3.26. $\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

3.26.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.12

method	result	size
default	$-\frac{x^2}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44
risch	$-\frac{x^2}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44

input
```
int(x^4*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2+1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.79

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(i\sqrt{-3}-i)+\sqrt{-3}-1\right)\left(i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(-i\sqrt{-3}-i)-\sqrt{-3}-1\right)\left(i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(i\sqrt{-3}+i)-\sqrt{-3}-1\right)\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(-i\sqrt{-3}+i)+\sqrt{-3}-1\right)\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)-\frac{1}{2}x^2+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}+3\right)^{\frac{2}{3}}\left(i\sqrt{3}+1\right)+12\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}\left(-i\sqrt{3}+1\right)+12\,x\right)$$

input
```
integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/108*18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)
*(I*sqrt(-3) - I) + sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) + 1/108*18
^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt
(-3) - I) - sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) + 1/108*18^(2/3)*(
-I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(I*sqrt(-3) + I
) - sqrt(-3) - 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) - 1/108*18^(2/3)*(-I*sqrt
(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) + I) + sq
rt(-3) - 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) - 1/2*x^2 + 1/54*18^(2/3)*(I*sq
rt(3) + 3)^(1/3)*log(18^(1/3)*(I*sqrt(3) + 3)^(2/3)*(I*sqrt(3) + 1) + 12*x
) + 1/54*18^(2/3)*(-I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(-I*sqrt(3) + 3)^(2/
3)*(-I*sqrt(3) + 1) + 12*x)
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.08

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$$
$$= -\frac{x^2}{2} - \text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-6561t^5 - 27t^2 + x\right)\right)\right)$$

input
```
integrate(x**4*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-x**2/2 - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t*
*5 - 27*_t**2 + x)))
```

3.26.7 Maxima [F]

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x^4}{x^6-x^3+1}\,dx$$

input
```
integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/2*x^2 + integrate(x/(x^6 - x^3 + 1), x)
```

3.26.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 820 vs. 2(246) = 492.

Time = 0.35 (sec) , antiderivative size = 820, normalized size of antiderivative = 2.15

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/2*x^2 - 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*p
i)^2 + 5*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) +
 10*cos(4/9*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 +
sqrt(3)*sin(4/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3)
 - 1)*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(
3)*cos(2/9*pi)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(
2/9*pi)*sin(2/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin
(2/9*pi)^3 - sin(2/9*pi)^5 - sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2
 + 2*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2
*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10
*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4
 + 5*cos(1/9*pi)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*
pi)^5 + sqrt(3)*cos(1/9*pi)^2 - sqrt(3)*sin(1/9*pi)^2 + 2*cos(1/9*pi)*sin(
1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3)
+ 1/2)*sin(1/9*pi))) - 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt
(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 -
10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 - 2*sqrt(3)*c
os(4/9*pi)*sin(4/9*pi) - cos(4/9*pi)^2 + sin(4/9*pi)^2)*log((-I*sqrt(3)*co
s(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(
2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^...
```

3.26. $\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$

3.26.9 Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.81

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\left(81\,x-\frac{27\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\left(81\,x-\frac{27\,\left(36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{x^2}{2} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(-(x^4*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/16
2))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)
^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - x^2/
2 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3
)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) +
3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3)
)/12 - (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/
3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(
1/2)*1i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^
(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)
^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.27 $\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$

3.27.1 Optimal result

Integrand size = 23, antiderivative size = 378

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx=-x-\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}$$

output
```
-x-1/3*I*2^(2/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))/(
1-I*3^(1/2))^(2/3)+1/3*I*2^(2/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(
1/3))*3^(1/2))/(1+I*3^(1/2))^(2/3)+1/9*I*2^(2/3)*ln(-2^(1/3)*x+(1-I*3^(1/2
))^(1/3))/(1-I*3^(1/2))^(2/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I
*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(2/3)/(1-I*3^(1/2))^(2/3)*3^(1/2)-1
/9*I*2^(2/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(2/3)*3^(1/2
)+1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))
*2^(2/3)/(1+I*3^(1/2))^(2/3)*3^(1/2)
```

3.27.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.12

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -x+\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1^2+2\#1^5}\&\right]$$

input
```
Integrate[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-x + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

3.27.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1826, 25, 1685, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(1-x^3)}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{ 1826}$$

$$-\int -\frac{1}{x^6-x^3+1}dx-x$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{1}{x^6-x^3+1}dx-x$$

$$\Big\downarrow \text{1685}$$

$$-\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx}{\sqrt{3}}+\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx}{\sqrt{3}}-x$$

$$\Big\downarrow \text{750}$$

$$\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-x$$

$$\Big\downarrow \text{16}$$

$$\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-x$$

$$\Big\downarrow \text{25}$$

3.27. $\int \frac{x^3\left(1-x^3\right)}{1-x^3+x^6}\,dx$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-x$$

$\downarrow$ 1142

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\cdots}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1\cdots\right.\right.}\cdots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\cdots\right.}{\sqrt{3}}\cdots$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\cdots}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1\cdots\right.\right.}\cdots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\cdots\right.}{\sqrt{3}}\cdots$$

x

$\downarrow$ 1082

3.27. $\int \frac{x^3\left(1-x^3\right)}{1-x^3+x^6}\,dx$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{\cdots}\right.}}\cdots\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\cdots\right.}{\sqrt{3}}\cdots$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{\cdots}\right.}}\cdots\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\cdots\right.}{\sqrt{3}}\cdots$$

$$x$$

$$\downarrow 217$$

3.27. $\int \frac{x^3\left(1-x^3\right)}{1-x^3+x^6}\,dx$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-$$

$$x$$

$$\Big\downarrow \text{1103}$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-$$

$$x$$

3.27. $\int \frac{x^3\left(1-x^3\right)}{1-x^3+x^6}\,dx$

input
```
Int[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-x + (I*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 - I*Sqrt[3])/2)^(2/
3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log
[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*
((1 - I*Sqrt[3])/2)^(2/3))))/Sqrt[3] - (I*(Log[(1 + I*Sqrt[3])^(1/3) - 2^(
1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I
*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[
3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[3])/2)^(2/3))))/Sqrt[3]
```

3.27.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

3.27.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.11

method	result	size
default	$-x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	41
risch	$-x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	41

input
```
int(x^3*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.76

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}-i)+3\sqrt{-3}-3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}-i)-3\sqrt{-3}-3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}+i)-3\sqrt{-3}-3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}+i)+3\sqrt{-3}-3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{4}{3}}+36\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{4}{3}}+36\,x\right)-x$$

input
```
integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/108*18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*
(I*sqrt(-3) - I) + 3*sqrt(-3) - 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*1
8^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(-I*sqr
t(-3) - I) - 3*sqrt(-3) - 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*18^(2/3
)*(-I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3)
+ I) - 3*sqrt(-3) - 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/108*18^(2/3)*(-I
*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) + I)
 + 3*sqrt(-3) - 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/54*18^(2/3)*(I*sqrt(
3) + 3)^(1/3)*log(18^(2/3)*(I*sqrt(3) + 3)^(4/3) + 36*x) + 1/54*18^(2/3)*(
-I*sqrt(3) + 3)^(1/3)*log(18^(2/3)*(-I*sqrt(3) + 3)^(4/3) + 36*x) - x
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -x - \operatorname{RootSum}\left(19683t^6+243t^3+1, \left(t \mapsto t\log\left(729t^4+x\right)\right)\right)$$

input
```
integrate(x**3*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-x - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x)
))
```

3.27.7 Maxima [F]

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x^3}{x^6-x^3+1}\,dx$$

input
```
integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-x + integrate(1/(x^6 - x^3 + 1), x)
```

3.27.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 635 vs. 2(244) = 488.

Time = 0.37 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.68

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt
(3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi
)^3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(
4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9
*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4
*cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9
*pi) - sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*
I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*co
s(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/
9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*a
rctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin
(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9
*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4
/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi)
- cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) -
4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(
2/9*pi)^2 - sin(2/9*pi)^4 - sqrt(3)*sin(2/9*pi) + cos(2/9*pi))*log((-I*sqr
t(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)
^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*c
os(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3)*sin(1/9*pi) + cos(...
```

3.27. $\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$

3.27.9 Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.87

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$$
$$= -x + \frac{\ln\left(x + \frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4} - \frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{12}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+ \frac{\ln\left(x + \frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4} + \frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{12}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$- \frac{2^{2/3}\ln\left(x - \frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2} + \frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\ln\left(x - \frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2} + \frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\ln\left(x + \frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\ln\left(x - \frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(-(x^3*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3
- 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 - x + (log(x + (2
^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i +
3)^(1/3)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*
3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3
))/12)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x
- (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1
i + 3)^(4/3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(
2/3)*log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1
i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3
^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))
/36
```

3.28 $\int \frac{x(1-x^3)}{1-x^3+x^6} \, dx$

3.28.1 Optimal result

Integrand size = 21, antiderivative size = 411

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^
(1/3)/(1-I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3^(
1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/
2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1
/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))
^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/
3))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(1/3)*
x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1+I*3^(1/2))^(1/3)
```

3.28.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.13

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1+2\#1^4}\&\right]$$

input
```
Integrate[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1 + 
2*#1^4) & ]
```

3.28.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1834, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-x^3)}{x^6-x^3+1}\,dx$$

$$\Big\downarrow \text{1834}$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{x}{-2x^3-i\sqrt{3}+1}dx+\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{x}{-2x^3+i\sqrt{3}+1}dx$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)+$$
$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)$$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+$$
$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)$$

$$\Big\downarrow 1142$$

3.28. $\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\text{lo}}{}\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\text{lo}}{}\right.$$

$\downarrow$ 1082

3.28. $\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\cdots\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\cdots\right.$$

$$\downarrow \ 217$$

3.28. $\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\;2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right.}{3\;2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\;2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right.}{3\;2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right.$$

$\downarrow$ 1103

3.28. $\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1-i\cdots}}\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1+i\cdots}}\right.$$

input
```
Int[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
((3 - I*Sqrt[3])*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1
- I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3
))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/
3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3))))/3 + ((3 +
 I*Sqrt[3])*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1 + I*S
qrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
+ 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/3
```

3.28. $\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$

3.28.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.28. $\int \frac{x(1-x^3)}{1-x^3+x^6} \, dx$

3.28.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

method	result	size
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(\_R^{4}-\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(-\_R^{4}+\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44

input
```
int(x*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.58

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}-3\right)^{\frac{2}{3}}(\sqrt{-3}+1)+12\,x\right) + \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(18^{\frac{1}{3}}\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}(\sqrt{-3}+1)+12\,x\right) - \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(-18^{\frac{1}{3}}\left(i\sqrt{3}-3\right)^{\frac{2}{3}}(\sqrt{-3}-1)+12\,x\right) - \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(-18^{\frac{1}{3}}\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}(\sqrt{-3}-1)+12\,x\right) + \frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(6\,x-18^{\frac{1}{3}}\left(i\sqrt{3}-3\right)^{\frac{2}{3}}\right) + \frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(6\,x-18^{\frac{1}{3}}\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}\right)$$

input
```
integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(I*sqrt(3
) - 3)^(2/3)*(sqrt(-3) + 1) + 12*x) + 1/108*18^(2/3)*(-I*sqrt(3) - 3)^(1/3
)*(sqrt(-3) - 1)*log(18^(1/3)*(-I*sqrt(3) - 3)^(2/3)*(sqrt(-3) + 1) + 12*x
) - 1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(-18^(1/3)*(I*s
qrt(3) - 3)^(2/3)*(sqrt(-3) - 1) + 12*x) - 1/108*18^(2/3)*(-I*sqrt(3) - 3)
^(1/3)*(sqrt(-3) + 1)*log(-18^(1/3)*(-I*sqrt(3) - 3)^(2/3)*(sqrt(-3) - 1)
+ 12*x) + 1/54*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*log(6*x - 18^(1/3)*(I*sqrt(3
) - 3)^(2/3)) + 1/54*18^(2/3)*(-I*sqrt(3) - 3)^(1/3)*log(6*x - 18^(1/3)*(-
I*sqrt(3) - 3)^(2/3))
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.05

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = -\operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(-27t^2+x\right)\right)\right)$$

input
```
integrate(x*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x)))
```

3.28.7 Maxima [F]

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x}{x^6-x^3+1}\,dx$$

input
```
integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-integrate((x^3 - 1)*x/(x^6 - x^3 + 1), x)
```

3.28.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(267) = 534.

Time = 0.37 (sec) , antiderivative size = 824, normalized size of antiderivative = 2.00

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output

```
1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sq
rt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9
*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)
*sin(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*c
os(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(
2/9*pi)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)
*sin(2/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi
)^3 - sin(2/9*pi)^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 -
4*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)
/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sq
rt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 +
5*cos(1/9*pi)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)
^5 - 2*sqrt(3)*cos(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin
(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3)
 + 1/2)*sin(1/9*pi))) + 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqr
t(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 -
 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 + 4*sqrt(3)*
cos(4/9*pi)*sin(4/9*pi) + 2*cos(4/9*pi)^2 - 2*sin(4/9*pi)^2)*log((-I*sqrt(
3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(2/9*pi)^4
*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9...
```

3.28. $\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$

3.28.9 Mupad [B] (verification not implemented)

Time = 10.35 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.68

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\frac{\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{12}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(-(x*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
(log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1
/3))/18 + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 36)^(1/
3))/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(
5/6)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(
2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/2
4)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x -
(2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i
- 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)
*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/
3) + 3^(5/6)*1i))/36
```

3.29 $\int \frac{1-x^3}{1-x^3+x^6} \, dx$

3.29.1 Optimal result

Integrand size = 20, antiderivative size = 411

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = -\frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output

```
-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2
^(2/3)/(1-I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3^
(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1
/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-
1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2)
)^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2
/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2*2^(1/3)
*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

3.29.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.14

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = -\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1^2+2\#1^5}\&\right]$$

input
```
Integrate[(1 - x^3)/(1 - x^3 + x^6),x]
```

output
```
-1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2
+ 2*#1^5) & ]
```

3.29.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1752, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^3}{x^6-x^3+1}\,dx$$

$$\Big\downarrow \text{1752}$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx-\frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx$$

$$\Big\downarrow \text{750}$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$\downarrow$ 16

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$\downarrow$ 25

3.29. $\int \frac{1-x^3}{1-x^3+x^6}\,dx$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$\downarrow$ 1142

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{}{x^2+}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{}{x^2+\sqrt[3]{}}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$\downarrow$ 1082

3.29. $\int \frac{1-x^3}{1-x^3+x^6}\,dx$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\cdots\right.}\cdots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\cdots\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\cdots\right.}\cdots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\cdots\right.$$

↓ 217

3.29. $\int \frac{1-x^3}{1-x^3+x^6}\,dx$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\ldots}{\ldots}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\ldots}{\sqrt{3}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\ldots\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x\ldots\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

input
```
Int[(1 - x^3)/(1 - x^3 + x^6),x]
```

3.29. $\int \frac{1-x^3}{1-x^3+x^6}\,dx$

output
```
-1/6*((3 - I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 - I*S
qrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/
Sqrt[3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/
3)*x^2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3)))) - ((3 + I*Sqrt[3])*(Log[(1 + I*
Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTa
n[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt[3])^(2/
3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[3])/2)^
(2/3))))/6
```

3.29.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.29.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(-\_R^{3}+1\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(-\_R^{3}+1\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44

input
```
int((-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.73

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}-i)+3\sqrt{-3}-3\right)\left(i\sqrt{3}-3\right)^{\frac{1}{3}}+72x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}-i)-3\sqrt{-3}-3\right)\left(i\sqrt{3}-3\right)^{\frac{1}{3}}+72x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}+i)-3\sqrt{-3}-3\right)\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}+72x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}+i)+3\sqrt{-3}-3\right)\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}+72x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)\left(i\sqrt{3}-3\right)^{\frac{1}{3}}+36x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}+36x\right)$$

input
```
integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*
(I*sqrt(-3) - I) + 3*sqrt(-3) - 3)*(I*sqrt(3) - 3)^(1/3) + 72*x) - 1/108*1
8^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(-I*sqr
t(-3) - I) - 3*sqrt(-3) - 3)*(I*sqrt(3) - 3)^(1/3) + 72*x) - 1/108*18^(2/3
)*(-I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3)
+ I) - 3*sqrt(-3) - 3)*(-I*sqrt(3) - 3)^(1/3) + 72*x) + 1/108*18^(2/3)*(-I
*sqrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) + I)
 + 3*sqrt(-3) - 3)*(-I*sqrt(3) - 3)^(1/3) + 72*x) + 1/54*18^(2/3)*(I*sqrt(
3) - 3)^(1/3)*log(18^(2/3)*(I*sqrt(3) + 3)*(I*sqrt(3) - 3)^(1/3) + 36*x) +
 1/54*18^(2/3)*(-I*sqrt(3) - 3)^(1/3)*log(18^(2/3)*(-I*sqrt(3) + 3)*(-I*sq
rt(3) - 3)^(1/3) + 36*x)
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = -\operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(729t^4-9t+x\right)\right)\right)$$

input
```
integrate((-x**3+1)/(x**6-x**3+1),x)
```

output
```
-RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t +
 x)))
```

3.29.7 Maxima [F]

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \int -\frac{x^3-1}{x^6-x^3+1}\,dx$$

input
```
integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

3.29.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(267) = 534.

Time = 0.44 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.56

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)
^3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*c
os(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(
2/9*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4
+ 4*cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*co
s(2/9*pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)
/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqr
t(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3
*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin
(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3)
 + 1/2)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt
(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi
)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt(
3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(2/9*pi)^3
*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos
(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2
/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(4*
sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 +
cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt(...
```

3.29. $\int \frac{1-x^3}{1-x^3+x^6} \, dx$

3.29.9 Mupad [B] (verification not implemented)

Time = 10.35 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.78

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+\frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(-(x^3 - 1)/(x^6 - x^3 + 1),x)
```

output
```
(log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/
2)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i -
36)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(-
 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i + 3)*(3^(1/3
) + 3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5
/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) -
3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16 - 27))/108)*
(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2
/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1
/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)
^(1/3)*1i)/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.30 $\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx$

3.30.1 Optimal result

Integrand size = 23, antiderivative size = 416

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = -\frac{1}{x} - \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
-1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2
))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-
I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*
3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1
/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(
1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2)
)^(2/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(
1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1/2))^(1/
3)
```

3.30.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.11

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx=-\frac{1}{x}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

output
```
-x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]
/3
```

3.30.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1828, 1710, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^3}{x^2\left(x^6-x^3+1\right)}dx$$

$$\Big\downarrow \ 1828$$

$$-\int \frac{x^4}{x^6-x^3+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{1710}$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx-\frac{1}{6}\left(3-i\sqrt{3}\right)\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{x}{-2x^3-i\sqrt{3}+1}dx+\frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)+\frac{1}{3}\left(3-i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)-\frac{1}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)-\frac{1}{x}$$

$$\Big\downarrow \text{1142}$$

3.30. $\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\ldots\text{lo}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\ldots\text{lo}\right.$$

$$\frac{1}{x}$$

$\downarrow$ 1082

3.30. $\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{x}$$

$$\Big\downarrow \text{ 217}$$

3.30. $\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{}}\right.}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{}}\right.}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right.$$

$$\frac{1}{x}$$

$$\downarrow 1103$$

3.30. $\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-\right.}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{}}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+\right.}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{}}}\right.$$

$$\frac{1}{x}$$

input
```
Int[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

output
```
-x^(-1) + ((3 + I*Sqrt[3])*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2
^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3
])/2)^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqr
t[3]))^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3))))
/3 + ((3 - I*Sqrt[3])*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3
)*(1 + I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)
^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/3
```

3.30. $\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx$

3.30.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.30.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.10

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^{6}-9\_Z^{3}+1\right)}\_R\ln\left(-27\_R^{5}+6\_R^{2}+x\right)\right)}{3}$	40
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}-\frac{1}{x}$	46

input
```
int((-x^3+1)/x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/3*sum(_R*ln(-27*_R^5+6*_R^2+x),_R=RootOf(27*_Z^6-9*_Z^3+1))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.75

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx$$
$$= \frac{18^{\frac{2}{3}}\left(\sqrt{-3}x-x\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(i\sqrt{-3}+i\right)-\sqrt{-3}-1\right)\left(i\sqrt{3}+3\right)^{\frac{2}{3}}+24x\right)-18^{\frac{2}{3}}\left(\sqrt{-3}x+\right.}{}$$

input
```
integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/108*(18^(2/3)*(sqrt(-3)*x - x)*(I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(
3)*(I*sqrt(-3) + I) - sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) - 18^(2/
3)*(sqrt(-3)*x + x)*(I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-
3) + I) + sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) - 18^(2/3)*(sqrt(-3)
*x + x)*(-I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(I*sqrt(-3) - I) + sq
rt(-3) - 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) + 18^(2/3)*(sqrt(-3)*x - x)*(-I
*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) - I) - sqrt(-3) - 1
)*(-I*sqrt(3) + 3)^(2/3) + 24*x) + 2*18^(2/3)*x*(-I*sqrt(3) + 3)^(1/3)*log
(18^(1/3)*(I*sqrt(3) + 1)*(-I*sqrt(3) + 3)^(2/3) + 12*x) + 2*18^(2/3)*x*(I
*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(I*sqrt(3) + 3)^(2/3)*(-I*sqrt(3) + 1) +
12*x) - 108)/x
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx = -\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right) - \frac{1}{x}$$

input
```
integrate((-x**3+1)/x**2/(x**6-x**3+1),x)
```

output
```
-RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t
**2 + x))) - 1/x
```

3.30.7 Maxima [F]

$$\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx = \int -\frac{x^3-1}{(x^6-x^3+1)x^2}\,dx$$

input
```
integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/x - integrate(x^4/(x^6 - x^3 + 1), x)
```

3.30.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 832 vs. 2(272) = 544.

Time = 0.32 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.00

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = \text{Too large to display}$$

input
```
integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos
(4/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(
3)*sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)
*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*
cos(2/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/
9*pi)*sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(
2/9*pi)^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^
2 - 2*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) +
2*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 -
 20*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*p
i)^4 + 10*cos(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*s
in(1/9*pi)^5 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*p
i)*sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*s
qrt(3) + 1/2)*sin(1/9*pi))) + 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) -
 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos(
4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4
+ 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log((
-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(
2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqrt...
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.75

$$\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx = \frac{\ln\left(-x+\left(162\,x+\frac{27\left(36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(-x-\left(162\,x+\frac{27\left(36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{1}{x} - \frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}1\mathrm{i}}{4}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}1\mathrm{i}}{4}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(-(x^3 - 1)/(x^2*(x^6 - x^3 + 1)),x)
```

output
```
(log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162)
- x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*
12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 -
1/x - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 - (2^(1
/3)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3)
- 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/
3))/12 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(
1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3
^(1/2)*1i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (
2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i +
3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

3.31 $\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx$

3.31.1 Optimal result

Integrand size = 23, antiderivative size = 418

$$\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx = -\frac{1}{2x^2} + \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output
```
-1/2/x^2-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^
(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))
*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(
1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2)
)^(2/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I
*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(
1/2))^(2/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2
*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))
^(2/3)
```

3.31. $\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx$

3.31.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.11

$$\int \frac{1-x^3}{x^3 (1-x^3+x^6)} \, dx = -\frac{1}{2x^2} - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

output
```
-1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) &
]/3
```

3.31.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1828, 27, 1710, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^3}{x^3 (x^6-x^3+1)} \, dx$$

$$\Big\downarrow \text{1828}$$

$$-\frac{1}{2}\int \frac{2x^3}{x^6-x^3+1}dx-\frac{1}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^3}{x^6-x^3+1}dx-\frac{1}{2x^2}$$

$$\Big\downarrow \text{1710}$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx-\frac{1}{2x^2}$$

$$\Big\downarrow \text{750}$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{2x^2}$$

$$\Big\downarrow \text{25}$$

3.31. $\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{2x^2}$$

$$\Big\downarrow \; 1142$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int \cdots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int \cdots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

$$\Big\downarrow \; 1082$$

3.31. $\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\cdots\right.}\cdots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.\cdots$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\cdots\right.}\cdots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.\cdots$$

$$\frac{1}{2x^2}$$

$\downarrow$ 217

3.31. $\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\cdots}{\cdots}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+\cdots\right.}}}{\sqrt{3}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

$\downarrow$ 1103

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)\cdots}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x\cdots\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

3.31. $\int\frac{1-x^3}{x^3\left(1-x^3+x^6\right)}dx$

input
```
Int[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

output
```
-1/2*1/x^2 - ((3 + I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*(
(1 - I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)
^(1/3))/Sqrt[3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x
 + 2^(2/3)*x^2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3))))/6 - ((3 - I*Sqrt[3])*(L
og[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqr
t[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sq
rt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sq
rt[3])/2)^(2/3))))/6
```

3.31.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.31.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.09

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^6-9\_Z^3+1\right)}\_R\ln\left(-18\_R^4+3\_R+x\right)\right)}{3}$	38
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}-\frac{1}{2x^2}$	46

input
```
int((-x^3+1)/x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+1/3*sum(_R*ln(-18*_R^4+3*_R+x),_R=RootOf(27*_Z^6-9*_Z^3+1))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.72

$$\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx$$
$$= \frac{2\cdot 18^{\frac{2}{3}}x^2\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(-i\cdot 18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+18\,x\right)+2\cdot 18^{\frac{2}{3}}x^2\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(i\cdot 18^{\frac{2}{3}}\sqrt{3}(-i\sqrt{3}\right.}{}$$

input
```
integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/108*(2*18^(2/3)*x^2*(I*sqrt(3) + 3)^(1/3)*log(-I*18^(2/3)*sqrt(3)*(I*sqr
t(3) + 3)^(1/3) + 18*x) + 2*18^(2/3)*x^2*(-I*sqrt(3) + 3)^(1/3)*log(I*18^(
2/3)*sqrt(3)*(-I*sqrt(3) + 3)^(1/3) + 18*x) - 18^(2/3)*(sqrt(-3)*x^2 + x^2
)*(I*sqrt(3) + 3)^(1/3)*log(18^(2/3)*sqrt(3)*(I*sqrt(3) + 3)^(1/3)*(I*sqrt
(-3) + I) + 36*x) + 18^(2/3)*(sqrt(-3)*x^2 - x^2)*(-I*sqrt(3) + 3)^(1/3)*l
og(18^(2/3)*sqrt(3)*(-I*sqrt(3) + 3)^(1/3)*(I*sqrt(-3) - I) + 36*x) + 18^(
2/3)*(sqrt(-3)*x^2 - x^2)*(I*sqrt(3) + 3)^(1/3)*log(18^(2/3)*sqrt(3)*(I*sq
rt(3) + 3)^(1/3)*(-I*sqrt(-3) + I) + 36*x) - 18^(2/3)*(sqrt(-3)*x^2 + x^2)
*(-I*sqrt(3) + 3)^(1/3)*log(18^(2/3)*sqrt(3)*(-I*sqrt(3) + 3)^(1/3)*(-I*sq
rt(-3) - I) + 36*x) - 54)/x^2
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.08

$$\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx = -\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(-1458t^4-9t+x\right)\right)\right) - \frac{1}{2x^2}$$

input
```
integrate((-x**3+1)/x**3/(x**6-x**3+1),x)
```

output
```
-RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t
 + x))) - 1/(2*x**2)
```

3.31.7 Maxima [F]

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = \int -\frac{x^3-1}{(x^6-x^3+1)x^3}\,dx$$

input
```
integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/2/x^2 - integrate(x^3/(x^6 - x^3 + 1), x)
```

3.31.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(272) = 544$.

Time = 0.32 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.54

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = \text{Too large to display}$$

input
```
integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*
sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/
9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*
cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*c
os(2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*
pi)^4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3
)*cos(2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*
x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 1
2*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/
9*pi)^3*sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) +
sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt
(3) + 1/2)*sin(1/9*pi))) + 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*s
qrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin(
4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi))*log((-I*s
qrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(2/9*p
i)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 +
 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) -
cos(2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/1
8*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)
^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sin(1/9*pi)^4...
```

3.31. $\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx$

3.31.9 Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.79

$$\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx$$
$$= \frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{1}{2\,x^2}$$
$$- \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(-(x^3 - 1)/(x^3*(x^6 - x^3 + 1)),x)
```

output
```
(log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(36 - 3^(1/2)*12i)
^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(1
/2)*12i + 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3
 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3
- 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)
*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/
3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x
 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 - 3^(1
/2)*1i)^(1/3)*1i)/12)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 -
(2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(
5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5
/6)*1i))/36
```

3.32 $\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx$

3.32.1 Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = \frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
1/6*ln(x^6-x^3+1)+1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = -\frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

input
```
Integrate[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

output
```
-(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

3.32.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1798, 25, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (x^3-2)}{x^6-x^3+1} \, dx$$

$$\Big\downarrow \text{ 1798}$$

$$\frac{1}{3} \int -\frac{2-x^3}{x^6-x^3+1}dx^3$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{3} \int \frac{2-x^3}{x^6-x^3+1}dx^3$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{3} \left(\frac{1}{2} \int -\frac{1-2x^3}{x^6-x^3+1}dx^3-\frac{3}{2} \int \frac{1}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3} \left(-\frac{3}{2} \int \frac{1}{x^6-x^3+1}dx^3-\frac{1}{2} \int \frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{3} \left(3 \int \frac{1}{-x^6-3}d\left(2x^3-1\right)-\frac{1}{2} \int \frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{3} \left(-\frac{1}{2} \int \frac{1-2x^3}{x^6-x^3+1}dx^3-\sqrt{3} \arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{3} \left(\frac{1}{2} \log\left(x^6-x^3+1\right)-\sqrt{3} \arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)\right)$$

input
```
Int[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

output
```
(-(Sqrt[3]*ArcTan[(-1 + 2*x^3)/Sqrt[3]]) + Log[1 - x^3 + x^6]/2)/3
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.32.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\ln(x^6-x^3+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3}$	33
risch	$\frac{\ln(4x^6-4x^3+4)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3}$	35

input
```
int(x^2*(x^3-2)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(x^6-x^3+1)-1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = \frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x**2*(x**3-2)/(x**6-x**3+1),x)
```

output
```
log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/3
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{3}$$

input
```
int((x^2*(x^3 - 2))/(x^6 - x^3 + 1),x)
```

output
```
log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/3
```

3.33 $\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx$

3.33.1 Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx=-\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
ln(x)-1/6*ln(x^6-x^3+1)-1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

3.33.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

$$\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx=\log(x)-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-2\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

output
```
Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(
-1 + 2*#1^3) & ]/3
```

3.33.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+1}{x\left(x^6-x^3+1\right)}dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{3}\int \frac{x^3+1}{x^3\left(x^6-x^3+1\right)}dx^3$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3}\int \left(\frac{2-x^3}{x^6-x^3+1}+\frac{1}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)+\log\left(x^3\right)-\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

output
```
(-(Sqrt[3]*ArcTan[(1 - 2*x^3)/Sqrt[3]]) + Log[x^3] - Log[1 - x^3 + x^6]/2)
/3
```

3.33.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.33.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{3}-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{3}+\ln\left(x\right)$	35

input
```
int((x^3+1)/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx=\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)-\frac{1}{6}\,\log\left(x^6-x^3+1\right)+\log\left(x\right)$$

input
```
integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(x)
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx = \log(x) - \frac{\log(x^6-x^3+1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**3+1)/x/(x**6-x**3+1),x)
```

output
```
log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/3
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \frac{1}{3}\log\left(x^3\right)$$

input
```
integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3
*log(x^3)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log(|x|)$$

input
```
integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(abs(x))
```

3.33.9 Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx = \ln(x) - \frac{\ln(x^6-x^3+1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^3}{3}\right)}{3}$$

input
```
int((x^3 + 1)/(x*(x^6 - x^3 + 1)),x)
```

output
```
log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/
3))/3
```

3.34 $\int \frac{1+x^3}{x-x^4+x^7} \, dx$

3.34.1 Optimal result

Integrand size = 18, antiderivative size = 39

$$\int \frac{1+x^3}{x-x^4+x^7} \, dx = -\frac{\arctan \left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{1}{6} \log \left(1-x^3+x^6\right)$$

output
```
ln(x)-1/6*ln(x^6-x^3+1)-1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

3.34.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

$$\int \frac{1+x^3}{x-x^4+x^7} \, dx = \log(x) - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{-2\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(1 + x^3)/(x - x^4 + x^7),x]
```

output
```
Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(
-1 + 2*#1^3) & ]/3
```

3.34.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1979, 1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+1}{x^7-x^4+x} \, dx$$

$$\Big\downarrow \text{1979}$$

$$\int \frac{x^3+1}{x \left(x^6-x^3+1\right)}dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{3} \int \frac{x^3+1}{x^3 \left(x^6-x^3+1\right)}dx^3$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3} \int \left(\frac{2-x^3}{x^6-x^3+1}+\frac{1}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\sqrt{3} \arctan\left(\frac{1-2 x^3}{\sqrt{3}}\right)+\log\left(x^3\right)-\frac{1}{2} \log\left(x^6-x^3+1\right)\right)$$

input
```
Int[(1 + x^3)/(x - x^4 + x^7),x]
```

output
```
(-(Sqrt[3]*ArcTan[(1 - 2*x^3)/Sqrt[3]]) + Log[x^3] - Log[1 - x^3 + x^6]/2)
/3
```

3.34.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1979
```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) +
(B_.)*(x_)^(r_.)), x_Symbol] :> Int[x^(p*q)*(A + B*x^(n - q))*(a + b*x^(n -
 q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[r, n
 - q] && EqQ[j, 2*n - q] && IntegerQ[p] && PosQ[n - q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.34.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{3}-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{3}+\ln\left(x\right)$	35

input
```
int((x^3+1)/(x^7-x^4+x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx=\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^{3}-1)\right)-\frac{1}{6}\,\log\left(x^{6}-x^{3}+1\right)+\log\left(x\right)$$

input
```
integrate((x^3+1)/(x^7-x^4+x),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(x)
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \log(x) - \frac{\log(x^6-x^3+1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**3+1)/(x**7-x**4+x),x)
```

output
```
log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/3
```

3.34.7 Maxima [F]

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \int \frac{x^3+1}{x^7-x^4+x}\,dx$$

input
```
integrate((x^3+1)/(x^7-x^4+x),x, algorithm="maxima")
```

output
```
-integrate((x^5 - 2*x^2)/(x^6 - x^3 + 1), x) + log(x)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log(|x|)$$

input
```
integrate((x^3+1)/(x^7-x^4+x),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(abs(x))
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \ln(x) - \frac{\ln(x^6-x^3+1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^3}{3}\right)}{3}$$

input
```
int((x^3 + 1)/(x - x^4 + x^7),x)
```

output
```
log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/
3))/3
```

3.35 $\int (d+ex^3)^{5/2} (a+bx^3+cx^6)\,dx$

3.35.1 Optimal result

Integrand size = 24, antiderivative size = 396

$$\int (d+ex^3)^{5/2} (a+bx^3+cx^6)\,dx = \frac{54d^2(16cd^2-58bde+667ae^2)\,x\sqrt{d+ex^3}}{124729e^2} + \frac{30d(16cd^2-58bde+667ae^2)\,x(d+ex^3)^{3/2}}{124729e^2} + \frac{2(16cd^2-58bde+667ae^2)\,x(d+ex^3)^{5/2}}{11339e^2} - \frac{2(8cd-29be)x(d+ex^3)^{7/2}}{667e^2} + \frac{2cx^4(d+ex^3)^{7/2}}{29e} + \frac{54\;3^{3/4}\sqrt{2+\sqrt{3}}d^3(16cd^2-58bde+667ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{(1-\sqrt{\ }}{(1+\sqrt{\ }}\right.\right.}{124729e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output

```
30/124729*d*(667*a*e^2-58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(3/2)/e^2+2/11339*(6
67*a*e^2-58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(5/2)/e^2-2/667*(-29*b*e+8*c*d)*x*
(e*x^3+d)^(7/2)/e^2+2/29*c*x^4*(e*x^3+d)^(7/2)/e+54/124729*d^2*(667*a*e^2-
58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+54/124729*3^(3/4)*d^3*(667*a*e^2-
58*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(
1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/
2)))^2)^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3
)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.35.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.26

$$\int \left(d+ex^3\right)^{5/2}\left(a+bx^3+cx^6\right)\,dx = \frac{x\sqrt{d+ex^3}\left(-2\left(d+ex^3\right)^3\left(8cd-29be-23cex^3\right)+\frac{\left(16cd^4+29d^2e(-2bd+23ae)\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{3},\frac{4}{3},\right.}{\sqrt{1+\frac{ex^3}{d}}}\right.}{667e^2}$$

input
```
Integrate[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```

output
```
(x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)^3*(8*c*d - 29*b*e - 23*c*e*x^3) + ((16*
c*d^4 + 29*d^2*e*(-2*b*d + 23*a*e))*Hypergeometric2F1[-5/2, 1/3, 4/3, -((e
*x^3)/d)])/Sqrt[1 + (e*x^3)/d]))/(667*e^2)
```

3.35.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1741, 27, 913, 748, 748, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^3\right)^{5/2}\left(a+bx^3+cx^6\right)\,dx$$

$$\Big\downarrow \ 1741$$

$$\frac{2\int \frac{1}{2}\left(ex^3+d\right)^{5/2}\left(29ae-(8cd-29be)x^3\right)dx}{29e}+\frac{2cx^4\left(d+ex^3\right)^{7/2}}{29e}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \left(ex^3+d\right)^{5/2}\left(29ae-(8cd-29be)x^3\right)dx}{29e}+\frac{2cx^4\left(d+ex^3\right)^{7/2}}{29e}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{\left(16cd^2-29e(2bd-23ae)\right)\int \left(ex^3+d\right)^{5/2}dx}{23e}-\frac{2x\left(d+ex^3\right)^{7/2}(8cd-29be)}{23e}}{29e}+\frac{2cx^4\left(d+ex^3\right)^{7/2}}{29e}$$

$$\Big\downarrow \ 748$$

$$\frac{\frac{(16cd^2-29e(2bd-23ae))\left(\frac{15}{17}d\int (ex^3+d)^{3/2}dx+\frac{2}{17}x(d+ex^3)^{5/2}\right)}{23e}-\frac{2x(d+ex^3)^{7/2}(8cd-29be)}{23e}}{29e}+\frac{2cx^4(d+ex^3)^{7/2}}{29e}$$

$$\Big\downarrow \ 748$$

$$\frac{\frac{(16cd^2-29e(2bd-23ae))\left(\frac{15}{17}d\left(\frac{9}{11}d\int \sqrt{ex^3+d}dx+\frac{2}{11}x(d+ex^3)^{3/2}\right)+\frac{2}{17}x(d+ex^3)^{5/2}\right)}{23e}-\frac{2x(d+ex^3)^{7/2}(8cd-29be)}{23e}}{29e}+\frac{2cx^4(d+ex^3)^{7/2}}{29e}$$

$$\Big\downarrow \ 748$$

$$\frac{\frac{(16cd^2-29e(2bd-23ae))\left(\frac{15}{17}d\left(\frac{9}{11}d\left(\frac{3}{5}d\int \frac{1}{\sqrt{ex^3+d}}dx+\frac{2}{5}x\sqrt{d+ex^3}\right)+\frac{2}{11}x(d+ex^3)^{3/2}\right)+\frac{2}{17}x(d+ex^3)^{5/2}\right)}{23e}-\frac{2x(d+ex^3)^{7/2}(8cd-29be)}{23e}}{29e}+\frac{2cx^4(d+ex^3)^{7/2}}{29e}$$

$$\Big\downarrow \ 759$$

$$\frac{\frac{(16cd^2-29e(2bd-23ae))\left(\frac{15}{17}d\left(\frac{9}{11}d\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}+\right.\right.\right.}{23e}}{29e}$$

$$\frac{2cx^4(d+ex^3)^{7/2}}{29e}$$

input
```
Int[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```

3.35. $\int (d+ex^3)^{5/2}(a+bx^3+cx^6)\,dx$

output
```
(2*c*x^4*(d + e*x^3)^(7/2))/(29*e) + ((-2*(8*c*d - 29*b*e)*x*(d + e*x^3)^(
7/2))/(23*e) + ((16*c*d^2 - 29*e*(2*b*d - 23*a*e))*((2*x*(d + e*x^3)^(5/2)
)/17 + (15*d*((2*x*(d + e*x^3)^(3/2))/11 + (9*d*((2*x*Sqrt[d + e*x^3])/5 +
 (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*d*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1
/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^
(1/3)*x)], -7 - 4*Sqrt[3]])/(5*e^(1/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x)
)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])))/11))/17))/(23*
e))/(29*e)
```

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

3.35.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.10

method	result
risch	$\frac{2x(4301e^4cx^{12}+5423be^4x^9+11407de^3cx^9+7337ae^4x^6+15631bde^3x^6+8591cd^2e^2x^6+24679de^3ax^3+141123bd^2e^2x^3+405d^3ecx^3-}{124729e^2}$
elliptic	$\frac{2ce^2x^{13}\sqrt{ex^3+d}}{29}+\frac{2\left(be^3+\frac{61}{29}cde^2\right)x^{10}\sqrt{ex^3+d}}{23e}+\frac{2\left(ae^3+3de^2b+3cd^2e-\frac{20d\left(be^3+\frac{61}{29}cde^2\right)}{23e}\right)x^7\sqrt{ex^3+d}}{17e}+\frac{2\left(3de^2a+3bd^2\right.}{}$
default	Expression too large to display

input
```
int((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/124729/e^2*x*(4301*c*e^4*x^12+5423*b*e^4*x^9+11407*c*d*e^3*x^9+7337*a*e^
4*x^6+15631*b*d*e^3*x^6+8591*c*d^2*e^2*x^6+24679*a*d*e^3*x^3+14123*b*d^2*e
^2*x^3+405*c*d^3*e*x^3+35351*a*d^2*e^2+2349*b*d^3*e-648*c*d^4)*(e*x^3+d)^(
1/2)-54/124729*I*d^3*(667*a*e^2-58*b*d*e+16*c*d^2)/e^3*3^(1/2)*(-d*e^2)^(1
/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(
-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*
3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)
/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*
3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^
2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))
```

3.35.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.43

$$\int \left(d+ex^3\right)^{5/2}\left(a+bx^3+cx^6\right)\,dx = \frac{2\left(81\left(16\,cd^5-58\,bd^4e+667\,ad^3e^2\right)\sqrt{e}\,\text{weierstrassPInverse}\left(0,-\frac{4d}{e},x\right)+\left(4301\,ce^5x^{13}+187\left(61\right.\right.\right.}{}$$

input
```
integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
2/124729*(81*(16*c*d^5 - 58*b*d^4*e + 667*a*d^3*e^2)*sqrt(e)*weierstrassPI
nverse(0, -4*d/e, x) + (4301*c*e^5*x^13 + 187*(61*c*d*e^4 + 29*b*e^5)*x^10
 + 11*(781*c*d^2*e^3 + 1421*b*d*e^4 + 667*a*e^5)*x^7 + (405*c*d^3*e^2 + 14
123*b*d^2*e^3 + 24679*a*d*e^4)*x^4 - (648*c*d^4*e - 2349*b*d^3*e^2 - 35351
*a*d^2*e^3)*x)*sqrt(e*x^3 + d))/e^3
```

3.35.6 Sympy [A] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.01

$$\int \left(d+ex^3\right)^{5/2}\left(a+bx^3+cx^6\right)\,dx = \frac{ad^{\frac{5}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2ad^{\frac{3}{2}}ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a\sqrt{d}e^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{bd^{\frac{5}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2bd^{\frac{3}{2}}ex^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{b\sqrt{d}e^2x^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{13}{3}\right)} + \frac{cd^{\frac{5}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{2cd^{\frac{3}{2}}ex^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{13}{3}\right)} + \frac{c\sqrt{d}e^2x^{13}\Gamma\left(\frac{13}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{3}\\ \frac{16}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{16}{3}\right)}$$

input
```
integrate((e*x**3+d)**(5/2)*(c*x**6+b*x**3+a),x)
```

output
```
a*d**(5/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/
d)/(3*gamma(4/3)) + 2*a*d**(3/2)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3
,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + a*sqrt(d)*e**2*x**7*gamma(7
/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3))
+ b*d**(5/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I
*pi)/d)/(3*gamma(7/3)) + 2*b*d**(3/2)*e*x**7*gamma(7/3)*hyper((-1/2, 7/3),
 (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*sqrt(d)*e**2*x**10
*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gam
ma(13/3)) + c*d**(5/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*
exp_polar(I*pi)/d)/(3*gamma(10/3)) + 2*c*d**(3/2)*e*x**10*gamma(10/3)*hype
r((-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3)) + c*sqr
t(d)*e**2*x**13*gamma(13/3)*hyper((-1/2, 13/3), (16/3,), e*x**3*exp_polar(
I*pi)/d)/(3*gamma(16/3))
```

3.35.7 Maxima [F]

$$\int \left(d+ex^3\right)^{5/2} \left(a+bx^3+cx^6\right) \, dx = \int \left(cx^6+bx^3+a\right) \left(ex^3+d\right)^{\frac{5}{2}} \, dx$$

input
```
integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)
```

3.35.8 Giac [F]

$$\int \left(d+ex^3\right)^{5/2} \left(a+bx^3+cx^6\right) \, dx = \int \left(cx^6+bx^3+a\right) \left(ex^3+d\right)^{\frac{5}{2}} \, dx$$

input
```
integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^3\right)^{5/2} \left(a+bx^3+cx^6\right) \, dx = \int \left(e\,x^3+d\right)^{5/2} \left(c\,x^6+b\,x^3+a\right) \, dx$$

input
```
int((d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x)
```

output
```
int((d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6), x)
```

3.36 $\int (d+ex^3)^{3/2}(a+bx^3+cx^6)\,dx$

3.36.1 Optimal result

Integrand size = 24, antiderivative size = 356

$$\int (d+ex^3)^{3/2}(a+bx^3+cx^6)\,dx = \frac{18d(16cd^2-46bde+391ae^2)\,x\sqrt{d+ex^3}}{21505e^2} + \frac{2(16cd^2-46bde+391ae^2)\,x(d+ex^3)^{3/2}}{4301e^2} - \frac{2(8cd-23be)x(d+ex^3)^{5/2}}{391e^2} + \frac{2cx^4(d+ex^3)^{5/2}}{23e} + \frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}d^2(16cd^2-46bde+391ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),\ldots\right)}{21505e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
2/4301*(391*a*e^2-46*b*d*e+16*c*d^2)*x*(e*x^3+d)^(3/2)/e^2-2/391*(-23*b*e+
8*c*d)*x*(e*x^3+d)^(5/2)/e^2+2/23*c*x^4*(e*x^3+d)^(5/2)/e+18/21505*d*(391*
a*e^2-46*b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+18/21505*3^(3/4)*d^2*(391*a
*e^2-46*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(
1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/
2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+
3^(1/2)))^2)^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e
^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.36.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.72 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.28

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx = \frac{x\sqrt{d+ex^3}\left(-2\left(d+ex^3\right)^2\left(8cd-23be-17cex^3\right)+\frac{\left(16cd^3+23de(-2bd+17ae)\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\right.}{\sqrt{1+\frac{ex^3}{d}}}\right.}{391e^2}$$

input
```
Integrate[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
```

output
```
(x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)^2*(8*c*d - 23*b*e - 17*c*e*x^3) + ((16*
c*d^3 + 23*d*e*(-2*b*d + 17*a*e))*Hypergeometric2F1[-3/2, 1/3, 4/3, -((e*x
^3)/d)])/Sqrt[1 + (e*x^3)/d]))/(391*e^2)
```

3.36.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1741, 27, 913, 748, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx$$

$$\Big\downarrow \ 1741$$

$$\frac{2\int \frac{1}{2}\left(ex^3+d\right)^{3/2}\left(23ae-(8cd-23be)x^3\right)dx}{23e}+\frac{2cx^4\left(d+ex^3\right)^{5/2}}{23e}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \left(ex^3+d\right)^{3/2}\left(23ae-(8cd-23be)x^3\right)dx}{23e}+\frac{2cx^4\left(d+ex^3\right)^{5/2}}{23e}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{\left(16cd^2-23e(2bd-17ae)\right)\int \left(ex^3+d\right)^{3/2}dx}{17e}-\frac{2x\left(d+ex^3\right)^{5/2}(8cd-23be)}{17e}}{23e}+\frac{2cx^4\left(d+ex^3\right)^{5/2}}{23e}$$

$$\Big\downarrow \ 748$$

$$\frac{\frac{(16cd^2-23e(2bd-17ae))\left(\frac{9}{11}d\int\sqrt{ex^3+d}dx+\frac{2}{11}x(d+ex^3)^{3/2}\right)}{17e}-\frac{2x(d+ex^3)^{5/2}(8cd-23be)}{17e}}{23e}+\frac{2cx^4(d+ex^3)^{5/2}}{23e}$$

$$\Big\downarrow \ 748$$

$$\frac{\frac{(16cd^2-23e(2bd-17ae))\left(\frac{9}{11}d\left(\frac{3}{5}d\int\frac{1}{\sqrt{ex^3+d}}dx+\frac{2}{5}x\sqrt{d+ex^3}\right)+\frac{2}{11}x(d+ex^3)^{3/2}\right)}{17e}-\frac{2x(d+ex^3)^{5/2}(8cd-23be)}{17e}}{23e}+\frac{2cx^4(d+ex^3)^{5/2}}{23e}$$

$$\Big\downarrow \ 759$$

$$\frac{\frac{(16cd^2-23e(2bd-17ae))\left(\frac{9}{11}d\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}+\frac{2}{5}x\sqrt{d\cdots}\right.\right.}{17e}}{23e}$$

$$\frac{2cx^4(d+ex^3)^{5/2}}{23e}$$

input
```
Int[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*x^4*(d + e*x^3)^(5/2))/(23*e) + ((-2*(8*c*d - 23*b*e)*x*(d + e*x^3)^(
5/2))/(17*e) + ((16*c*d^2 - 23*e*(2*b*d - 17*a*e))*((2*x*(d + e*x^3)^(3/2)
)/11 + (9*d*((2*x*Sqrt[d + e*x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*d*(d^(
1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + S
qrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) +
e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*e^(1/
3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x
)^2]*Sqrt[d + e*x^3])))/11))/(17*e))/(23*e)
```

3.36. $\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx$

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

3.36.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.12

method	result
risch	$\frac{2x(935e^3cx^9+1265be^3x^6+1430cde^2x^6+1955ae^3x^3+2300bde^2x^3+135cd^2ex^3+5474de^2a+621bd^2e-216d^3c)\sqrt{ex^3+d}}{21505e^2} - \frac{18id^2(\ldots}{\ldots}$
elliptic	$\frac{2ecx^{10}\sqrt{ex^3+d}}{23} + \frac{2(be^2+\frac{26}{23}dce)x^7\sqrt{ex^3+d}}{17e} + \frac{2\left(ae^2+2bde+cd^2-\frac{14d\left(be^2+\frac{26}{23}dce\right)}{17e}\right)x^4\sqrt{ex^3+d}}{11e} + \frac{2\left(2eda+bd^2-\frac{8d\left(ae^2+\ldots\right.}{\ldots}\right.}{\ldots}$
default	Expression too large to display

input
```
int((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/21505/e^2*x*(935*c*e^3*x^9+1265*b*e^3*x^6+1430*c*d*e^2*x^6+1955*a*e^3*x^
3+2300*b*d*e^2*x^3+135*c*d^2*e*x^3+5474*a*d*e^2+621*b*d^2*e-216*c*d^3)*(e*
x^3+d)^(1/2)-18/21505*I*d^2*(391*a*e^2-46*b*d*e+16*c*d^2)/e^3*3^(1/2)*(-d*
e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1
/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)
+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*
3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e
*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))
```

3.36.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.38

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx=\frac{2\left(27\left(16\,cd^4-46\,bd^3e+391\,ad^2e^2\right)\sqrt{e}\text{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)+\left(935\,ce^4x^{10}+55\left(26\,cd\right.\right.\right.}{2}$$

input
```
integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
2/21505*(27*(16*c*d^4 - 46*b*d^3*e + 391*a*d^2*e^2)*sqrt(e)*weierstrassPIn
verse(0, -4*d/e, x) + (935*c*e^4*x^10 + 55*(26*c*d*e^3 + 23*b*e^4)*x^7 + 5
*(27*c*d^2*e^2 + 460*b*d*e^3 + 391*a*e^4)*x^4 - (216*c*d^3*e - 621*b*d^2*e
^2 - 5474*a*d*e^3)*x)*sqrt(e*x^3 + d))/e^3
```

3.36.6 Sympy [A] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.72

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx=\frac{ad^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a\sqrt{d}ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{bd^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{b\sqrt{d}ex^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{cd^{\frac{3}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{c\sqrt{d}ex^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate((e*x**3+d)**(3/2)*(c*x**6+b*x**3+a),x)
```

output
```
a*d**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/
d)/(3*gamma(4/3)) + a*sqrt(d)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,),
 e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*d**(3/2)*x**4*gamma(4/3)*hyp
er((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*sqrt(
d)*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)
/(3*gamma(10/3)) + c*d**(3/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,),
e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + c*sqrt(d)*e*x**10*gamma(10/3)*
hyper((-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3))
```

3.36.7 Maxima [F]

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx=\int \left(cx^6+bx^3+a\right)\left(ex^3+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)
```

3.36.8 Giac [F]

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx=\int \left(cx^6+bx^3+a\right)\left(ex^3+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx=\int \left(e\,x^3+d\right)^{3/2}\left(c\,x^6+b\,x^3+a\right)\,dx$$

input
```
int((d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x)
```

output
```
int((d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6), x)
```

3.37 $\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$

3.37.1 Optimal result

Integrand size = 24, antiderivative size = 316

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$$

$$= \frac{2(16cd^2-34bde+187ae^2)\,x\sqrt{d+ex^3}}{935e^2} - \frac{2(8cd-17be)x(d+ex^3)^{3/2}}{187e^2} + \frac{2cx^4(d+ex^3)^{3/2}}{17e}$$

$$+ \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}d(16cd^2-34bde+187ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{(1-}{(1+}\right.\right.}{935e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
-2/187*(-17*b*e+8*c*d)*x*(e*x^3+d)^(3/2)/e^2+2/17*c*x^4*(e*x^3+d)^(3/2)/e+
2/935*(187*a*e^2-34*b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+2/935*3^(3/4)*d*
(187*a*e^2-34*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(
1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1
/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/
3)*(1+3^(1/2)))^2)^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)
*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.37.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.95 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.31

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$$
$$=\frac{x\sqrt{d+ex^3}\left(-2(d+ex^3)\left(8cd-17be-11cex^3\right)+\frac{(16cd^2+17e(-2bd+11ae))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{ex^3}{d}\right)}{\sqrt{1+\frac{ex^3}{d}}}\right)}{187e^2}$$

input
```
Integrate[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```

output
```
(x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)*(8*c*d - 17*b*e - 11*c*e*x^3) + ((16*c*
d^2 + 17*e*(-2*b*d + 11*a*e))*Hypergeometric2F1[-1/2, 1/3, 4/3, -((e*x^3)/
d)])/Sqrt[1 + (e*x^3)/d]))/(187*e^2)
```

3.37.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1741, 27, 913, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$$
$$\Big\downarrow \text{1741}$$
$$\frac{2\int \frac{1}{2}\sqrt{ex^3+d}\left(17ae-(8cd-17be)x^3\right)dx}{17e}+\frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \sqrt{ex^3+d}\left(17ae-(8cd-17be)x^3\right)dx}{17e}+\frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e}$$
$$\Big\downarrow \text{913}$$
$$\frac{\frac{(16cd^2-17e(2bd-11ae))\int \sqrt{ex^3+d}dx}{11e}-\frac{2x\left(d+ex^3\right)^{3/2}(8cd-17be)}{11e}}{17e}+\frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e}$$

$$\Big\downarrow \; 748$$

$$\frac{\frac{\left(16cd^2-17e(2bd-11ae)\right)\left(\frac{3}{5}d\int\frac{1}{\sqrt{ex^3+d}}dx+\frac{2}{5}x\sqrt{d+ex^3}\right)}{11e}-\frac{2x\left(d+ex^3\right)^{3/2}(8cd-17be)}{11e}}{17e}+\frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e}$$

$$\Big\downarrow \; 759$$

$$\frac{\frac{\left(16cd^2-17e(2bd-11ae)\right)\left(\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}+\frac{2}{5}x\sqrt{d+ex^3}\right)}{11e}}{17e}$$

$$\frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e}$$

input
```
Int[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*x^4*(d + e*x^3)^(3/2))/(17*e) + ((-2*(8*c*d - 17*b*e)*x*(d + e*x^3)^(
3/2))/(11*e) + ((16*c*d^2 - 17*e*(2*b*d - 11*a*e))*((2*x*Sqrt[d + e*x^3])/
5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*d*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d
^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) +
 e^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*e^(1/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)
*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])))/(11*e))/(17
*e)
```

3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

3.37.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.15

method	result
risch	$\frac{2x\left(55c\,e^2x^6+85b\,e^2x^3+15dc\,x^3e+187a\,e^2+51bde-24c\,d^2\right)\sqrt{e\,x^3+d}}{935e^2} - \frac{2id\left(187a\,e^2-34bde+16c\,d^2\right)\sqrt{3}\,\left(-d\,e^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\,e^2\right)^{\frac{1}{3}}}{2e}\cdots\right.}{\left(\cdots\right.}}}{\cdots}$
elliptic	$\frac{2c\,x^7\sqrt{e\,x^3+d}}{17}+\frac{2\left(be+\frac{3cd}{17}\right)x^4\sqrt{e\,x^3+d}}{11e}+\frac{2\left(ae+bd-\frac{8d\left(be+\frac{3cd}{17}\right)}{11e}\right)x\sqrt{e\,x^3+d}}{5e}-\frac{2i\left(da-\frac{2d\left(ae+bd-\frac{8d\left(be+\frac{3cd}{17}\right)}{11e}\right)}{5e}\right)\sqrt{3}\,\left(-d\,e\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/935*x*(55*c*e^2*x^6+85*b*e^2*x^3+15*c*d*e*x^3+187*a*e^2+51*b*d*e-24*c*d^
2)/e^2*(e*x^3+d)^(1/2)-2/935*I*d*(187*a*e^2-34*b*d*e+16*c*d^2)/e^3*3^(1/2)
*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))
*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^
(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1
/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*
e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-
3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))
```

3.37.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.33

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$$
$$=\frac{2\left(3\left(16\,cd^3-34\,bd^2e+187\,ade^2\right)\sqrt{e}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)+\left(55\,ce^3x^7+5\left(3\,cde^2+17\,be^3\right)x^4-\cdots\right.}{935\,e^3}$$

input
```
integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
2/935*(3*(16*c*d^3 - 34*b*d^2*e + 187*a*d*e^2)*sqrt(e)*weierstrassPInverse
(0, -4*d/e, x) + (55*c*e^3*x^7 + 5*(3*c*d*e^2 + 17*b*e^3)*x^4 - (24*c*d^2*
e - 51*b*d*e^2 - 187*a*e^3)*x)*sqrt(e*x^3 + d))/e^3
```

3.37.6 Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.39

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx = \frac{a\sqrt{d}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{b\sqrt{d}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{c\sqrt{d}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((e*x**3+d)**(1/2)*(c*x**6+b*x**3+a),x)
```

output
```
a*sqrt(d)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d
)/(3*gamma(4/3)) + b*sqrt(d)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*
x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + c*sqrt(d)*x**7*gamma(7/3)*hyper((
-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3))
```

3.37.7 Maxima [F]

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx=\int \left(cx^6+bx^3+a\right)\sqrt{ex^3+d}\,dx$$

input
```
integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

3.37.8 Giac [F]

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx=\int \left(cx^6+bx^3+a\right)\sqrt{ex^3+d}\,dx$$

input
```
integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx=\int \sqrt{e\,x^3+d}\,\left(c\,x^6+b\,x^3+a\right)\,dx$$

input
```
int((d + e*x^3)^(1/2)*(a + b*x^3 + c*x^6),x)
```

output
```
int((d + e*x^3)^(1/2)*(a + b*x^3 + c*x^6), x)
```

3.38 $\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$

3.38.1 Optimal result

Integrand size = 24, antiderivative size = 278

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = -\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2}+\frac{2cx^4\sqrt{d+ex^3}}{11e}$$
$$+\frac{2\sqrt{2+\sqrt{3}}(16cd^2-22bde+55ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right.\right.}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
-2/55*(-11*b*e+8*c*d)*x*(e*x^3+d)^(1/2)/e^2+2/11*c*x^4*(e*x^3+d)^(1/2)/e+2
/165*(55*a*e^2-22*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x
+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d
^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(
1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.38.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.35

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \frac{x\left(-2(d+ex^3)\left(8cd-11be-5cex^3\right)+\left(16cd^2+11e(-2bd+5ae)\right)\sqrt{1+\frac{ex^3}{d}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\right.\right.}{55e^2\sqrt{d+ex^3}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```

output
```
(x*(-2*(d + e*x^3)*(8*c*d - 11*b*e - 5*c*e*x^3) + (16*c*d^2 + 11*e*(-2*b*d
 + 5*a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/
d)]))/(55*e^2*Sqrt[d + e*x^3])
```

3.38.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1741, 27, 913, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$$

$$\Big\downarrow \text{1741}$$

$$\frac{2\int \frac{11ae-(8cd-11be)x^3}{2\sqrt{ex^3+d}}dx}{11e}+\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{11ae-(8cd-11be)x^3}{\sqrt{ex^3+d}}dx}{11e}+\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{(16cd^2-11e(2bd-5ae))\int \frac{1}{\sqrt{ex^3+d}}dx}{5e}-\frac{2x\sqrt{d+ex^3}(8cd-11be)}{5e}}{11e}+\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

$$\Big\downarrow \; 759$$

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}(16cd^2-11e(2bd-5ae))\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}e^{4/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2x\sqrt{d+ex^3}(}{5e}}{11e}$$

$$\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

input
```
Int[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```

output
```
(2*c*x^4*Sqrt[d + e*x^3])/(11*e) + ((-2*(8*c*d - 11*b*e)*x*Sqrt[d + e*x^3]
)/(5*e) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 11*e*(2*b*d - 5*a*e))*(d^(1/3)
+ e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3
])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/
3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*e^
(4/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3
)*x)^2]*Sqrt[d + e*x^3]))/(11*e)
```

3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

3.38.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.20

method	result
risch	$\frac{2x(5cx^3e+11be-8cd)\sqrt{ex^3+d}}{55e^2}-\frac{2i(55ae^2-22bde+16cd^2)\sqrt{3}\,(-de^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-de^2)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,(-de^2)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{(-de^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-d\cdots}{}}{-\frac{3(-de^2)^{\frac{1}{3}}}{2e}+\cdots}}\cdots}{\cdots}$
elliptic	$\frac{2cx^4\sqrt{ex^3+d}}{11e}+\frac{2\left(b-\frac{8cd}{11e}\right)x\sqrt{ex^3+d}}{5e}-\frac{2i\left(a-\frac{2d\left(b-\frac{8cd}{11e}\right)}{5e}\right)\sqrt{3}\,(-de^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-de^2)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,(-de^2)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{(-de^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(\cdots}{}}{-\frac{3(-de^2)^{\frac{1}{3}}}{2e}\cdots}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/55*x*(5*c*e*x^3+11*b*e-8*c*d)/e^2*(e*x^3+d)^(1/2)-2/165*I*(55*a*e^2-22*b
*d*e+16*c*d^2)/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I
*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2
)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I
*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2
)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^
(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3
^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1
/3)))^(1/2))
```

3.38.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.26

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \frac{2\left((16\,cd^2-22\,bde+55\,ae^2)\sqrt{e}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)+(5\,ce^2x^4-(8\,cde-11\,be^2)x)\sqrt{ex^3+d}\right)}{55\,e^3}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

output
```
2/55*((16*c*d^2 - 22*b*d*e + 55*a*e^2)*sqrt(e)*weierstrassPInverse(0, -4*d
/e, x) + (5*c*e^2*x^4 - (8*c*d*e - 11*b*e^2)*x)*sqrt(e*x^3 + d))/e^3
```

3.38.6 Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.43

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(1/2),x)
```

output
```
a*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt
(d)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), e*x**3*exp_p
olar(I*pi)/d)/(3*sqrt(d)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((1/2, 7/3),
 (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(10/3))
```

3.38.7 Maxima [F]

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \int \frac{cx^6+bx^3+a}{\sqrt{ex^3+d}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

3.38.8 Giac [F]

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \int \frac{cx^6+bx^3+a}{\sqrt{ex^3+d}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \int \frac{c\,x^6+b\,x^3+a}{\sqrt{e\,x^3+d}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2),x)
```

output
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2), x)
```

3.39 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx$

3.39.1 Optimal result

Integrand size = 24, antiderivative size = 289

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \frac{2(cd^2-bde+ae^2)\,x}{3de^2\sqrt{d+ex^3}}+\frac{2cx\sqrt{d+ex^3}}{5e^2}$$

$$-\frac{2\sqrt{2+\sqrt{3}}(16cd^2-5e(2bd+ae))\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\ldots}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\ldots}\right.\right.}{15\sqrt[4]{3}de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
2/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(1/2)+2/5*c*x*(e*x^3+d)^(1/2)/e^
2-2/45*(16*c*d^2-5*e*(a*e+2*b*d))*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x
+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d
^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d
^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.39.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.35

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \frac{x\left(2(5e(-bd+ae)+cd(8d+3ex^3))+(-16cd^2+5e(2bd+ae))\sqrt{1+\frac{ex^3}{d}}\,\text{Hypergeo}\right.}{15de^2\sqrt{d+ex^3}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
```

output
```
(x*(2*(5*e*(-(b*d) + a*e) + c*d*(8*d + 3*e*x^3)) + (-16*c*d^2 + 5*e*(2*b*d
 + a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)
]))/(15*d*e^2*Sqrt[d + e*x^3])
```

3.39.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1739, 27, 913, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}dx$$

$$\Big\downarrow \text{1739}$$

$$\frac{2x(ae^2-bde+cd^2)}{3de^2\sqrt{d+ex^3}}-\frac{2\int \frac{-3cdex^3+2cd^2-e(2bd+ae)}{2\sqrt{ex^3+d}}dx}{3de^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x(ae^2-bde+cd^2)}{3de^2\sqrt{d+ex^3}}-\frac{\int \frac{-3cdex^3+2cd^2-e(2bd+ae)}{\sqrt{ex^3+d}}dx}{3de^2}$$

$$\Big\downarrow \text{913}$$

$$\frac{2x(ae^2-bde+cd^2)}{3de^2\sqrt{d+ex^3}}-\frac{\frac{1}{5}(16cd^2-5e(ae+2bd))\int \frac{1}{\sqrt{ex^3+d}}dx-\frac{6}{5}cdx\sqrt{d+ex^3}}{3de^2}$$

$$\Big\downarrow \text{759}$$

$$\frac{\frac{2x(ae^2-bde+cd^2)}{3de^2\sqrt{d+ex^3}} - \frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}(16cd^2-5e(ae+2bd))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}} - \frac{6}{5}cdx\sqrt{d+e}}{3de^2}$$

input
```
Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
```

output
```
(2*(c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*Sqrt[d + e*x^3]) - ((-6*c*d*x*Sqrt[
d + e*x^3])/5 + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 5*e*(2*b*d + a*e))*(d^(1/
3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqr
t[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^
(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)
*e^(1/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(
1/3)*x)^2]*Sqrt[d + e*x^3]))/(3*d*e^2)
```

3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

3.39.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{2x\left(a\,e^{2}-bde+c\,d^{2}\right)}{3e^{2}d\sqrt{\left(x^{3}+\frac{d}{e}\right)e}}+\frac{2cx\sqrt{e\,x^{3}+d}}{5e^{2}}-\frac{2i\left(\frac{be-cd}{e^{2}}+\frac{a\,e^{2}-bde+c\,d^{2}}{3e^{2}d}-\frac{2cd}{5e^{2}}\right)\sqrt{3}\left(-d\,e^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{\left(-d\,e^{2}\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/e^2/d*x*(a*e^2-b*d*e+c*d^2)/((x^3+1/e*d)*e)^(1/2)+2/5*c*x*(e*x^3+d)^(1
/2)/e^2-2/3*I*((b*e-c*d)/e^2+1/3*(a*e^2-b*d*e+c*d^2)/e^2/d-2/5*c/e^2*d)*3^
(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)
^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-
d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^
(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x
^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)
/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(
1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))
```

3.39.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.43

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = -\frac{2\left((16\,cd^3-10\,bd^2e-5\,ade^2+(16\,cd^2e-10\,bde^2-5\,ae^3)x^3)\sqrt{e}\text{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)-(3\,cde^2\right.}{15\,(de^4x^3+d^2e^3)}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*((16*c*d^3 - 10*b*d^2*e - 5*a*d*e^2 + (16*c*d^2*e - 10*b*d*e^2 - 5*a
*e^3)*x^3)*sqrt(e)*weierstrassPInverse(0, -4*d/e, x) - (3*c*d*e^2*x^4 + (8
*c*d^2*e - 5*b*d*e^2 + 5*a*e^3)*x)*sqrt(e*x^3 + d))/(d*e^4*x^3 + d^2*e^3)
```

3.39.6 Sympy [A] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.41

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(3/2),x)
```

output
```
a*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(
3/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), e*x**3*exp_
polar(I*pi)/d)/(3*d**(3/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((3/2, 7/3
), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3/2)*gamma(10/3))
```

3.39.7 Maxima [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)
```

3.39.8 Giac [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \int \frac{c\,x^6+b\,x^3+a}{(e\,x^3+d)^{3/2}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x)
```

output
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2), x)
```

3.40 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{5/2}}\,dx$

3.40.1 Optimal result

Integrand size = 24, antiderivative size = 309

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{5/2}}\,dx = \frac{2\left(cd^2-bde+ae^2\right)x}{9de^2\left(d+ex^3\right)^{3/2}} - \frac{2\left(11cd^2-2bde-7ae^2\right)x}{27d^2e^2\sqrt{d+ex^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(16cd^2+e(2bd+7ae)\right)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right.\right.}{27\sqrt[4]{3}d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
2/9*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(3/2)-2/27*(-7*a*e^2-2*b*d*e+11*
c*d^2)*x/d^2/e^2/(e*x^3+d)^(1/2)+2/81*(16*c*d^2+e*(7*a*e+2*b*d))*(d^(1/3)+
e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1
/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^2/e^
(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

3.40.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.42

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \frac{-2x(cd^2(8d+11ex^3)+e(bd(d-2ex^3)-ae(10d+7ex^3)))+(16cd^2+e(2bd+7ae)}{27d^2e^2(d+ex^3)^{3/2}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]
```

output
```
(-2*x*(c*d^2*(8*d + 11*e*x^3) + e*(b*d*(d - 2*e*x^3) - a*e*(10*d + 7*e*x^3
))) + (16*c*d^2 + e*(2*b*d + 7*a*e))*x*(d + e*x^3)*Sqrt[1 + (e*x^3)/d]*Hyp
ergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])/(27*d^2*e^2*(d + e*x^3)^(3/2)
)
```

3.40.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1739, 27, 910, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx$$

$$\Big\downarrow \ 1739$$

$$\frac{2x(ae^2-bde+cd^2)}{9de^2(d+ex^3)^{3/2}}-\frac{2\int \frac{-9cdex^3+2cd^2-e(2bd+7ae)}{2(ex^3+d)^{3/2}}dx}{9de^2}$$

$$\Big\downarrow \ 27$$

$$\frac{2x(ae^2-bde+cd^2)}{9de^2(d+ex^3)^{3/2}}-\frac{\int \frac{-9cdex^3+2cd^2-7ae^2-2bde}{(ex^3+d)^{3/2}}dx}{9de^2}$$

$$\Big\downarrow \ 910$$

$$\frac{2x(ae^2-bde+cd^2)}{9de^2(d+ex^3)^{3/2}}-\frac{\frac{2x(-7ae^2-2bde+11cd^2)}{3d\sqrt{d+ex^3}}-\frac{(e(7ae+2bd)+16cd^2)\int \frac{1}{\sqrt{ex^3+d}}dx}{3d}}{9de^2}$$

$$\Big\downarrow \; 759$$

$$\frac{2x\left(ae^2-bde+cd^2\right)}{9de^2\left(d+ex^3\right)^{3/2}}-\frac{\frac{2x\left(-7ae^2-2bde+11cd^2\right)}{3d\sqrt{d+ex^3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(e(7ae+2bd)+16cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-\cdots\right.}{3\sqrt[4]{3}d\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}}{9de^2}$$

input
```
Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]
```

output
```
(2*(c*d^2 - b*d*e + a*e^2)*x)/(9*d*e^2*(d + e*x^3)^(3/2)) - ((2*(11*c*d^2
- 2*b*d*e - 7*a*e^2)*x)/(3*d*Sqrt[d + e*x^3]) - (2*Sqrt[2 + Sqrt[3]]*(16*c
*d^2 + e*(2*b*d + 7*a*e))*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^
(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*
x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d*e^(1/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3
)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3]))/(9*d*e^2)
```

3.40.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

3.40.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.30

method	result
elliptic	$\frac{2x(a\,e^2-bde+c\,d^2)\sqrt{e\,x^3+d}}{9d\,e^4\left(x^3+\frac{d}{e}\right)^2}+\frac{2x(7a\,e^2+2bde-11c\,d^2)}{27e^2d^2\sqrt{\left(x^3+\frac{d}{e}\right)e}}-\frac{2i\left(\frac{c}{e^2}+\frac{7a\,e^2+2bde-11c\,d^2}{27e^2d^2}\right)\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-d\,e^2)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,(-d\,e^2}{2e}\right.}{(-d\,e^2)^{\frac{1}{3}}}}}{}$
default	Expression too large to display

input
```
int((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9/d*x/e^4*(a*e^2-b*d*e+c*d^2)*(e*x^3+d)^(1/2)/(x^3+1/e*d)^2+2/27/e^2/d^2
*x*(7*a*e^2+2*b*d*e-11*c*d^2)/((x^3+1/e*d)*e)^(1/2)-2/3*I*(c/e^2+1/27/e^2/
d^2*(7*a*e^2+2*b*d*e-11*c*d^2))*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e
^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*
((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1
/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^
(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1
/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2
)/e*(-d*e^2)^(1/3)))^(1/2))
```

3.40.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.61

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \frac{2\left(\left(\left(16\,cd^2e^2+2\,bde^3+7\,ae^4\right)x^6+16\,cd^4+2\,bd^3e+7\,ad^2e^2+2\left(16\,cd^3e+2\,bd^2e^2+\right.\right.\right.}{}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="fricas")
```

output
```
2/27*(((16*c*d^2*e^2 + 2*b*d*e^3 + 7*a*e^4)*x^6 + 16*c*d^4 + 2*b*d^3*e + 7
*a*d^2*e^2 + 2*(16*c*d^3*e + 2*b*d^2*e^2 + 7*a*d*e^3)*x^3)*sqrt(e)*weierst
rassPInverse(0, -4*d/e, x) - ((11*c*d^2*e^2 - 2*b*d*e^3 - 7*a*e^4)*x^4 + (
8*c*d^3*e + b*d^2*e^2 - 10*a*d*e^3)*x)*sqrt(e*x^3 + d))/(d^2*e^5*x^6 + 2*d
^3*e^4*x^3 + d^4*e^3)
```

3.40.6 Sympy [A] (verification not implemented)

Time = 34.98 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.39

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{3},\frac{5}{2}\\ \frac{10}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(5/2),x)
```

output
```
a*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(
5/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), e*x**3*exp_
polar(I*pi)/d)/(3*d**(5/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((7/3, 5/2
), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(5/2)*gamma(10/3))
```

3.40.7 Maxima [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)
```

3.40.8 Giac [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \int \frac{c\,x^6+b\,x^3+a}{(e\,x^3+d)^{5/2}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x)
```

output
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2), x)
```

3.41 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx$

3.41.1 Optimal result

Integrand size = 24, antiderivative size = 349

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \frac{2(cd^2-bde+ae^2)\,x}{15de^2\,(d+ex^3)^{5/2}}$$
$$-\frac{2(17cd^2-2bde-13ae^2)\,x}{135d^2e^2\,(d+ex^3)^{3/2}}+\frac{2(16cd^2+14bde+91ae^2)\,x}{405d^3e^2\sqrt{d+ex^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}(16cd^2+14bde+91ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{}}\right.\right.}{405\sqrt[4]{3}d^3e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
2/15*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(5/2)-2/135*(-13*a*e^2-2*b*d*e+
17*c*d^2)*x/d^2/e^2/(e*x^3+d)^(3/2)+2/405*(91*a*e^2+14*b*d*e+16*c*d^2)*x/d
^3/e^2/(e*x^3+d)^(1/2)+2/1215*(91*a*e^2+14*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3
)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+
e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^3/e^(7/3)/
(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)
))^2)^(1/2)
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.48

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \frac{2x(cd^2(-8d^2-19dex^3+16e^2x^6)+e(bd(-7d^2+34dex^3+14e^2x^6)+ae(157d^2+221}{4}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]
```

output
```
(2*x*(c*d^2*(-8*d^2 - 19*d*e*x^3 + 16*e^2*x^6) + e*(b*d*(-7*d^2 + 34*d*e*x
^3 + 14*e^2*x^6) + a*e*(157*d^2 + 221*d*e*x^3 + 91*e^2*x^6))) + (16*c*d^2
+ 7*e*(2*b*d + 13*a*e))*x*(d + e*x^3)^2*Sqrt[1 + (e*x^3)/d]*Hypergeometric
2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])/(405*d^3*e^2*(d + e*x^3)^(5/2))
```

3.41.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1739, 27, 910, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}dx$$

$$\Big\downarrow \text{1739}$$

$$\frac{2x(ae^2-bde+cd^2)}{15de^2(d+ex^3)^{5/2}}-\frac{2\int \frac{-15cdex^3+2cd^2-e(2bd+13ae)}{2(ex^3+d)^{5/2}}dx}{15de^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x(ae^2-bde+cd^2)}{15de^2(d+ex^3)^{5/2}}-\frac{\int \frac{-15cdex^3+2cd^2-13ae^2-2bde}{(ex^3+d)^{5/2}}dx}{15de^2}$$

$$\Big\downarrow \text{910}$$

$$\frac{2x\left(ae^2-bde+cd^2\right)}{15de^2\left(d+ex^3\right)^{5/2}}-\frac{\frac{2x\left(-13ae^2-2bde+17cd^2\right)}{9d\left(d+ex^3\right)^{3/2}}-\frac{\left(91ae^2+14bde+16cd^2\right)\int\frac{1}{\left(ex^3+d\right)^{3/2}}dx}{9d}}{15de^2}$$

$\downarrow$ 749

$$\frac{2x\left(ae^2-bde+cd^2\right)}{15de^2\left(d+ex^3\right)^{5/2}}-\frac{\frac{2x\left(-13ae^2-2bde+17cd^2\right)}{9d\left(d+ex^3\right)^{3/2}}-\frac{\left(91ae^2+14bde+16cd^2\right)\left(\frac{\int\frac{1}{\sqrt{ex^3+d}}dx}{3d}+\frac{2x}{3d\sqrt{d+ex^3}}\right)}{9d}}{15de^2}$$

$\downarrow$ 759

$$\frac{2x\left(ae^2-bde+cd^2\right)}{15de^2\left(d+ex^3\right)^{5/2}}-\frac{\frac{2x\left(-13ae^2-2bde+17cd^2\right)}{9d\left(d+ex^3\right)^{3/2}}-\frac{\left(91ae^2+14bde+16cd^2\right)\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-\right.}{3\sqrt[4]{3}d\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}\right.}{9d}}{15de^2}$$

input
```
Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]
```

output
```
(2*(c*d^2 - b*d*e + a*e^2)*x)/(15*d*e^2*(d + e*x^3)^(5/2)) - ((2*(17*c*d^2
 - 2*b*d*e - 13*a*e^2)*x)/(9*d*(d + e*x^3)^(3/2)) - ((16*c*d^2 + 14*b*d*e
+ 91*a*e^2)*((2*x)/(3*d*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(d^(1/3) +
 e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3]
)*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3
)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d*e
^(1/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/
3)*x)^2]*Sqrt[d + e*x^3])))/(9*d))/(15*d*e^2)
```

3.41. $\int\frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{7/2}}dx$

3.41.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

3.41.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{2x(a\,e^2-bde+c\,d^2)\sqrt{e\,x^3+d}}{15d\,e^5\left(x^3+\frac{d}{e}\right)^3}+\frac{2x(13a\,e^2+2bde-17c\,d^2)\sqrt{e\,x^3+d}}{135d^2e^4\left(x^3+\frac{d}{e}\right)^2}+\frac{2x(91a\,e^2+14bde+16c\,d^2)}{405e^2d^3\sqrt{\left(x^3+\frac{d}{e}\right)e}}-\frac{2i(91a\,e^2+14bde+16c\,d^2)\sqrt{3}}{}$
default	Expression too large to display

input
```
int((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*x/e^5*(a*e^2-b*d*e+c*d^2)*(e*x^3+d)^(1/2)/(x^3+1/e*d)^3+2/135/d^2*x
*(13*a*e^2+2*b*d*e-17*c*d^2)/e^4*(e*x^3+d)^(1/2)/(x^3+1/e*d)^2+2/405/e^2/d
^3*x*(91*a*e^2+14*b*d*e+16*c*d^2)/((x^3+1/e*d)*e)^(1/2)-2/1215*I*(91*a*e^2
+14*b*d*e+16*c*d^2)/d^3/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1
/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/
e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^
(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*
e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*
(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(
1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-
d*e^2)^(1/3)))^(1/2))
```

3.41.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.77

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \frac{2\left(\left(\left(16\,cd^2e^3+14\,bde^4+91\,ae^5\right)x^9+3\left(16\,cd^3e^2+14\,bd^2e^3+91\,ade^4\right)x^6+16\,cd^5+\right.\right.}{}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="fricas")
```

output
```
2/405*(((16*c*d^2*e^3 + 14*b*d*e^4 + 91*a*e^5)*x^9 + 3*(16*c*d^3*e^2 + 14*
b*d^2*e^3 + 91*a*d*e^4)*x^6 + 16*c*d^5 + 14*b*d^4*e + 91*a*d^3*e^2 + 3*(16
*c*d^4*e + 14*b*d^3*e^2 + 91*a*d^2*e^3)*x^3)*sqrt(e)*weierstrassPInverse(0
, -4*d/e, x) + ((16*c*d^2*e^3 + 14*b*d*e^4 + 91*a*e^5)*x^7 - (19*c*d^3*e^2
 - 34*b*d^2*e^3 - 221*a*d*e^4)*x^4 - (8*c*d^4*e + 7*b*d^3*e^2 - 157*a*d^2*
e^3)*x)*sqrt(e*x^3 + d))/(d^3*e^6*x^9 + 3*d^4*e^5*x^6 + 3*d^5*e^4*x^3 + d^
6*e^3)
```

3.41.6 Sympy [A] (verification not implemented)

Time = 146.82 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.34

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{7}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{7}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{7}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{7}{3},\frac{7}{2}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{7}{2}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(7/2),x)
```

output
```
a*x*gamma(1/3)*hyper((1/3, 7/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(
7/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 7/2), (7/3,), e*x**3*exp_
polar(I*pi)/d)/(3*d**(7/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((7/3, 7/2
), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(7/2)*gamma(10/3))
```

3.41.7 Maxima [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)
```

3.41.8 Giac [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \int \frac{c\,x^6+b\,x^3+a}{(e\,x^3+d)^{7/2}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x)
```

output
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2), x)
```

3.42 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}} dx$

3.42.1 Optimal result

Integrand size = 24, antiderivative size = 389

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}} dx = \frac{2(cd^2-bde+ae^2)x}{21de^2(d+ex^3)^{7/2}} - \frac{2(23cd^2-2bde-19ae^2)x}{315d^2e^2(d+ex^3)^{5/2}}$$
$$+ \frac{2(16cd^2+26bde+247ae^2)x}{2835d^3e^2(d+ex^3)^{3/2}} + \frac{2(16cd^2+26bde+247ae^2)x}{1215d^4e^2\sqrt{d+ex^3}}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}(16cd^2+26bde+247ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{}}\right.\right.}{1215\sqrt[4]{3}d^4e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
2/21*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(7/2)-2/315*(-19*a*e^2-2*b*d*e+
23*c*d^2)*x/d^2/e^2/(e*x^3+d)^(5/2)+2/2835*(247*a*e^2+26*b*d*e+16*c*d^2)*x
/d^3/e^2/(e*x^3+d)^(3/2)+2/1215*(247*a*e^2+26*b*d*e+16*c*d^2)*x/d^4/e^2/(e
*x^3+d)^(1/2)+2/3645*(247*a*e^2+26*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*Ell
ipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*
x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^4/e^(7/3)/(e*x^3+d
)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

3.42.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.51

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx = \frac{2x(cd^2(-56d^3-189d^2ex^3+384de^2x^6+112e^3x^9)+e(bd(-91d^3+756d^2ex^3+624de\ldots}{}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```

output
```
(2*x*(c*d^2*(-56*d^3 - 189*d^2*e*x^3 + 384*d*e^2*x^6 + 112*e^3*x^9) + e*(b
*d*(-91*d^3 + 756*d^2*e*x^3 + 624*d*e^2*x^6 + 182*e^3*x^9) + a*e*(3388*d^3
 + 7182*d^2*e*x^3 + 5928*d*e^2*x^6 + 1729*e^3*x^9))) + 7*(16*c*d^2 + 13*e*
(2*b*d + 19*a*e))*x*(d + e*x^3)^3*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/
3, 1/2, 4/3, -((e*x^3)/d)])/(8505*d^4*e^2*(d + e*x^3)^(7/2))
```

3.42.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1739, 27, 910, 749, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx$$

$$\Big\downarrow \text{1739}$$

$$\frac{2x(ae^2-bde+cd^2)}{21de^2(d+ex^3)^{7/2}}-\frac{2\int \frac{-21cdex^3+2cd^2-e(2bd+19ae)}{2(ex^3+d)^{7/2}}dx}{21de^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x(ae^2-bde+cd^2)}{21de^2(d+ex^3)^{7/2}}-\frac{\int \frac{-21cdex^3+2cd^2-19ae^2-2bde}{(ex^3+d)^{7/2}}dx}{21de^2}$$

$$\Big\downarrow \text{910}$$

$$\frac{2x\left(ae^2-bde+cd^2\right)}{21de^2\left(d+ex^3\right)^{7/2}}-\frac{\frac{2x\left(-19ae^2-2bde+23cd^2\right)}{15d(d+ex^3)^{5/2}}-\frac{\left(247ae^2+26bde+16cd^2\right)\int\frac{1}{\left(ex^3+d\right)^{5/2}}dx}{15d}}{21de^2}$$

$$\Big\downarrow \; 749$$

$$\frac{2x\left(ae^2-bde+cd^2\right)}{21de^2\left(d+ex^3\right)^{7/2}}-\frac{\frac{2x\left(-19ae^2-2bde+23cd^2\right)}{15d(d+ex^3)^{5/2}}-\frac{\left(247ae^2+26bde+16cd^2\right)\left(\frac{7\int\frac{1}{\left(ex^3+d\right)^{3/2}}dx}{9d}+\frac{2x}{9d\left(d+ex^3\right)^{3/2}}\right)}{15d}}{21de^2}$$

$$\Big\downarrow \; 749$$

$$\frac{2x\left(ae^2-bde+cd^2\right)}{21de^2\left(d+ex^3\right)^{7/2}}-\frac{\frac{2x\left(-19ae^2-2bde+23cd^2\right)}{15d(d+ex^3)^{5/2}}-\frac{\left(247ae^2+26bde+16cd^2\right)\left(\frac{7\left(\frac{\int\frac{1}{\sqrt{ex^3+d}}dx}{3d}+\frac{2x}{3d\sqrt{d+ex^3}}\right)}{9d}+\frac{2x}{9d\left(d+ex^3\right)^{3/2}}\right)}{15d}}{21de^2}$$

$$\Big\downarrow \; 759$$

$$\frac{2x\left(ae^2-bde+cd^2\right)}{21de^2\left(d+ex^3\right)^{7/2}}-\frac{\frac{2x\left(-19ae^2-2bde+23cd^2\right)}{15d(d+ex^3)^{5/2}}-\frac{\left(247ae^2+26bde+16cd^2\right)\left(\frac{7\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right.\right.}{3\sqrt[4]{3}d\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}\right.}{9d}\right.}{15d}}{21de^2}$$

input
```
Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```

3.42. $\int\frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{9/2}}\,dx$

output
```
(2*(c*d^2 - b*d*e + a*e^2)*x)/(21*d*e^2*(d + e*x^3)^(7/2)) - ((2*(23*c*d^2
 - 2*b*d*e - 19*a*e^2)*x)/(15*d*(d + e*x^3)^(5/2)) - ((16*c*d^2 + 26*b*d*e
 + 247*a*e^2)*((2*x)/(9*d*(d + e*x^3)^(3/2)) + (7*((2*x)/(3*d*Sqrt[d + e*x
^3]) + (2*Sqrt[2 + Sqrt[3]]*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*
e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3
)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d*e^(1/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1
/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])))/(9*d)))/
(15*d))/(21*d*e^2)
```

3.42.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

3.42.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{2x\left(a\,e^{2}-bde+c\,d^{2}\right)\sqrt{e\,x^{3}+d}}{21d\,e^{6}\left(x^{3}+\frac{d}{e}\right)^{4}}+\frac{2x\left(19a\,e^{2}+2bde-23c\,d^{2}\right)\sqrt{e\,x^{3}+d}}{315d^{2}e^{5}\left(x^{3}+\frac{d}{e}\right)^{3}}+\frac{2x\left(247a\,e^{2}+26bde+16c\,d^{2}\right)\sqrt{e\,x^{3}+d}}{2835d^{3}e^{4}\left(x^{3}+\frac{d}{e}\right)^{2}}+\frac{2x\left(247a\,e^{2}+26bde\right.}{1215e^{2}d^{4}\sqrt{\left(x^{3}\right.}}$
default	Expression too large to display

input
```
int((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d*x/e^6*(a*e^2-b*d*e+c*d^2)*(e*x^3+d)^(1/2)/(x^3+1/e*d)^4+2/315/d^2*x
*(19*a*e^2+2*b*d*e-23*c*d^2)/e^5*(e*x^3+d)^(1/2)/(x^3+1/e*d)^3+2/2835/d^3*
x*(247*a*e^2+26*b*d*e+16*c*d^2)/e^4*(e*x^3+d)^(1/2)/(x^3+1/e*d)^2+2/1215/e
^2/d^4*x*(247*a*e^2+26*b*d*e+16*c*d^2)/((x^3+1/e*d)*e)^(1/2)-2/3645*I*(247
*a*e^2+26*b*d*e+16*c*d^2)/d^4/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e
^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*
((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1
/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^
(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1
/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2
)/e*(-d*e^2)^(1/3)))^(1/2))
```

3.42.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{9/2}}\,dx = \frac{2\left(7\left(\left(16\,cd^2e^4+26\,bde^5+247\,ae^6\right)x^{12}+4\left(16\,cd^3e^3+26\,bd^2e^4+247\,ade^5\right)x^9+16\,c\right.\right.}{}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="fricas")
```

output
```
2/8505*(7*((16*c*d^2*e^4 + 26*b*d*e^5 + 247*a*e^6)*x^12 + 4*(16*c*d^3*e^3
+ 26*b*d^2*e^4 + 247*a*d*e^5)*x^9 + 16*c*d^6 + 26*b*d^5*e + 247*a*d^4*e^2
+ 6*(16*c*d^4*e^2 + 26*b*d^3*e^3 + 247*a*d^2*e^4)*x^6 + 4*(16*c*d^5*e + 26
*b*d^4*e^2 + 247*a*d^3*e^3)*x^3)*sqrt(e)*weierstrassPInverse(0, -4*d/e, x)
 + (7*(16*c*d^2*e^4 + 26*b*d*e^5 + 247*a*e^6)*x^10 + 24*(16*c*d^3*e^3 + 26
*b*d^2*e^4 + 247*a*d*e^5)*x^7 - 189*(c*d^4*e^2 - 4*b*d^3*e^3 - 38*a*d^2*e^
4)*x^4 - 7*(8*c*d^5*e + 13*b*d^4*e^2 - 484*a*d^3*e^3)*x)*sqrt(e*x^3 + d))/
(d^4*e^7*x^12 + 4*d^5*e^6*x^9 + 6*d^6*e^5*x^6 + 4*d^7*e^4*x^3 + d^8*e^3)
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(9/2),x)
```

output
```
Timed out
```

3.42.7 Maxima [F]

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{9/2}}\,dx = \int \frac{cx^6+bx^3+a}{\left(ex^3+d\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)
```

3.42.8 Giac [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx = \int \frac{c\,x^6+b\,x^3+a}{(e\,x^3+d)^{9/2}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x)
```

output
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2), x)
```

3.43 $\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx$

3.43.1 Optimal result

Integrand size = 25, antiderivative size = 433

$$\begin{aligned}\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx &= \frac{ex}{c} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\ &- \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} \\ &- \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\ &- \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}\end{aligned}$$

output
```
e*x/c-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(c*d-b*e
+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2
)^(1/2))^(3/4)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
)*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(
-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/
2))^(1/4))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(
5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a
*c+b^2)^(1/2))^(1/4))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*
2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

3.43.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.20

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{ex}{c} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{ae\log(x-\#1)-cd\log(x-\#1)\#1^4+be\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

input
```
Integrate[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(e*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*e*Log[x - #1] - c*d*Log[x - #
1]*#1^4 + b*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

3.43.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1826, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1826}$$

$$\frac{ex}{c}-\frac{\int \frac{ae-(cd-be)x^4}{cx^8+bx^4+a}dx}{c}$$

$\Big\downarrow$ 1752

$$\frac{ex}{c}-\frac{-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$\Big\downarrow$ 756

$$\frac{ex}{c}-\frac{-\frac{1}{2}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be\right.}{c}$$

$\Big\downarrow$ 218

$$\frac{ex}{c}-\frac{-\frac{1}{2}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}\right.}{c}$$

$\Big\downarrow$ 221

$$\frac{ex}{c}-\frac{-\frac{1}{2}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-\frac{1}{2}\left(-\frac{2ace+b}{\sqrt{b}}\right.}{c}$$

input
```
Int[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

3.43. $\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx$

output
```
(e*x)/c - (-1/2*((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])
*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^
(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)
))) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan
[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b
+ Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2
- 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/c
```

3.43.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

3.43.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left((-be+cd)\_R^{4}-ae\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	67
risch	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left((-be+cd)\_R^{4}-ae\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	67

input
```
int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
e*x/c+1/4/c*sum(((-b*e+c*d)*_R^4-a*e)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf
(_Z^8*c+_Z^4*b+a))
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12866 vs. 2(353) = 706.

Time = 7.77 (sec) , antiderivative size = 12866, normalized size of antiderivative = 29.71

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx=\text{Too large to display}$$

input
```
integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.43.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.43.7 Maxima [F]

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{(ex^4+d)x^4}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
e*x/c - integrate(-((c*d - b*e)*x^4 - a*e)/(c*x^8 + b*x^4 + a), x)/c
```

3.43.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

3.43.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 50213, normalized size of antiderivative = 115.97

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
atan((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d
^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 1
6384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(
-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^
4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c
^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^
7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 +
 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2
)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^
3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*
d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
 + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d
 - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^
9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 +
128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e
^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7...
```

3.44 $\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx$

3.44.1 Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx=-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{e\log\left(a+bx^4+cx^8\right)}{8c}$$

output
```
1/8*e*ln(c*x^8+b*x^4+a)/c-1/4*(-b*e+2*c*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2
)^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx=\frac{-\frac{2(-2cd+be)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+e\log\left(a+bx^4+cx^8\right)}{8c}$$

input
```
Integrate[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] + e*Log[a + b*x^4 + c*x^8])/(8*c)
```

3.44.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1798, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (d+e x^4)}{a+b x^4+c x^8} \, dx$$

$$\Big\downarrow \text{ 1798}$$

$$\frac{1}{4} \int \frac{e x^4+d}{c x^8+b x^4+a}dx^4$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{4} \left(\frac{(2 c d-b e) \int \frac{1}{c x^8+b x^4+a}dx^4}{2 c}+\frac{e \int \frac{2 c x^4+b}{c x^8+b x^4+a}dx^4}{2 c}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{4} \left(\frac{e \int \frac{2 c x^4+b}{c x^8+b x^4+a}dx^4}{2 c}-\frac{(2 c d-b e) \int \frac{1}{-x^8+b^2-4 a c}d\left(2 c x^4+b\right)}{c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4} \left(\frac{e \int \frac{2 c x^4+b}{c x^8+b x^4+a}dx^4}{2 c}-\frac{(2 c d-b e) \text{arctanh}\left(\frac{b+2 c x^4}{\sqrt{b^2-4 a c}}\right)}{c \sqrt{b^2-4 a c}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{4} \left(\frac{e \log \left(a+b x^4+c x^8\right)}{2 c}-\frac{(2 c d-b e) \text{arctanh}\left(\frac{b+2 c x^4}{\sqrt{b^2-4 a c}}\right)}{c \sqrt{b^2-4 a c}}\right)$$

input
```
Int[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(-(((2*c*d - b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 -
4*a*c])) + (e*Log[a + b*x^4 + c*x^8])/(2*c))/4
```

3.44.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.44.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

method	result
default	$\frac{e\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{8c}+\frac{\left(d-\frac{be}{2c}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}}$
risch	$\frac{\ln\left(\left(\left(-4abce+8a\,c^{2}d+b^{3}e-2b^{2}cd+\sqrt{-\left(be-2cd\right)^{2}\left(4ac-b^{2}\right)}\,b\right)x^{4}+2\sqrt{-\left(be-2cd\right)^{2}\left(4ac-b^{2}\right)}\,a\right)ae}{8ac-2b^{2}}-\frac{\ln\left(\left(-4abce+8a\,c^{2}d+b^{3}e-2b\right.\right.}{}$

input
```
int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*e*ln(c*x^8+b*x^4+a)/c+1/2*(d-1/2/c*b*e)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x^4+b)/(4*a*c-b^2)^(1/2))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.00

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx$$
$$=\left[\frac{(b^2-4\,ac)e\log\left(cx^8+bx^4+a\right)-\sqrt{b^2-4\,ac}(2\,cd-be)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{8\,(b^2c-4\,ac^2)},\ \frac{(b^2-\cdots}{\cdots}\right.$$

input
```
integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*((b^2 - 4*a*c)*e*log(c*x^8 + b*x^4 + a) - sqrt(b^2 - 4*a*c)*(2*c*d -
b*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4
*a*c))/(c*x^8 + b*x^4 + a)))/(b^2*c - 4*a*c^2), 1/8*((b^2 - 4*a*c)*e*log(c
*x^8 + b*x^4 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^4 +
b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(63) = 126.

Time = 119.86 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.99

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx=\left(\frac{e}{8c}\right.$$
$$\left.-\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c\,(4ac-b^2)}\right)\log\left(x^4+\frac{-16ac\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)+2ae+4b^2\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)\cdots}{be-2cd}\right.$$
$$+\left(\frac{e}{8c}\right.$$
$$\left.+\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c\,(4ac-b^2)}\right)\log\left(x^4+\frac{-16ac\left(\frac{e}{8c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)+2ae+4b^2\left(\frac{e}{8c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)\cdots}{be-2cd}\right.$$

input
```
integrate(x**3*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2)))*log(x**
4 + (-16*a*c*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b*
*2))) + 2*a*e + 4*b**2*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(
4*a*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(8*c) + sqrt(-4*a*c + b**2)*(b*
e - 2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4 + (-16*a*c*(e/(8*c) + sqrt(-4*a*
c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) + 2*a*e + 4*b**2*(e/(8*c) +
sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*
d))
```

3.44.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.44.8 Giac [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{e\log\left(cx^8+bx^4+a\right)}{8\,c} + \frac{(2\,cd-be)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c}$$

input
```
integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*e*log(c*x^8 + b*x^4 + a)/c + 1/4*(2*c*d - b*e)*arctan((2*c*x^4 + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 3704, normalized size of antiderivative = 51.44

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
- (log(a + b*x^4 + c*x^8)*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c))
- (atan((8*x^4*(((a*c - b^2)*(((((4*b^2*e - 16*a*c*e)*(((b*e - 2*c*d)*(448
*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2
- 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)
*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2
 - 16*b^2*c)) + ((b*e - 2*c*d)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*
b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 -
 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e
))/(8*c*(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*
c)) - ((((b*e - 2*c*d)*(((b*e - 2*c*d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (2
56*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2
)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16
*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b
^2*e - 16*a*c*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)))*(
b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) + ((b*e - 2*c*d)*(((4*b^2*e - 16*a
*c*e)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d
 + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2
 - 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*
c)) - 8*c^4*d^3 + 20*b^3*c*e^3 - 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e))/(8*c*
(4*a*c - b^2)^(1/2)) - (b^3*c*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^3)/(2*...
```

3.45 $\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx$

3.45.1 Optimal result

Integrand size = 25, antiderivative size = 375

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)
/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*arc
tanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)/(-4*a
*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*arctan(2^
(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2
)^(1/2))*2^(1/4)/c^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/4*arctanh(2^(1/4)
*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/
2))*2^(1/4)/c^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

3.45.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.16

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]$$

input
```
Integrate[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1
 + 2*c*#1^5) & ]/4
```

3.45.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1834}$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \; 827$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)$$

$$\Big\downarrow \; 218$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)$$

$$\Big\downarrow \; 221$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)$$

input
```
Int[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

3.45. $\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx$

output
```
(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
 - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*
c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (e + (2*c*d - b*e)/Sqrt[b^2 - 4
*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3
/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c]
)^(1/4)))
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.45.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{6}e+\_R^{2}d\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	51
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{6}e+\_R^{2}d\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	51

input
```
int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^6*e+_R^2*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b
+a))
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15561 vs. 2(295) = 590.

Time = 49.64 (sec) , antiderivative size = 15561, normalized size of antiderivative = 41.50

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.45.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.45.7 Maxima [F]

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{(ex^4+d)x^2}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a), x)
```

3.45.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

3.45.9 Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 29445, normalized size of antiderivative = 78.52

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
2*atan(((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4
*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 +
 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b
^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^
4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*
c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^
5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c
^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*
c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2
- 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*
b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2
*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^
2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^
(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)
^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*
b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2
*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2
- 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*...
```

3.46 $\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx$

3.46.1 Optimal result

Integrand size = 23, antiderivative size = 184

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*
d)/(-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*ar
ctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*
a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

input
```
Integrate[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt
[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

3.46.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1814, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}dx$$

$$\Big\downarrow \text{ 1814}$$

$$\frac{1}{2}\int \frac{ex^4+d}{cx^8+bx^4+a}dx^2$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

input
```
Int[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/2
```

3.46.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.46.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.91

method	result
default	$2c\left(\frac{\left(e\sqrt{-4ac+b^2}+be-2cd\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16c^3a^3-8a^2b^2c^2+a\,b^4c\right)\_Z^4+\left(-4a^2bc\,e^2+16a^2c^2de+b^3e^2a-4b^2cdea-4b\,c^2d^2a+b^3c\,d^2\right)\_Z^2+a^2e^4-2abd\,e^3+2ac\,d^2e^2+b^2d^2\right.}\right.$

input
```
int(x*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
2*c*(1/8*(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))-1/8*(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)))
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1535 vs. 2(144) = 288.

Time = 0.41 (sec) , antiderivative size = 1535, normalized size of antiderivative = 8.34

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)
*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b
^2*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 +
1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c
 - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a
*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*sqrt(-(b
*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c
*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log
(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 - 1/2*sqrt(1/2)*((b^2*c
- 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*
(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b
^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*
a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2))) + 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a
*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e
 + a*b*d*e^3 - a^2*e^4)*x^2 + 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^
2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*...
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.46.7 Maxima [F]

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{(ex^4+d)x}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((e*x^4 + d)*x/(c*x^8 + b*x^4 + a), x)
```

3.46.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(144) = 288$.

Time = 2.36 (sec) , antiderivative size = 1402, normalized size of antiderivative = 7.62

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^
2 - 4*a*c)*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arctan(2*sqrt(1/2)*x^2/sq
rt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*
c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/8*((sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sq...
```

3.46.9 Mupad [B] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 4501, normalized size of antiderivative = 24.46

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
atan((b^4*c*d^3*x^2*1i + a^2*b^3*e^3*x^2*1i + a^2*c^3*d^3*x^2*8i - a^2*e^3
*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a^3*b*c*e
^3*x^2*4i - a*b^4*d*e^2*x^2*1i - b*c*d^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^
2*c^2 - 12*a*b^4*c)^(1/2)*1i - a*b^2*c^2*d^3*x^2*6i - a^3*c^2*d*e^2*x^2*8i
 + a*b*d*e^2*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i
 + a*c*d^2*e*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i
 + a^2*b*c^2*d^2*e*x^2*4i + a^2*b^2*c*d*e^2*x^2*6i - a*b^3*c*d^2*e*x^2*1i)
/(8*a^2*b^4*e^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*
b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 1024*a^3*
b^3*c^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*
e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 64*a^3*c^3*d^2*(-
(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)
 - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^
3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 64*a^4*c^2*e^2*(-(a*b^3*e^2
 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(...
```

3.47 $\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$

3.47.1 Optimal result

Integrand size = 22, antiderivative size = 375

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d
)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/4*ar
ctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)/(-4*
a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctan(2
^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^
2)^(1/2))*2^(3/4)/c^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctanh(2^(1/4
)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1
/2))*2^(3/4)/c^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

3.47.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]$$

input
```
Integrate[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1
^3 + 2*c*#1^7) & ]/4
```

3.47.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \; 1752$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+ \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$$\Big\downarrow \ 756$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+ \frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{\sqrt{\sqrt{b^2-4ac}-b}}\right)$$

$$\Big\downarrow \ 218$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+ \frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+ \frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)$$

input
```
Int[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

3.47. $\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$

output
```
((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)
)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*
c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(
2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a
*c])^(3/4))))/2
```

3.47.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

3.47.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4}$	47
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4}$	47

input
```
int((e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^4*e+d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9245 vs. 2(295) = 590.

Time = 2.70 (sec) , antiderivative size = 9245, normalized size of antiderivative = 24.65

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.47.7 Maxima [F]

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \int \frac{ex^4+d}{cx^8+bx^4+a}\,dx$$

input
```
integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a), x)
```

3.47.8 Giac [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

3.47.9 Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 36707, normalized size of antiderivative = 97.89

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(a + b*x^4 + c*x^8),x)
```

output
```
- atan((((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 1
1*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3
*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6
*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4
*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 +
 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 +
a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b
^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
+ 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^
4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d
^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*
a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^
7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(
1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096
*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c
^4*d - 262144*a^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 ...
```

3.48 $\int \frac{d+ex^4}{x(a+bx^4+cx^8)}\,dx$

3.48.1 Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx=\frac{(bd-2ae)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}$$

output
```
d*ln(x)/a-1/8*d*ln(c*x^8+b*x^4+a)/a+1/4*(-2*a*e+b*d)*arctanh((2*c*x^4+b)/(
-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

3.48.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx=\frac{d\log(x)}{a}-\frac{\operatorname{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a}$$

input
```
Integrate[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

output
```
(d*Log[x])/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[
x - #1] + c*d*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

3.48.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{4}\int \frac{ex^4+d}{x^4\left(cx^8+bx^4+a\right)}dx^4$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4}\int \left(\frac{d}{ax^4}+\frac{-cdx^4-bd+ae}{a\left(cx^8+bx^4+a\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^4+cx^8\right)}{2a}+\frac{d\log\left(x^4\right)}{a}\right)$$

input
```
Int[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

output
```
(((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x^4])/a - (d*Log[a + b*x^4 + c*x^8])/(2*a))/4
```

3.48.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.48.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

method	result
default	$\frac{d\ln(x)}{a}+\frac{-\frac{d\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4}+\frac{\left(ae-\frac{bd}{2}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a}$
risch	$\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4c\,a^{2}-b^{2}a\right)\_Z^{2}+\left(4acd-b^{2}d\right)\_Z+a\,e^{2}-bde+c\,d^{2}\right)}\_R\ln\left(\left(\left(18ac-5b^{2}\right)\_R^{2}+\left(-be+9cd\right)\_R+4e^{2}\right)x^{4}-b\right.\right.}{4}$

input
```
int((e*x^4+d)/x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
d*ln(x)/a+1/2/a*(-1/4*d*ln(c*x^8+b*x^4+a)+(a*e-1/2*b*d)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.08

$$\int\frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx$$
$$=\left[-\frac{(b^2-4\,ac)d\log\left(cx^8+bx^4+a\right)-8\left(b^2-4\,ac\right)d\log\left(x\right)+\sqrt{b^2-4\,ac}(bd-2\,ae)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac}{cx^8+b}\right.}{8\left(ab^2-4\,a^2c\right)}\right.$$
$$-\frac{(b^2-4\,ac)d\log\left(cx^8+bx^4+a\right)-8\left(b^2-4\,ac\right)d\log\left(x\right)-2\sqrt{-b^2+4\,ac}(bd-2\,ae)\arctan\left(-\frac{(2\,cx^4+b)\sqrt{}}{b^2-4}\right.}{8\left(ab^2-4\,a^2c\right)}$$

input
```
integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[-1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) +
 sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c
- (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)))/(a*b^2 - 4*a^2*c)
, -1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x)
- 2*sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a
*c)/(b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

input
```
integrate((e*x**4+d)/x/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.48.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.48.8 Giac [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx = -\frac{d\log\left(cx^8+bx^4+a\right)}{8\,a}+\frac{d\log\left(x^4\right)}{4\,a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a}$$

input
```
integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/8*d*log(c*x^8 + b*x^4 + a)/a + 1/4*d*log(x^4)/a - 1/4*(b*d - 2*a*e)*arc
tan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 8454, normalized size of antiderivative = 108.38

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x)
```

output
```
(d*log(x))/a - (log(a + b*x^4 + c*x^8)*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2
- 64*a^2*c)) + (atan((128*a^5*x^4*(((c^4*e^5 - ((4*b^2*d - 16*a*c*d)*(11*b
*c^4*e^4 + 9*c^5*d*e^3 - ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4
*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))
/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c
^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432
*b^2*c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*c^4*e^3 +
 108*b*c^5*d*e^2))/(2*(16*a*b^2 - 64*a^2*c))))/(2*(16*a*b^2 - 64*a^2*c)) -
 ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d
 - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) +
576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/
2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))
/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*
a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*
(2*a*e - b*d)^2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*
b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)
*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*
d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^
2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a
*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*b...
```

3.48. $\int \frac{d+ex^4}{x(a+bx^4+cx^8)}\,dx$

3.49 $\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx$

3.49.1 Optimal result

Integrand size = 25, antiderivative size = 392

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{ax} - \frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
-d/a/x-1/4*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))
*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4
)+1/4*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(d+
(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/
4*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*a
*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^
(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*a*e+
b*d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

3.49.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.22

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{ax}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{4a}$$

input
```
Integrate[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

output
```
-(d/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x
- #1] + c*d*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

3.49.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1828, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx$$

$$\Big\downarrow \text{1828}$$

$$-\frac{\int \frac{x^2\left(cdx^4+bd-ae\right)}{cx^8+bx^4+a}dx}{a}-\frac{d}{ax}$$

$\Big\downarrow$ 1834

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{a}-\frac{d}{ax}$$

$\Big\downarrow$ 27

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{a}-\frac{d}{ax}$$

$\Big\downarrow$ 827

$$-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{2\sqrt{2}\sqrt{c}}\cdots\right)}{a}$$
$$\frac{d}{ax}$$

$\Big\downarrow$ 218

$$-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\cdots}{\sqrt[4]{\sqrt{b^2-\cdots}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2\cdots}}}\cdots\right)}{a}$$
$$\frac{d}{ax}$$

$\Big\downarrow$ 221

$$-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\arctan\left(\cdots\right)}{2\ 2^{3/\cdots}}\cdots\right)}{a}$$
$$\frac{d}{ax}$$

input
```
Int[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

3.49. $\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx$

output
```
-(d/(a*x)) - (c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(
1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1
/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + c*(d + (b*d - 2
*a*e)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*
c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2
^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b +
 Sqrt[b^2 - 4*a*c])^(1/4))))/a
```

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

3.49.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.19

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-cd\_R^{6}+(ae-bd)\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4a}-\frac{d}{ax}$	73
risch	Expression too large to display	1333

input
```
int((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*sum((-c*d*_R^6+(a*e-b*d)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(
_Z^8*c+_Z^4*b+a))-d/a/x
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21400 vs. 2(312) = 624.

Time = 137.79 (sec) , antiderivative size = 21400, normalized size of antiderivative = 54.59

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

input
```
integrate((e*x**4+d)/x**2/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.49.7 Maxima [F]

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx=\int \frac{ex^4+d}{(cx^8+bx^4+a)x^2}\,dx$$

input
```
integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*d*x^6 + (b*d - a*e)*x^2)/(c*x^8 + b*x^4 + a), x)/a - d/(a*x)
```

3.49.8 Giac [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

input
```
integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

3.49.9 Mupad [B] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 39028, normalized size of antiderivative = 99.56

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x)
```

output
```
atan((((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d
^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*
b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a
^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200
*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^
5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a
^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^
9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 -
4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d
^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^
7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c -
 b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*...
```

3.50 $\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx$

3.50.1 Optimal result

Integrand size = 25, antiderivative size = 199

$$\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx = -\frac{d}{2ax^2} - \frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/2*d/a/x^2-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*
c^(1/2)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1
/2)*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1
/2)
```

3.50.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.45

$$\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx = -\frac{d}{2ax^2} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1^2+2c\#1^6}\&\right]}{4a}$$

input
```
Integrate[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/2*d/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Lo
g[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

3.50.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1814, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int \frac{ex^4+d}{x^4\left(cx^8+bx^4+a\right)}dx^2$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{2}\left(-\frac{\int \frac{cdx^4+bd-ae}{cx^8+bx^4+a}dx^2}{a}-\frac{d}{ax^2}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2}{a}-\frac{d}{ax^2}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(-\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{d}{ax^2}\right)$$

input
```
Int[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(d/(a*x^2)) - ((Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/a)/2
```

3.50.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.50.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.89

method	result
default	$\frac{2c\left(\frac{\left(bd-2ae-d\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-d\sqrt{-4ac+b^2}+2ae-bd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a}-\frac{d}{2a\,x^2}$
risch	$-\frac{d}{2a\,x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^5c^2-8a^4b^2c+b^4a^3\right)\_Z^4+\left(-4a^3b\,e^2c-16a^3de\,c^2+a^2b^3e^2+12a^2b^2dec+12a^2b\,c^2d^2-2a\,b^4de-7a\,b^3c\,d^2+b^5d^2\right)\right.}\right.}{}$

input
```
int((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
2/a*c*(1/8*(b*d-2*a*e-d*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))-1/8*(-d*(-4*a*c+b^2)^(1/2)+2*a*e-b*d)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)))-1/2*d/a/x^2
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2772 vs. 2(157) = 314.

Time = 1.06 (sec) , antiderivative size = 2772, normalized size of antiderivative = 13.93

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2
*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 -
2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*
c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*
e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c
^2)*d^3*e)*x^2 + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a
*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (
a^3*b^2 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3
 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c
^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*
b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2
*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*
b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d
^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt
(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 -
4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4
+ 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*
a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a
^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt
(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + ...
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/x**3/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.50.7 Maxima [F]

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \int \frac{ex^4+d}{(cx^8+bx^4+a)x^3}\,dx$$

input
```
integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*d*x^4 + b*d - a*e)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2*d/(a*x^
2)
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3003 vs. $2(157) = 314$.

Time = 2.11 (sec) , antiderivative size = 3003, normalized size of antiderivative = 15.09

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^
3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a
*c)*a*c^3)*d*x^4*abs(a) - (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2
*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^
2)*d*abs(a) - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^3*c - 2*a*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c...
```

3.50.9 Mupad [B] (verification not implemented)

Time = 12.12 (sec) , antiderivative size = 15013, normalized size of antiderivative = 75.44

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x)
```

output
```
- atan((((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2
*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c
*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e +
 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*
a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a
*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 -
 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*
c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*
b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*
d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(
1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(
9216*a^11*b^5*c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^
11*b^6*c^4*e - 8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c
^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 1
2*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16...
```

3.50. $\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx$

3.51 $\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx$

3.51.1 Optimal result

Integrand size = 25, antiderivative size = 394

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx=-\frac{d}{3ax^3}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
-1/3*d/a/x^3+1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^
(1/4))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2)
)^(3/4)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4
))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3
/4)+1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d
+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+
1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-
2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

3.51.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.22

$$\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx = -\frac{\frac{4d}{x^3}+3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{12a}$$

input
```
Integrate[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/12*((4*d)/x^3 + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*
e*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/a
```

3.51.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1828, 27, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx$$

$$\Big\downarrow \ 1828$$

$$-\frac{\int \frac{3(cdx^4+bd-ae)}{cx^8+bx^4+a}dx}{3a}-\frac{d}{3ax^3}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{cdx^4+bd-ae}{cx^8+bx^4+a}dx}{a}-\frac{d}{3ax^3}$$

$\Big\downarrow$ 1752

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{d}{3ax^3}$$

$\Big\downarrow$ 756

$$-\frac{\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}}}}{\sqrt{\sqrt{b^2-4ac}-}}\right.}{a}$$

$$\frac{d}{3ax^3}$$

$\Big\downarrow$ 218

$$-\frac{\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4a}}}}{\sqrt{\sqrt{b}}}\right.}{a}$$

$$\frac{d}{3ax^3}$$

$\Big\downarrow$ 221

$$-\frac{\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(-\frac{a}{-}\right.}{a}$$

$$\frac{d}{3ax^3}$$

input
```
Int[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

3.51. $\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx$

output
```
-1/3*d/(a*x^3) - ((c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1
/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt
[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*
c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + (c*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[
b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - A
rcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4
)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/a
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.51.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-cd\_R^{4}+ae-bd\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4a}-\frac{d}{3a\,x^{3}}$	68
risch	Expression too large to display	1633

input
```
int((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*sum((-_R^4*c*d+a*e-b*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+
_Z^4*b+a))-1/3*d/a/x^3
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20184 vs. 2(312) = 624.

Time = 52.64 (sec) , antiderivative size = 20184, normalized size of antiderivative = 51.23

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/x**4/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

3.51.7 Maxima [F]

$$\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx = \int \frac{ex^4+d}{(cx^8+bx^4+a)x^4}\,dx$$

input
```
integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*d*x^4 + b*d - a*e)/(c*x^8 + b*x^4 + a), x)/a - 1/3*d/(a*x^3)
```

3.51.8 Giac [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

3.51.9 Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 65350, normalized size of antiderivative = 165.86

$$\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x)
```

output
```
atan((((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*
a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c -
 b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 +
 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*
d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a
^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a
^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2)
 - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^
6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3
*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e
- 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 +
16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^
6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e
^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e
^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e
^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a...
```

3.52 $\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx$

3.52.1 Optimal result

Integrand size = 23, antiderivative size = 278

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

output
```
-x-1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6
^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2))
)*6^(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1
/2)))*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2
^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*l
n(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+
1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

3.52.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x+\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
-x + RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

3.52.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.49, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1826, 25, 1684, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(1-x^4)}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{ 1826}$$

$$-\int -\frac{1}{x^8-x^4+1}dx-x$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{1}{x^8-x^4+1}dx-x$$

$$\Big\downarrow \text{ 1684}$$

$$\frac{\int \frac{\sqrt{3}-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-x$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{\frac{\int \frac{(1-\sqrt{3})x+\sqrt{3(2-\sqrt{3})}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3(2-\sqrt{3})}-(1-\sqrt{3})x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{3(2+\sqrt{3})}-(1+\sqrt{3})x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{(1+\sqrt{3})x+\sqrt{3(2+\sqrt{3})}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-x$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-x$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-x$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$x$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-x$$

$$\Big\downarrow \text{1103}$$

3.52. $\int \frac{x^4\left(1-x^4\right)}{1-x^4+x^8}\,dx$

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-x$$

input
```
Int[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
-x + ((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sq
rt[3]]] + ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2
- Sqrt[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt
[2 + Sqrt[3]]] - ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*
Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((Sqrt[2/(2 - Sqrt[3])]*ArcTan[(-Sqrt[2
+ Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((1 + Sqrt[3])*Log[1 - Sqrt[2 + Sqr
t[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (Sqrt[2/(2 - Sqrt[3])]*ArcTan[(
Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + ((1 + Sqrt[3])*Log[1 + Sqrt[
2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1684
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x^(
n/2))/(q - r*x^(n/2) + x^n), x], x] + Simp[1/(2*c*q*r)   Int[(r + x^(n/2))/
(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

3.52.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.12

method	result	size
default	$-x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{4}+1\right)}\_R\ln\left(3\_R^{2}+3\_Rx+x^{2}\right)\right)}{4}$	34
risch	$-x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{4}+1\right)}\_R\ln\left(3\_R^{2}+3\_Rx+x^{2}\right)\right)}{4}$	34

input
```
int(x^4*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

3.52.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.37

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = \left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right) - \left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right) + \left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right) - \left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right) - x$$

input
```
integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
(1/24*I + 1/24)*sqrt(3)*sqrt(2)*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2 +
6*I) - (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log(-(3*I - 3)*sqrt(3)*sqrt(2)*x +
6*x^2 - 6*I) + (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log((3*I - 3)*sqrt(3)*sqrt(
2)*x + 6*x^2 - 6*I) - (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log(-(3*I + 3)*sqrt(
3)*sqrt(2)*x + 6*x^2 + 6*I) - x
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.61

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x - \frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} - \frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

input
```
integrate(x**4*(-x**4+1)/(x**8-x**4+1),x)
```

output
```
-x - sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 +
2*sqrt(6)*x - 3))/24 - sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6
)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 +
 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + s
qrt(6)*x + 1)/24
```

3.52.7 Maxima [F]

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = \int -\frac{(x^4-1)x^4}{x^8-x^4+1}\,dx$$

input
```
integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-x + integrate(1/(x^8 - x^4 + 1), x)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx &= \frac{1}{12}\,\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\,\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &\quad+\frac{1}{12}\,\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\,\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &\quad+\frac{1}{24}\,\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &\quad-\frac{1}{24}\,\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &\quad+\frac{1}{24}\,\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ &\quad-\frac{1}{24}\,\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-x\end{aligned}$$

input
```
integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(
6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*ar
ctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2
 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
+ sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) -
1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - x
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

input
```
int(-(x^4*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

output
```
- x - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i
/12) - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1
i/12)
```

3.53 $\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx$

3.53.1 Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
-1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^4+x^8\right)$$

input
```
Integrate[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

3.53.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1798, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(1-x^4)}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{ 1798}$$

$$\frac{1}{4}\int \frac{1-x^4}{x^8-x^4+1}dx^4$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^4-\frac{1}{2}\int -\frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^4+\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4-\int \frac{1}{-x^8-3}d(2x^4-1)\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4+\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^8-x^4+1\right)\right)$$

input
```
Int[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
(ArcTan[(-1 + 2*x^4)/Sqrt[3]]/Sqrt[3] - Log[1 - x^4 + x^8]/2)/4
```

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.53.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)}{12}$	33
risch	$-\frac{\ln\left(4x^{8}-4x^{4}+4\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)}{12}$	35

input
```
int(x^3*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) - \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\log\left(x^8-x^4+1\right)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**3*(-x**4+1)/(x**8-x**4+1),x)
```

output
```
-log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) - \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) - \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\ln\left(x^8-x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

input
```
int(-(x^3*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

output
```
- log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

3.54 $\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$

3.54.1 Optimal result

Integrand size = 23, antiderivative size = 355

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right) + \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$

output
```
1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln
(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((
-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*
6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)
))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/
2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1
/2)+1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/
2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1
/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.15

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1+2\#1^5}\&\right]$$

input
```
Integrate[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
-1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1 +
2*#1^5) & ]
```

3.54.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1830, 1602, 25, 1483, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-x^4)}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \; 1830$$

$$\frac{\int \frac{x^2\left(\sqrt{3}-2x^2\right)}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2\left(2x^2+\sqrt{3}\right)}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \; 1602$$

$$\frac{-\int -\frac{2-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx-2x}{2\sqrt{3}}+\frac{2x-\int \frac{\sqrt{3}x^2+2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx-2x}{2\sqrt{3}}+\frac{2x-\int \frac{\sqrt{3}x^2+2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \; 1483$$

$$\frac{-\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}\left(\sqrt{2-\sqrt{3}}x+2\right)}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+2x}{2\sqrt{3}}+\frac{\frac{\int \frac{2\sqrt{2+\sqrt{3}}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}\left(\sqrt{2+\sqrt{3}}x+2\right)}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}-2x}{2\sqrt{3}}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}x+2}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+2x}{2\sqrt{3}}+\frac{\frac{\int \frac{2\sqrt{2+\sqrt{3}}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}x+2}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-2x}{2\sqrt{3}}$$

$\downarrow$ 1142

$$\frac{-\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\sqrt{\cdots}}{x^2+\sqrt{2-\cdots}}\cdots\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\left(2+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{\cdots}}}{x^2+\sqrt{2+\sqrt{3}}\cdots}\cdots\right.}{2\sqrt{3}}$$

$\downarrow$ 25

$$\frac{-\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\sqrt{2-\cdots}}{x^2+\sqrt{2-\cdots}}\cdots\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+\cdots}\cdots\right.}{2\sqrt{3}}$$

$\downarrow$ 1083

$$\frac{-\frac{\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(\left(2+\sqrt{3}\right)\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x+\cdots\right)\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\left(2-\sqrt{3}\right)\cdots\right.}{2\sqrt{3}}$$

$\downarrow$ 217

3.54. $\int \frac{x^2\left(1-x^4\right)}{1-x^4+x^8}\,dx$

$$\frac{-\frac{\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right.\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)\right)}{2\sqrt{3}}$$

$\downarrow$ 1103

$$\frac{-\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\sqrt{2+\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\sqrt{2-\sqrt{3}}\log\left(x^2+\sqrt{2}\right.\right.}{2\sqrt{3}}$$

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\log\left(x^2+\sqrt{2+}\right.\right.}{2\sqrt{3}}$$

input
```
Int[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
(2*x - (ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] - ((2 - Sqrt[
3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]) + (-(Sqrt
[2 + Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]) - (Sqrt
[2 - Sqrt[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/2)/(2*Sqrt[3]) + (-2*
x + (ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((2 + Sqrt[3])
*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (Sqrt[2 -
Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + (Sqrt[2 + S
qrt[3]]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/2)/2)/(2*Sqrt[3])
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1830
```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]
/(q - r*x^(n/2) + c*x^n)), x], x] + Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r
+ (c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q
- b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a
*c, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

3.54. $\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$

3.54.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.13

method	result	size
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(\_R^6-\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	46
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^6+\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	46

input
```
int(x^2*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

3.54.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.54

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$

3.54. $\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$

input
```
integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(I*sqrt(3)*sqr
t(2) - 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3) + 1) +
24*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqr
t(3)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3)
 + 1) + 24*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6
)*(I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(
I*sqrt(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*l
og(sqrt(6)*(-I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) +
 1))*sqrt(I*sqrt(3) + 1) + 24*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(
3) + 1))*log(sqrt(6)*(I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(2)*sqrt(-I*
sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(sqrt(2)*sqr
t(-I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(
2)*sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(
-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*(I*sqrt(3)*sqrt(2) + 3*sqrt(2))
*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) + 1/24*s
qrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqrt(3)*sqrt(2)
 - 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 2
4*x)
```

3.54.6 Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.08

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = -\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(442368t^7-384t^3+x\right)\right)\right)$$

input
```
integrate(x**2*(-x**4+1)/(x**8-x**4+1),x)
```

output
```
-RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(442368*_t**7 -
384*_t**3 + x)))
```

3.54.7 Maxima [F]

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \int -\frac{(x^4-1)x^2}{x^8-x^4+1}\,dx$$

input
```
integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)*x^2/(x^8 - x^4 + 1), x)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = & -\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\ & -\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\ & -\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\ & -\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\ & +\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\ & -\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\ & +\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) \\ & -\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

input
```
integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sq
rt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt
(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(
6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*
x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(s
qrt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.70

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{\sqrt{3}\,x\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\sqrt{3}\,x\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}-\frac{2^{3/4}\,\sqrt{3}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} - \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}+\frac{2^{3/4}\,\sqrt{3}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(-(x^2*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

output
```
(3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x
*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1
i)/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) -
 (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)
^(1/4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(3/4))
- (2^(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4
)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4))
 + (2^(3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4))
/12
```

3.55 $\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx$

3.55.1 Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

output
```
-1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = \frac{-\log\left(-1+\sqrt{3}x^2-x^4\right)+\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

input
```
Integrate[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
(-Log[-1 + Sqrt[3]*x^2 - x^4] + Log[1 + Sqrt[3]*x^2 + x^4])/(4*Sqrt[3])
```

3.55.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1814, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-x^4)}{x^8-x^4+1} \, dx$$

$$\Big\downarrow \text{ 1814}$$

$$\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2$$

$$\Big\downarrow \text{ 1478}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
(-1/2*Log[1 - Sqrt[3]*x^2 + x^4]/Sqrt[3] + Log[1 + Sqrt[3]*x^2 + x^4]/(2*S
qrt[3]))/2
```

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.55.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	39
risch	$-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	39

input
```
int(x*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{x^8+5\,x^4+2\,\sqrt{3}(x^6+x^2)+1}{x^8-x^4+1}\right)$$

input
```
integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4 + 1)
)
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

input
```
integrate(x*(-x**4+1)/(x**8-x**4+1),x)
```

output
```
-sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
 + 1)/12
```

3.55.7 Maxima [F]

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = \int -\frac{(x^4-1)x}{x^8-x^4+1}\,dx$$

input
```
integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = -\frac{1}{12}\sqrt{3}\log\left(\frac{x^2-\sqrt{3}+\frac{1}{x^2}}{x^2+\sqrt{3}+\frac{1}{x^2}}\right)$$

input
```
integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*log((x^2 - sqrt(3) + 1/x^2)/(x^2 + sqrt(3) + 1/x^2))
```

3.55.9 Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x^2}{x^4+1}\right)}{6}$$

input
```
int(-(x*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

output
```
(3^(1/2)*atanh((3^(1/2)*x^2)/(x^4 + 1)))/6
```

3.56 $\int \frac{1-x^4}{1-x^4+x^8} \, dx$

3.56.1 Optimal result

Integrand size = 20, antiderivative size = 355

$$\begin{aligned}\int \frac{1-x^4}{1-x^4+x^8} \, dx = &-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} \\ &-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) \\ &-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) \\ &-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right) \\ &+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)\end{aligned}$$

output
```
1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln
(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((
-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*
6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)
))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/
2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1
/2)+1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/
2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1
/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

3.56.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.16

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx=-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
```

output
```
-1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3
+ 2*#1^7) & ]
```

3.56.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1751, 25, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \ 1751$$

$$-\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \ 1483$$

$$\frac{\frac{\int \frac{\left(2-\sqrt{3}\right)x+\sqrt{3\left(2-\sqrt{3}\right)}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(2-\sqrt{3}\right)x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(2+\sqrt{3}\right)x+\sqrt{3\left(2+\sqrt{3}\right)}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\left(2+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{}{-\left(2x+\sqrt{2-\right.}}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right.}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

3.56. $\int \frac{1-x^4}{1-x^4+x^8}\,dx$

input
```
Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

output
```
((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] + ((2 - Sqrt[3])*Lo
g[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]) + (ArcTan[(Sqrt
[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] - ((2 - Sqrt[3])*Log[1 + Sqrt[2 -
Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((-ArcTan[(-Sqr
t[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((2 + Sqrt[3])*Log[1 - Sqrt[2 +
 Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-ArcTan[(Sqrt[2 + Sqrt[3]]
 + 2*x)/Sqrt[2 - Sqrt[3]]] + ((2 + Sqrt[3])*Log[1 + Sqrt[2 + Sqrt[3]]*x +
x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1751
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x
^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Simp[e/(2*c*q)   Int[(q +
2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGt
Q[n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.56.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44

input
```
int((-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

3.56.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{1-x^4}{1-x^4+x^8}\,dx &= \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)+12\,x\right)\\
&\quad+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)+12\,x\right)\\
&\quad-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)+12\,x\right)\\
&\quad-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)+12\,x\right)\\
&\quad+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)+12\,x\right)\\
&\quad+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)+12\,x\right)\\
&\quad-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)+12\,x\right)\\
&\quad-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)+12\,x\right)\end{aligned}$$

input
```
integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*s
qrt(-I*sqrt(3) + 1))*(I*sqrt(3) + 3) + 12*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*
sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(I*s
qrt(3) + 3) + 12*x) - 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(s
qrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) - 1/24*sq
rt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(I*
sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*s
qrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3
) + 12*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sq
rt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*x) - 1/24*sqrt(6)*s
qrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3)
 + 1))*(-I*sqrt(3) - 3) + 12*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(
3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I*sqrt(3) - 3)
+ 12*x)
```

3.56.6 Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.07

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(9216t^5 - 8t + x\right)\right)\right)$$

input
```
integrate((-x**4+1)/(x**8-x**4+1),x)
```

output
```
-RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*
_t + x)))
```

3.56.7 Maxima [F]

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-x^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{1-x^4}{1-x^4+x^8}\,dx &= \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ &-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

input
```
integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(
2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6
) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6
) - sqrt(2)) + 1)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.59

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(-(x^4 - 1)/(x^8 - x^4 + 1),x)
```

output
```
(2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^
(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3
^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(
1/4))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4)
+ (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (
2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*
3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.57 $\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx$

3.57.1 Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx=\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
ln(x)-1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

3.57.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx=\log(x)-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]$$

input
```
Integrate[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

output
```
Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 + 2*#1^4) & ]/
4
```

3.57.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x\left(x^8-x^4+1\right)}dx$$

$$\Big\downarrow \ 1802$$

$$\frac{1}{4}\int \frac{1-x^4}{x^4\left(x^8-x^4+1\right)}dx^4$$

$$\Big\downarrow \ 1200$$

$$\frac{1}{4}\int \left(\frac{1}{x^4}-\frac{x^4}{x^8-x^4+1}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^4\right)-\frac{1}{2}\log\left(x^8-x^4+1\right)\right)$$

input
```
Int[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

output
```
(ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[x^4] - Log[1 - x^4 + x^8]/2)/4
```

3.57.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.57.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x)-\frac{\ln(x^{8}-x^{4}+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{4}-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$	33
default	$\ln(x)-\frac{\ln(x^{8}-x^{4}+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)}{12}$	35

input
```
int((-x^4+1)/x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(2/3*(x^4-1/2)*3^(1/2))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx=-\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)-\frac{1}{8}\,\log\left(x^8-x^4+1\right)+\log\left(x\right)$$

input
```
integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + l
og(x)
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx = \log(x) - \frac{\log(x^8-x^4+1)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate((-x**4+1)/x/(x**8-x**4+1),x)
```

output
```
log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/
3)/12
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) - \frac{1}{8}\log\left(x^8-x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1
/4*log(x^4)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) - \frac{1}{8}\log\left(x^8-x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1
/4*log(x^4)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx = \ln\left(x\right)-\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

input
```
int(-(x^4 - 1)/(x*(x^8 - x^4 + 1)),x)
```

output
```
log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/
3))/12
```

3.58 $\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx$

3.58.1 Optimal result

Integrand size = 23, antiderivative size = 280

$$\begin{aligned}\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx = & -\frac{1}{x}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}\\ & -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}\\ & -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}\\ & -\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}\end{aligned}$$

output
```
-1/x+1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))
*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2
)))*6^(1/2)+1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^
(1/2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2
*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24
*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2
)+1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

3.58.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.17

$$\int \frac{1-x^4}{x^2 (1-x^4+x^8)} \, dx = -\frac{1}{x} - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1^3}{-1+2\#1^4}\&\right]$$

input
```
Integrate[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

output
```
-x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]
/4
```

3.58.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.49, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1828, 1708, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^2 (x^8-x^4+1)} \, dx$$

$$\Big\downarrow \text{ 1828}$$

$$-\int \frac{x^6}{x^8-x^4+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{ 1708}$$

$$\frac{\int \frac{1-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{\sqrt{3}x^2+1}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{ 1483}$$

$$-\frac{\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\left(1-\sqrt{3}\right)x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(1+\sqrt{3}\right)x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{ 1142}$$

$$-\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{x}$$

$\downarrow$ 25

$$-\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{x}$$

$\downarrow$ 1083

$$-\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-\cdots}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\cdots}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{1}{x}$$

$\downarrow$ 217

$$-\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{x}$$

$\downarrow$ 1103

3.58. $\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx$

$$-\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{x}$$

input
```
Int[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

output
```
-x^(-1) - ((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2
 + Sqrt[3]]] - ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sq
rt[2 - Sqrt[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)
/Sqrt[2 + Sqrt[3]]] + ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2
)/(2*Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((-(Sqrt[2/(2 - Sqrt[3])]*ArcTan[(-
Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) - ((1 + Sqrt[3])*Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-(Sqrt[2/(2 - Sqrt[3])
]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) + ((1 + Sqrt[3])*Lo
g[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1708
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Simp[1/(2*c*r)   Int[x^
(m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Simp[1/(2*c
*r)   Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]]
 /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2,
 0] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.58.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.14

method	result	size
default	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+1\right)} \_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}-\frac{1}{x}$	38
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+1\right)} \_R\ln\left(-9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	38

input
```
int((-x^4+1)/x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))-1/x
```

3.58.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.40

$$\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx$$
$$= \frac{-(i-1)\,\sqrt{3}\sqrt{2}x\log\left((3i+3)\,\sqrt{3}\sqrt{2}x+6\,x^2+6i\right)+(i+1)\,\sqrt{3}\sqrt{2}x\log\left(-(3i-3)\,\sqrt{3}\sqrt{2}x+6\,x^2-6i\right.}{}$$

input
```
integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/24*(-(I - 1)*sqrt(3)*sqrt(2)*x*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2 +
 6*I) + (I + 1)*sqrt(3)*sqrt(2)*x*log(-(3*I - 3)*sqrt(3)*sqrt(2)*x + 6*x^2
 - 6*I) - (I + 1)*sqrt(3)*sqrt(2)*x*log((3*I - 3)*sqrt(3)*sqrt(2)*x + 6*x^
2 - 6*I) + (I - 1)*sqrt(3)*sqrt(2)*x*log(-(3*I + 3)*sqrt(3)*sqrt(2)*x + 6*
x^2 + 6*I) - 24)/x
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.60

$$\begin{aligned}\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx = &-\frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}\\ &-\frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}\\ &-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}\\ &+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}-\frac{1}{x}\end{aligned}$$

input
```
integrate((-x**4+1)/x**2/(x**8-x**4+1),x)
```

output
```
-sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqr
t(6)*x - 3))/24 - sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3
 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**
2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)
*x + 1)/24 - 1/x
```

3.58.7 Maxima [F]

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx = \int -\frac{x^4-1}{(x^8-x^4+1)x^2}\,dx$$

input
```
integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/x - integrate(x^6/(x^8 - x^4 + 1), x)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.75

$$\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{x}$$

input
```
integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12
*sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt
(6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*a
rctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^
2 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
 + sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) -
 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/x
```

3.58.9 Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.21

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx = -\frac{1}{x}+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

input
```
int(-(x^4 - 1)/(x^2*(x^8 - x^4 + 1)),x)
```

output
```
6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12)
- 1/x
```

3.59 $\int \frac{1-x^4}{x^3(1-x^4+x^8)}\,dx$

3.59.1 Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx=-\frac{1}{2x^2}+\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}$$

output
```
-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4
-x^2*3^(1/2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

3.59.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.55

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx=-\frac{1}{2x^2}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^4}\&\right]$$

input
```
Integrate[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

output
```
-1/2*1/x^2 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^4)
& ]/4
```

3.59.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1814, 1604, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^3\left(x^8-x^4+1\right)}dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int \frac{1-x^4}{x^4\left(x^8-x^4+1\right)}dx^2$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{2}\left(-\int \frac{x^4}{x^8-x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{2}\int \frac{x^4+1}{x^8-x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx^2-\frac{1}{2}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx^2\right)+\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{-x^4-1}d\left(2x^2-\sqrt{3}\right)+\int \frac{1}{-x^4-1}d\left(2x^2+\sqrt{3}\right)\right)+\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{x^2}+\frac{1}{2}\left(\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}\right)\right)$$

input
```
Int[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

output
```
(-x^(-2) + (ArcTan[Sqrt[3] - 2*x^2] - ArcTan[Sqrt[3] + 2*x^2])/2 + (-1/2*L
og[1 - Sqrt[3]*x^2 + x^4]/Sqrt[3] + Log[1 + Sqrt[3]*x^2 + x^4]/(2*Sqrt[3])
)/2)/2
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

3.59.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-6\_R^3+x^2-\_R\right)\right)}{4}$	40
default	$-\frac{1}{2x^2}+\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2-\sqrt{3}\right)\right)}{12}+\frac{\sqrt{3}\left(\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2+\sqrt{3}\right)\right)}{12}$	82

input
```
int((-x^4+1)/x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+1/4*sum(_R*ln(-6*_R^3+x^2-_R),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

3.59.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.90

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = -\frac{\sqrt{6}x^2\sqrt{i\sqrt{3}-1}\log\left(6x^2+i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right)-\sqrt{6}x^2\sqrt{i\sqrt{3}-1}\log\left(6x^2-i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right)-}{}$$

input
```
integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(sqrt(6)*x^2*sqrt(I*sqrt(3) - 1)*log(6*x^2 + I*sqrt(6)*sqrt(3)*sqrt(
I*sqrt(3) - 1)) - sqrt(6)*x^2*sqrt(I*sqrt(3) - 1)*log(6*x^2 - I*sqrt(6)*sq
rt(3)*sqrt(I*sqrt(3) - 1)) - sqrt(6)*x^2*sqrt(-I*sqrt(3) - 1)*log(6*x^2 +
I*sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1)) + sqrt(6)*x^2*sqrt(-I*sqrt(3) - 1)
*log(6*x^2 - I*sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1)) + 12)/x^2
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = -\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24} - \frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{1}{2x^2}$$

input
```
integrate((-x**4+1)/x**3/(x**8-x**4+1),x)
```

output
```
-sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
 + 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 - 1/(2*x**2
)
```

3.59.7 Maxima [F]

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = \int -\frac{x^4-1}{(x^8-x^4+1)x^3}\,dx$$

input
```
integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/2/x^2 - integrate(x^5/(x^8 - x^4 + 1), x)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = -\frac{1}{24}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right)+\frac{1}{24}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right) - \frac{1}{4}\,x^4\arctan\left(2\,x^2+\sqrt{3}\right)-\frac{1}{4}\,x^4\arctan\left(2\,x^2-\sqrt{3}\right)-\frac{1}{2\,x^2}$$

input
```
integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*sqrt(3)*x^4*log(x^4 -
sqrt(3)*x^2 + 1) - 1/4*x^4*arctan(2*x^2 + sqrt(3)) - 1/4*x^4*arctan(2*x^2
- sqrt(3)) - 1/2/x^2
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = \mathrm{atan}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \mathrm{atan}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{1}{2\,x^2}$$

input
```
int(-(x^4 - 1)/(x^3*(x^8 - x^4 + 1)),x)
```

output
```
atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(
1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - 1/(2*x^2)
```

3.60 $\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx$

3.60.1 Optimal result

Integrand size = 23, antiderivative size = 370

$$\begin{aligned}\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx = &-\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)\\ &-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\ &+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)\\ &-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)\\ &-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)\\ &+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)\end{aligned}$$

output
```
-1/3/x^3-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2
)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/
2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1
/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/
2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(
1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/4*arctan((2*x
+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1
/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1
/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

3.60.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.13

$$\int \frac{1-x^4}{x^4 (1-x^4+x^8)} \, dx = -\frac{1}{3x^3} - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]$$

input
```
Integrate[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) &
]/4
```

3.60.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1828, 27, 1709, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^4 (x^8-x^4+1)} \, dx$$

$$\Big\downarrow \text{1828}$$

$$-\frac{1}{3}\int \frac{3x^4}{x^8-x^4+1}dx - \frac{1}{3x^3}$$

$$\Big\downarrow \ 27$$

$$-\int \frac{x^4}{x^8-x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \ 1709$$

$$-\frac{\int \frac{x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 1447$$

$$-\frac{\frac{1}{2}\int \frac{x^2+1}{x^4-\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{x^2+1}{x^4+\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 1475$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+$$
$$\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 1083$$

$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x+\sqrt{2-\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$
$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x+\sqrt{2+\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$
$$\frac{1}{3x^3}$$

$$\Big\downarrow \ 217$$

$$-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+$$
$$\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 1478$$

3.60. $\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx$

$$\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int -\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int -\frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}-\frac{\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{1}{3x^3}$$

input
```
Int[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 + ((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[2
 + Sqrt[3]] + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[2 +
 Sqrt[3]])/2 + (Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 - Sqrt[3]]) -
 Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 - Sqrt[3]]))/2)/(2*Sqrt[3])
- ((ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2 - Sqrt[3]]
 + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2 - Sqrt[3]])/
2 + (Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]]) - Log[1 + Sq
rt[2 + Sqrt[3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]]))/2)/(2*Sqrt[3])
```

3.60. $\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx$

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

3.60.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.10

method	result	size
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(18\_R^5-\_R+x\right)\right)}{4}$	38
default	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}-\frac{1}{3x^3}$	46

input
```
int((-x^4+1)/x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+1/4*sum(_R*ln(18*_R^5-_R+x),_R=RootOf(81*_Z^8-9*_Z^4+1))
```

3.60.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.13

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx$$
$$= \frac{\sqrt{6}x^3\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)-\sqrt{6}x^3\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\right.}}{}$$

input
```
integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/24*(sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(3)*
sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) - sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(I
*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1)) +
6*x) + sqrt(6)*x^3*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(3
)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) - sqrt(6)*x^3*sqrt(-sqrt(2)*sq
rt(I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1
)) + 6*x) - sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(I*sqrt(6)*s
qrt(3)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) + sqrt(6)*x^3*sqrt(sqrt(2
)*sqrt(-I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(-I*sqrt(3
) + 1)) + 6*x) - sqrt(6)*x^3*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(I*sqr
t(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) + sqrt(6)*x^3*sqrt
(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(
-I*sqrt(3) + 1)) + 6*x) - 8)/x^3
```

3.60.6 Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.09

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx$$
$$= -\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-18432t^5+4t+x\right)\right)\right)-\frac{1}{3x^3}$$

input
```
integrate((-x**4+1)/x**4/(x**8-x**4+1),x)
```

output
```
-RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 +
4*_t + x))) - 1/(3*x**3)
```

3.60.7 Maxima [F]

$$\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx = \int -\frac{x^4-1}{(x^8-x^4+1)x^4}\,dx$$

input
```
integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/3/x^3 - integrate(x^4/(x^8 - x^4 + 1), x)
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.70

$$\begin{aligned}\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx = & -\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ & -\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ & -\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ & -\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ & -\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ & +\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ & -\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ & +\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{3x^3}\end{aligned}$$

input
```
integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sq
rt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt
(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(
6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*
x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(s
qrt(6) - sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1) - 1/3/x^3
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.29

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx$$

$$= -\frac{1}{3x^3} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)} + \frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12}$$
$$+ \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)} - \frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12}$$
$$- \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)} - \frac{2^{3/4}\,\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12}$$
$$- \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)} + \frac{2^{3/4}\,\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(-(x^4 - 1)/(x^4*(x^8 - x^4 + 1)),x)
```

output
```
(3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/
2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*
1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4))
)*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - 1/(3*x^3) + (3^(1/2)*atan((x*(8 - 3^(1/2
)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8
i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2
)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12
 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(
3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2))
 - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i
+ 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*
1i + 1)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(
1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2
)*1i + 1)^(1/2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((
2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2
)*1i)/2)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.60. $\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx$

3.61 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

3.61.1 Optimal result

Integrand size = 25, antiderivative size = 280

$$\begin{aligned}\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx &= \frac{(a^2d^2+b^2e^2+ae(bd-ce))\,x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}\\ &+\frac{(b^5d-5ab^3cd+5a^2bc^2d-b^4ce+4ab^2c^2e-2a^2c^3e)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}\\ &-\frac{d^5\log(d+ex)}{e^4\,(ad^2-e(bd-ce))}+\frac{(b^4d-3ab^2cd+a^2c^2d-b^3ce+2abc^2e)\log\left(c+bx+ax^2\right)}{2a^4\,(ad^2-e(bd-ce))}\end{aligned}$$

output
```
(a^2*d^2+b^2*e^2+a*e*(b*d-c*e))*x/a^3/e^3-1/2*(a*d+b*e)*x^2/a^2/e^2+1/3*x^
3/a/e-d^5*ln(e*x+d)/e^4/(a*d^2-e*(b*d-c*e))+1/2*(a^2*c^2*d-3*a*b^2*c*d+2*a
*b*c^2*e+b^4*d-b^3*c*e)*ln(a*x^2+b*x+c)/a^4/(a*d^2-e*(b*d-c*e))+(5*a^2*b*c
^2*d-2*a^2*c^3*e-5*a*b^3*c*d+4*a*b^2*c^2*e+b^5*d-b^4*c*e)*arctanh((2*a*x+b
)/(-4*a*c+b^2)^(1/2))/a^4/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$
$$=\frac{(a^2d^2+abde+b^2e^2-ace^2)\,x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}$$
$$+\frac{(b^5d-5ab^3cd+5a^2bc^2d-b^4ce+4ab^2c^2e-2a^2c^3e)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^4\sqrt{-b^2+4ac}\,(-ad^2+bde-ce^2)}$$
$$-\frac{d^5\log(d+ex)}{e^4\,(ad^2-bde+ce^2)}+\frac{(b^4d-3ab^2cd+a^2c^2d-b^3ce+2abc^2e)\log\left(c+bx+ax^2\right)}{2a^4\,(ad^2-bde+ce^2)}$$

input
```
Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

output
```
((a^2*d^2 + a*b*d*e + b^2*e^2 - a*c*e^2)*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/
(2*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*
e + 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(
a^4*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^5*Log[d + e*x])/(e
^4*(a*d^2 - b*d*e + c*e^2)) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e
+ 2*a*b*c^2*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - b*d*e + c*e^2))
```

3.61.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$
$$\Big\downarrow \; 1893$$
$$\int \frac{x^5}{(d+ex)\,(ax^2+bx+c)}dx$$
$$\Big\downarrow \; 1200$$

$$\int \left(-\frac{x(ad+be)}{a^2e^2}+\frac{a^2d^2+ae(bd-ce)+b^2e^2}{a^3e^3}+\frac{x(a^2c^2d-3ab^2cd+2abc^2e+b^4d-b^3ce)+c(-2abcd+ac^2e+}{a^3\left(ax^2+bx+c\right)\left(ad^2-e(bd-ce)\right)}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{x^2(ad+be)}{2a^2e^2}+\frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(5a^2bc^2d-2a^2c^3e-5ab^3cd+4ab^2c^2e+b^5d-b^4ce\right)}{a^4\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(a^2c^2d-3ab^2cd+2abc^2e+b^4d-b^3ce\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-e(bd-ce)\right)}+\frac{x\left(a^2d^2+ae(bd-ce)+b^2e^2\right)}{a^3e^3}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}+\frac{x^3}{3ae}$$

input
```
Int[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

output
```
((a^2*d^2 + b^2*e^2 + a*e*(b*d - c*e))*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(2
*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e
+ 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^
4*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^5*Log[d + e*x])/(e^4*(a*
d^2 - e*(b*d - c*e))) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*
b*c^2*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - e*(b*d - c*e)))
```

3.61.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.61.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.02

method	result
default	$\frac{\frac{1}{3}a^2e^2x^3-\frac{1}{2}a^2dex^2-\frac{1}{2}abe^2x^2+a^2d^2x+abdex-e^2acx+b^2e^2x}{a^3e^3}+\frac{\frac{\left(a^2c^2d-3ab^2cd+2abc^2e+db^4-b^3ce\right)\ln\left(ax^2+bx+c\right)}{2a}+\frac{2\left(-2abc^2d+a\right.}{}}{(ad^2-}$
risch	$\frac{x^3}{3ae}-\frac{dx^2}{2ae^2}-\frac{bx^2}{2a^2e}+\frac{d^2x}{ae^3}+\frac{bdx}{a^2e^2}-\frac{cx}{a^2e}+\frac{b^2x}{a^3e}-\frac{d^5\ln(ex+d)}{e^4(ad^2-bde+ce^2)}+\frac{\_R{=}\mathrm{RootOf}\left(\left(4a^3cd^2-b^2d^2a^2-4a^2bcde+4a^2c^2e^2\right.\right.}{}$

input
```
int(x^3/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/a^3/e^3*(1/3*a^2*e^2*x^3-1/2*a^2*d*e*x^2-1/2*a*b*e^2*x^2+a^2*d^2*x+a*b*d
*e*x-e^2*a*c*x+b^2*e^2*x)+1/(a*d^2-b*d*e+c*e^2)/a^3*(1/2*(a^2*c^2*d-3*a*b^
2*c*d+2*a*b*c^2*e+b^4*d-b^3*c*e)/a*ln(a*x^2+b*x+c)+2*(-2*a*b*c^2*d+a*c^3*e
+b^3*c*d-b^2*c^2*e-1/2*(a^2*c^2*d-3*a*b^2*c*d+2*a*b*c^2*e+b^4*d-b^3*c*e)*b
/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-1/e^4*d^5/(a*d^
2-b*d*e+c*e^2)*ln(e*x+d)
```

3.61.5 Fricas [A] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 1027, normalized size of antiderivative = 3.67

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$

$$= \left[-\frac{6\,(a^4b^2-4\,a^5c)d^5\log\left(ex+d\right)-2\,((a^4b^2-4\,a^5c)d^2e^3-(a^3b^3-4\,a^4bc)de^4+(a^3b^2c-4\,a^4c^2)e^5)x^3+3}{}\right.$$

$$-\frac{6\,(a^4b^2-4\,a^5c)d^5\log\left(ex+d\right)-2\,((a^4b^2-4\,a^5c)d^2e^3-(a^3b^3-4\,a^4bc)de^4+(a^3b^2c-4\,a^4c^2)e^5)x^3+3}{}$$

input
```
integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

output
```
[-1/6*(6*(a^4*b^2 - 4*a^5*c)*d^5*log(e*x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2
*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*
((a^4*b^2 - 4*a^5*c)*d^3*e^2 - (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 +
 (a^2*b^3*c - 4*a^3*b*c^2)*e^5)*x^2 + 3*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d
*e^4 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^5)*sqrt(b^2 - 4*a*c)*log((2*a^2
*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x
 + c)) - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2
)*d*e^4 + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6 - 7*a*b^4
*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^
3)*e^5)*log(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 - 4*
a^5*b*c)*d*e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6), -1/6*(6*(a^4*b^2 - 4*a^5*c)
*d^5*log(e*x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)
*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*((a^4*b^2 - 4*a^5*c)*d^3*e^2
 - (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 + (a^2*b^3*c - 4*a^3*b*c^2)*e
^5)*x^2 - 6*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4 - (b^4*c - 4*a*b^2*c^2
+ 2*a^2*c^3)*e^5)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b
)/(b^2 - 4*a*c)) - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8
*a^3*b*c^2)*d*e^4 + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6
 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 +
8*a^2*b*c^3)*e^5)*log(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4 - ...
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(a+c/x**2+b/x)/(e*x+d),x)
```

output
```
Timed out
```

3.61.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx &= -\frac{d^5\log\left(|ex+d|\right)}{ad^2e^4-bde^5+ce^6}+\frac{(b^4d-3\,ab^2cd+a^2c^2d-b^3ce+2\,abc^2e)\log\left(ax^2+bx+c\right)}{2\,(a^5d^2-a^4bde+a^4ce^2)}\\&\quad-\frac{(b^5d-5\,ab^3cd+5\,a^2bc^2d-b^4ce+4\,ab^2c^2e-2\,a^2c^3e)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^5d^2-a^4bde+a^4ce^2)\sqrt{-b^2+4\,ac}}\\&\quad+\frac{2\,a^2e^2x^3-3\,a^2dex^2-3\,abe^2x^2+6\,a^2d^2x+6\,abdex+6\,b^2e^2x-6\,ace^2x}{6\,a^3e^3}\end{aligned}$$

input
```
integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

output
```
-d^5*log(abs(e*x + d))/(a*d^2*e^4 - b*d*e^5 + c*e^6) + 1/2*(b^4*d - 3*a*b^
2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*log(a*x^2 + b*x + c)/(a^5*d^2 -
 a^4*b*d*e + a^4*c*e^2) - (b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e +
 4*a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^5
*d^2 - a^4*b*d*e + a^4*c*e^2)*sqrt(-b^2 + 4*a*c)) + 1/6*(2*a^2*e^2*x^3 - 3
*a^2*d*e*x^2 - 3*a*b*e^2*x^2 + 6*a^2*d^2*x + 6*a*b*d*e*x + 6*b^2*e^2*x - 6
*a*c*e^2*x)/(a^3*e^3)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 2490, normalized size of antiderivative = 8.89

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Too large to display}$$

input
```
int(x^3/((d + e*x)*(a + b/x + c/x^2)),x)
```

output
```
(log(4*a^5*c*d^7 - a^4*b^2*d^7 + b^3*c^3*e^7 - b^6*d^3*e^4 - 6*a^2*c^4*d*e
^6 - 3*b^4*c^2*d*e^6 + 3*b^5*c*d^2*e^5 - 2*a^2*c^4*e^7*x - b^2*c^3*e^7*(b^
2 - 4*a*c)^(1/2) + b^5*d^3*e^4*(b^2 - 4*a*c)^(1/2) + 2*a^3*c^3*d^3*e^4 - 4
*a^4*c^2*d^5*e^2 - 3*a*b*c^4*e^7 + a^4*b*d^7*(b^2 - 4*a*c)^(1/2) + a*c^4*e
^7*(b^2 - 4*a*c)^(1/2) + 2*a^5*d^7*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c^3*d^2*e
^5*(b^2 - 4*a*c)^(1/2) + 8*a^5*c*d^6*e*x - 9*a^2*b^2*c^2*d^3*e^4 - 4*a^4*c
*d^6*e*(b^2 - 4*a*c)^(1/2) + 12*a*b^2*c^3*d*e^6 + 6*a*b^4*c*d^3*e^4 + a*b^
2*c^3*e^7*x - a*b^5*d^3*e^4*x - 2*a^4*b^2*d^6*e*x + 3*b^3*c^2*d*e^6*(b^2 -
 4*a*c)^(1/2) - 3*b^4*c*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 15*a*b^3*c^2*d^2*e^5
 + 15*a^2*b*c^3*d^2*e^5 + a^3*b^2*c*d^5*e^2 + a^3*b^3*d^5*e^2*x + 6*a^3*c^
3*d^2*e^5*x - 4*a*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b*c*d^5*e^2*(b^2
 - 4*a*c)^(1/2) + a*b^4*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c^3*d*e^6*x*
(b^2 - 4*a*c)^(1/2) - 2*a^4*c*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 5*a^2*b^3*c*
d^3*e^4*x - 5*a^3*b*c^2*d^3*e^4*x + 9*a*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2
) + 3*a^2*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b^2*d^5*e^2*x*(b^2 - 4*a
*c)^(1/2) + a^3*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) - 12*a^2*b^2*c^2*d^2*e^5
*x - 6*a*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^3*e^7*x*(b^2 - 4*a*c)^(1/
2) - 2*a^4*b*d^6*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^3*c^2*d*e^6*x + 3*a*b^4*c
*d^2*e^5*x + 9*a^2*b*c^3*d*e^6*x - 4*a^4*b*c*d^5*e^2*x + 3*a*b^2*c^2*d*e^6
*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^3*c*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) + 6*a^...
```

3.62 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

3.62.1 Optimal result

Integrand size = 25, antiderivative size = 218

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae} - \frac{(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e)\,\text{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} + \frac{d^4\log(d+ex)}{e^3\,(ad^2-e(bd-ce))} - \frac{(b^3d-2abcd-b^2ce+ac^2e)\log\left(c+bx+ax^2\right)}{2a^3\,(ad^2-e(bd-ce))}$$

output
```
-(a*d+b*e)*x/a^2/e^2+1/2*x^2/a/e+d^4*ln(e*x+d)/e^3/(a*d^2-e*(b*d-c*e))-1/2
*(-2*a*b*c*d+a*c^2*e+b^3*d-b^2*c*e)*ln(a*x^2+b*x+c)/a^3/(a*d^2-e*(b*d-c*e)
)-(2*a^2*c^2*d-4*a*b^2*c*d+3*a*b*c^2*e+b^4*d-b^3*c*e)*arctanh((2*a*x+b)/(-
4*a*c+b^2)^(1/2))/a^3/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae} + \frac{(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^3\sqrt{-b^2+4ac}\,(ad^2+e(-bd+ce))} + \frac{d^4\log(d+ex)}{e^3\,(ad^2+e(-bd+ce))} + \frac{(-b^3d+2abcd+b^2ce-ac^2e)\log(c+x(b+ax))}{2a^3\,(ad^2+e(-bd+ce))}$$

input
```
Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

output
```
-(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) + ((b^4*d - 4*a*b^2*c*d + 2*a^2
*c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a
^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))) + (d^4*Log[d + e*x])/(e^
3*(a*d^2 + e*(-(b*d) + c*e))) + ((-(b^3*d) + 2*a*b*c*d + b^2*c*e - a*c^2*e
)*Log[c + x*(b + a*x)])/(2*a^3*(a*d^2 + e*(-(b*d) + c*e)))
```

3.62.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \; 1893$$

$$\int \frac{x^4}{(d+ex)(ax^2+bx+c)}dx$$

$$\Big\downarrow \; 1200$$

$$\int \left(\frac{-c(-acd + b^2d - bce) - x(-2abcd + ac^2e + b^3d - b^2ce)}{a^2 (ax^2 + bx + c) (ad^2 - e(bd - ce))} + \frac{-ad - be}{a^2e^2} + \frac{d^4}{e^2(d + ex) (ad^2 - e(bd - ce))} + \frac{x}{ae} \right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(-2abcd + ac^2e + b^3d - b^2ce) \log (ax^2 + bx + c)}{2a^3 (ad^2 - e(bd - ce))} - \frac{x(ad + be)}{a^2e^2} - \frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) (2a^2c^2d - 4ab^2cd + 3abc^2e + b^4d - b^3ce)}{a^3\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))} + \frac{d^4 \log(d + ex)}{e^3 (ad^2 - e(bd - ce))} + \frac{x^2}{2ae}$$

input
```
Int[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

output
```
-(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^2
*c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a
^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + (d^4*Log[d + e*x])/(e^3*(a
*d^2 - e*(b*d - c*e))) - ((b^3*d - 2*a*b*c*d - b^2*c*e + a*c^2*e)*Log[c +
b*x + a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*e)))
```

3.62.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.62.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.95

method	result
default	$-\frac{-\frac{1}{2}ae\,x^2+adx+bex}{e^2a^2}+\frac{\frac{\left(2abcd-a\,c^2e-b^3d+b^2ce\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(a\,c^2d-b^2cd+b\,c^2e-\frac{\left(2abcd-a\,c^2e-b^3d+b^2ce\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right.}{\sqrt{4ac-b^2}}}{(a\,d^2-bde+c\,e^2)a^2}$
risch	$\frac{x^2}{2ae}-\frac{dx}{e^2a}-\frac{bx}{e\,a^2}+\frac{d^4\ln(ex+d)}{e^3(a\,d^2-bde+c\,e^2)}+\frac{\sum_{\_R=\text{RootOf}\left(\left(4a^3c\,d^2-b^2d^2a^2-4a^2bcde+4a^2c^2e^2+a\,b^3de-a\,b^2c\,e^2\right)\_Z^2+\left(-8a^2b\,c^2d\,e^2+\right.\right.}}{}$

input
```
int(x^2/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/e^2/a^2*(-1/2*a*e*x^2+a*d*x+b*e*x)+1/(a*d^2-b*d*e+c*e^2)/a^2*(1/2*(2*a*
b*c*d-a*c^2*e-b^3*d+b^2*c*e)/a*ln(a*x^2+b*x+c)+2*(a*c^2*d-b^2*c*d+b*c^2*e-
1/2*(2*a*b*c*d-a*c^2*e-b^3*d+b^2*c*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x
+b)/(4*a*c-b^2)^(1/2)))+1/e^3*d^4/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 798, normalized size of antiderivative = 3.66

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$

$$= \left[\frac{2\,(a^3b^2-4\,a^4c)d^4\log\left(ex+d\right)+((a^3b^2-4\,a^4c)d^2e^2-(a^2b^3-4\,a^3bc)de^3+(a^2b^2c-4\,a^3c^2)e^4)x^2+((b^4}{}\right.$$

input
```
integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*log(e*x + d) + ((a^3*b^2 - 4*a^4*c)*d^2*e^
2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^4
 - 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)*sqrt(b^2 - 4*a*
c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))
/(a*x^2 + b*x + c)) - 2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c
+ 4*a^3*c^2)*d*e^3 + (a*b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c +
8*a^2*b*c^2)*d*e^3 - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b*
x + c))/((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*
b^2*c - 4*a^4*c^2)*e^5), 1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*log(e*x + d) + ((a
^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a
^3*c^2)*e^4)*x^2 - 2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b
*c^2)*e^4)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2
- 4*a*c)) - 2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^
2)*d*e^3 + (a*b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c + 8*a^2*b*c^
2)*d*e^3 - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b*x + c))/((
a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*
a^4*c^2)*e^5)]
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(a+c/x**2+b/x)/(e*x+d),x)
```

output
```
Timed out
```

3.62.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \frac{d^4\log\left(|ex+d|\right)}{ad^2e^3-bde^4+ce^5} - \frac{(b^3d-2\,abcd-b^2ce+ac^2e)\log\left(ax^2+bx+c\right)}{2\left(a^4d^2-a^3bde+a^3ce^2\right)} + \frac{(b^4d-4\,ab^2cd+2\,a^2c^2d-b^3ce+3\,abc^2e)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4d^2-a^3bde+a^3ce^2)\sqrt{-b^2+4\,ac}} + \frac{aex^2-2\,adx-2\,bex}{2\,a^2e^2}$$

input
```
integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

output
```
d^4*log(abs(e*x + d))/(a*d^2*e^3 - b*d*e^4 + c*e^5) - 1/2*(b^3*d - 2*a*b*c
*d - b^2*c*e + a*c^2*e)*log(a*x^2 + b*x + c)/(a^4*d^2 - a^3*b*d*e + a^3*c*
e^2) + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*arctan(
(2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^4*d^2 - a^3*b*d*e + a^3*c*e^2)*sqrt(-b
^2 + 4*a*c)) + 1/2*(a*e*x^2 - 2*a*d*x - 2*b*e*x)/(a^2*e^2)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 2051, normalized size of antiderivative = 9.41

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Too large to display}$$

input
```
int(x^2/((d + e*x)*(a + b/x + c/x^2)),x)
```

output

```
(d^4*log(d + e*x))/(c*e^5 + a*d^2*e^3 - b*d*e^4) - (log(4*a^4*c*d^6 - 2*a*
c^4*e^6 - a^3*b^2*d^6 + b^2*c^3*e^6 - b^5*d^3*e^3 - 3*b^3*c^2*d*e^5 + 3*b^
4*c*d^2*e^4 + b^4*d^3*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^3*d^2*e^4 - 4*a^3*
c^2*d^4*e^2 + a^3*b*d^6*(b^2 - 4*a*c)^(1/2) - b*c^3*e^6*(b^2 - 4*a*c)^(1/2
) + 2*a^4*d^6*x*(b^2 - 4*a*c)^(1/2) + 9*a*b*c^3*d*e^5 + a^2*c^2*d^3*e^3*(b
^2 - 4*a*c)^(1/2) + a*b*c^3*e^6*x + 8*a^4*c*d^5*e*x - 3*a*c^3*d*e^5*(b^2 -
 4*a*c)^(1/2) - 4*a^3*c*d^5*e*(b^2 - 4*a*c)^(1/2) - a*c^3*e^6*x*(b^2 - 4*a
*c)^(1/2) + 5*a*b^3*c*d^3*e^3 - a*b^4*d^3*e^3*x - 2*a^3*b^2*d^5*e*x + 6*a^
2*c^3*d*e^5*x + 3*b^2*c^2*d*e^5*(b^2 - 4*a*c)^(1/2) - 3*b^3*c*d^2*e^4*(b^2
 - 4*a*c)^(1/2) - 12*a*b^2*c^2*d^2*e^4 - 5*a^2*b*c^2*d^3*e^3 + a^2*b^2*c*d
^4*e^2 + a^2*b^3*d^4*e^2*x - 2*a^3*c^2*d^3*e^3*x + 6*a*b*c^2*d^2*e^4*(b^2
- 4*a*c)^(1/2) - 3*a*b^2*c*d^3*e^3*(b^2 - 4*a*c)^(1/2) + a^2*b*c*d^4*e^2*(
b^2 - 4*a*c)^(1/2) + a*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*c*d^4*e^2
*x*(b^2 - 4*a*c)^(1/2) - 9*a^2*b*c^2*d^2*e^4*x + 4*a^2*b^2*c*d^3*e^3*x + a
^2*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(
1/2) - 2*a^3*b*d^5*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c^2*d*e^5*x + 3*a*b^3
*c*d^2*e^4*x - 4*a^3*b*c*d^4*e^2*x + 3*a*b*c^2*d*e^5*x*(b^2 - 4*a*c)^(1/2)
 - 3*a*b^2*c*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*c*d^3*e^3*x*(b^2 - 4*
a*c)^(1/2))*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^2*c^3*e + b^4*c*e + 6
*a*b^3*c*d - b^3*c*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a*b^2*c^2*...
```

3.62. $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

3.63 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

3.63.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \frac{x}{ae}+\frac{(b^3d-3abcd-b^2ce+2ac^2e)\,\text{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} - \frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{(b^2d-acd-bce)\log\left(c+bx+ax^2\right)}{2a^2\left(ad^2-e(bd-ce)\right)}$$

output
```
x/a/e-d^3*ln(e*x+d)/e^2/(a*d^2-e*(b*d-c*e))+1/2*(-a*c*d+b^2*d-b*c*e)*ln(a*
x^2+b*x+c)/a^2/(a*d^2-e*(b*d-c*e))+(-3*a*b*c*d+2*a*c^2*e+b^3*d-b^2*c*e)*ar
ctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(
1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \frac{x}{ae}+\frac{(b^3d-3abcd-b^2ce+2ac^2e)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^2\sqrt{-b^2+4ac}\,(-ad^2+bde-ce^2)} - \frac{d^3\log(d+ex)}{e^2\left(ad^2-bde+ce^2\right)}+\frac{(b^2d-acd-bce)\log\left(c+bx+ax^2\right)}{2a^2\left(ad^2-bde+ce^2\right)}$$

input
```
Integrate[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

output
```
x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTan[(b + 2*a*x)/Sq
rt[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (
d^3*Log[d + e*x])/(e^2*(a*d^2 - b*d*e + c*e^2)) + ((b^2*d - a*c*d - b*c*e)
*Log[c + b*x + a*x^2])/(2*a^2*(a*d^2 - b*d*e + c*e^2))
```

3.63.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \text{ 1893}$$

$$\int \frac{x^3}{(d+ex)\left(ax^2+bx+c\right)}dx$$

$$\Big\downarrow \text{ 1200}$$

$$\int \left(\frac{x\left(-acd+b^2d-bce\right)+c(bd-ce)}{a\left(ax^2+bx+c\right)\left(ad^2-e(bd-ce)\right)}+\frac{d^3}{e(d+ex)\left(e(bd-ce)-ad^2\right)}+\frac{1}{ae}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(-3abcd+2ac^2e+b^3d-b^2ce\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{x}{ae}$$

input
```
Int[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

output
```
x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTanh[(b + 2*a*x)/S
qrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^3*
Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))) + ((b^2*d - a*c*d - b*c*e)*Log
[c + b*x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e)))
```

3.63.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.63.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x}{ae}+\frac{\frac{\left(-acd+b^2d-ebc\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(bcd-e\,c^2-\frac{\left(-acd+b^2d-ebc\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{\left(a\,d^2-bde+c\,e^2\right)a}-\frac{d^3\ln\left(ex+d\right)}{e^2\left(a\,d^2-bde+c\,e^2\right)}$	164
risch	Expression too large to display	15838

input
```
int(x/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
x/a/e+1/(a*d^2-b*d*e+c*e^2)/a*(1/2*(-a*c*d+b^2*d-b*c*e)/a*ln(a*x^2+b*x+c)+
2*(b*c*d-e*c^2-1/2*(-a*c*d+b^2*d-b*c*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a
*x+b)/(4*a*c-b^2)^(1/2)))-1/e^2*d^3/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

3.63.5 Fricas [A] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 596, normalized size of antiderivative = 3.39

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$
$$= \left[-\frac{2\,(a^2b^2-4\,a^3c)d^3\log\left(ex+d\right)-((b^3-3\,abc)de^2-(b^2c-2\,ac^2)e^3)\sqrt{b^2-4\,ac}\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+}{ax^2+bx+}\right.}{2\,((a^3b^2-}\right.$$
$$-\frac{2\,(a^2b^2-4\,a^3c)d^3\log\left(ex+d\right)-2\,((b^3-3\,abc)de^2-(b^2c-2\,ac^2)e^3)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2}{b^2-4\,ac}\right.}{2\,((a^3b^2-4\,a^4}$$

input
```
integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

output
```
[-1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*log(e*x + d) - ((b^3 - 3*a*b*c)*d*e^2 - (
b^2*c - 2*a*c^2)*e^3)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2
*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*((a^2*b^2 - 4
*a^3*c)*d^2*e - (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x -
 ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2
 + b*x + c))/((a^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 +
(a^2*b^2*c - 4*a^3*c^2)*e^4), -1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*log(e*x + d)
 - 2*((b^3 - 3*a*b*c)*d*e^2 - (b^2*c - 2*a*c^2)*e^3)*sqrt(-b^2 + 4*a*c)*ar
ctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - 2*((a^2*b^2 - 4*a^3*
c)*d^2*e - (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x - ((b^
4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2 + b*
x + c))/((a^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*
b^2*c - 4*a^3*c^2)*e^4)]
```

3.63.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(x/(a+c/x**2+b/x)/(e*x+d),x)
```

output
```
Timed out
```

3.63.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{d^3\log\left(|ex+d|\right)}{ad^2e^2-bde^3+ce^4}+\frac{(b^2d-acd-bce)\log\left(ax^2+bx+c\right)}{2\,(a^3d^2-a^2bde+a^2ce^2)} - \frac{(b^3d-3\,abcd-b^2ce+2\,ac^2e)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3d^2-a^2bde+a^2ce^2)\sqrt{-b^2+4\,ac}}+\frac{x}{ae}$$

input
```
integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

output
```
-d^3*log(abs(e*x + d))/(a*d^2*e^2 - b*d*e^3 + c*e^4) + 1/2*(b^2*d - a*c*d
- b*c*e)*log(a*x^2 + b*x + c)/(a^3*d^2 - a^2*b*d*e + a^2*c*e^2) - (b^3*d -
 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/(
(a^3*d^2 - a^2*b*d*e + a^2*c*e^2)*sqrt(-b^2 + 4*a*c)) + x/(a*e)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 1367, normalized size of antiderivative = 7.77

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \frac{x}{a\,e}$$
$$-\frac{\ln\left(c^3\,e^5\,\sqrt{b^2-4\,a\,c}-b\,c^3\,e^5-4\,a^3\,c\,d^5+a^2\,b^2\,d^5+b^4\,d^3\,e^2+3\,b^2\,c^2\,d\,e^4-3\,b^3\,c\,d^2\,e^3-b^3\,d^3\,e^2\,\sqrt{b^2-4}\right.}{}$$
$$-\frac{\ln\left(a^2\,b^2\,d^5-b\,c^3\,e^5-c^3\,e^5\,\sqrt{b^2-4\,a\,c}-4\,a^3\,c\,d^5+b^4\,d^3\,e^2+3\,b^2\,c^2\,d\,e^4-3\,b^3\,c\,d^2\,e^3+b^3\,d^3\,e^2\,\sqrt{b^2-4}\right.}{}$$
$$-\frac{d^3\,\ln\left(d+e\,x\right)}{a\,d^2\,e^2-b\,d\,e^3+c\,e^4}$$

input
```
int(x/((d + e*x)*(a + b/x + c/x^2)),x)
```

output
```
x/(a*e) - (log(c^3*e^5*(b^2 - 4*a*c)^(1/2) - b*c^3*e^5 - 4*a^3*c*d^5 + a^2
*b^2*d^5 + b^4*d^3*e^2 + 3*b^2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 - b^3*d^3*e^2*(
b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x - a
^2*b*d^5*(b^2 - 4*a*c)^(1/2) - 2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d
^4*e*x + 4*a^2*c*d^4*e*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d*e^4*(b^2 - 4*a*c)^(
1/2) + 9*a*b*c^2*d^2*e^3 - 5*a*b^2*c*d^3*e^2 + 2*a^2*b^2*d^4*e*x - 3*a*c^2
*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2
*c^2*d^2*e^3*x - 2*a*b^2*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c*d^3*e^2*x
*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*e^4*x + a*b*c*d^3*e^2*(b^2 - 4*a*c)^(1/
2) + 2*a^2*b*d^4*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*c^2*d*e^4*x*(b^2 - 4*a*c)^(
1/2) - 3*a*b^2*c*d^2*e^3*x + a^2*b*c*d^3*e^2*x + 3*a*b*c*d^2*e^3*x*(b^2 -
4*a*c)^(1/2))*(b^4*d - b^3*d*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d - b^3*c*e -
 5*a*b^2*c*d + 4*a*b*c^2*e - 2*a*c^2*e*(b^2 - 4*a*c)^(1/2) + b^2*c*e*(b^2
- 4*a*c)^(1/2) + 3*a*b*c*d*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^4*c*d^2 - a^3*b^2
*d^2 + 4*a^3*c^2*e^2 - a^2*b^2*c*e^2 + a^2*b^3*d*e - 4*a^3*b*c*d*e)) - (lo
g(a^2*b^2*d^5 - b*c^3*e^5 - c^3*e^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5 + b^
4*d^3*e^2 + 3*b^2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 + b^3*d^3*e^2*(b^2 - 4*a*c)^
(1/2) + 6*a^2*c^2*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x + a^2*b*d^5*(b^2
 - 4*a*c)^(1/2) + 2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d^4*e*x - 4*a^
2*c*d^4*e*(b^2 - 4*a*c)^(1/2) + 3*b*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) + 9*a...
```

3.64 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

3.64.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{(b^2d-2acd-bce)\,\text{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} + \frac{d^2\log(d+ex)}{e\,(ad^2-bde+ce^2)} - \frac{(bd-ce)\log\left(c+bx+ax^2\right)}{2a\,(ad^2-e(bd-ce))}$$

output
```
d^2*ln(e*x+d)/e/(a*d^2-b*d*e+c*e^2)-1/2*(b*d-c*e)*ln(a*x^2+b*x+c)/a/(a*d^2
-e*(b*d-c*e))-(-2*a*c*d+b^2*d-b*c*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))
/a/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{2e(-b^2d+2acd+bce)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}(-2ad^2\log(d+ex)+e(bd-ce)\log(c+x(b+}{2a\sqrt{-b^2+4ace}\,(ad^2+e(-bd+ce))}$$

input
```
Integrate[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

output
```
-1/2*(2*e*(-(b^2*d) + 2*a*c*d + b*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*
c]] + Sqrt[-b^2 + 4*a*c]*(-2*a*d^2*Log[d + e*x] + e*(b*d - c*e)*Log[c + x*
(b + a*x)]))/(a*Sqrt[-b^2 + 4*a*c]*e*(a*d^2 + e*(-(b*d) + c*e)))
```

3.64.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1775, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \text{1775}$$

$$\int \frac{x^2}{(d+ex)\left(ax^2+bx+c\right)}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{-x(bd-ce)-cd}{\left(ax^2+bx+c\right)\left(ad^2-e(bd-ce)\right)}+\frac{d^2}{(d+ex)\left(ad^2-e(bd-ce)\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(-2acd+b^2d-bce\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)}$$

input
```
Int[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

output
```
-(((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a*Sq
rt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (d^2*Log[d + e*x])/(e*(a*d^2 -
 b*d*e + c*e^2)) - ((b*d - c*e)*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d
 - c*e)))
```

3.64.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1775
```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.64.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\frac{(-bd+ec)\ln\left(a\,x^{2}+bx+c\right)}{2a}+\frac{2\left(-cd-\frac{(-bd+ec)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a\,d^{2}-bde+c\,e^{2}}+\frac{d^{2}\ln(ex+d)}{e(a\,d^{2}-bde+c\,e^{2})}$	130
risch	Expression too large to display	7752

input
```
int(1/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2-b*d*e+c*e^2)*(1/2*(-b*d+c*e)/a*ln(a*x^2+b*x+c)+2*(-c*d-1/2*(-b*d+
c*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+d^2*ln(e*
x+d)/e/(a*d^2-b*d*e+c*e^2)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.72

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$
$$= \left[\frac{2\,(ab^2-4\,a^2c)d^2\log{(ex+d)}+(bce^2-(b^2-2\,ac)de)\sqrt{b^2-4\,ac}\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,ax+b)}{ax^2+bx+c}\right)}{2\,((a^2b^2-4\,a^3c)d^2e-(ab^3-4\,a^2bc)de^2+(ab^2c-4\,a^2c\right.$$

input
```
integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/2*(2*(a*b^2 - 4*a^2*c)*d^2*log(e*x + d) + (b*c*e^2 - (b^2 - 2*a*c)*d*e)
*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a
*c)*(2*a*x + b))/(a*x^2 + b*x + c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*c - 4*a*
c^2)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^
2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/2*(2*(a*b^2 - 4*a^2*c)*d^2*lo
g(e*x + d) + 2*(b*c*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*
c - 4*a*c^2)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^
3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/(e*x+d),x)
```

output
```
Timed out
```

3.64.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \frac{d^2\log\left(|ex+d|\right)}{ad^2e-bde^2+ce^3} - \frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2\left(a^2d^2-abde+ace^2\right)} + \frac{\left(b^2d-2\,acd-bce\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^2d^2-abde+ace^2\right)\sqrt{-b^2+4\,ac}}$$

input
```
integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

output
```
d^2*log(abs(e*x + d))/(a*d^2*e - b*d*e^2 + c*e^3) - 1/2*(b*d - c*e)*log(a*
x^2 + b*x + c)/(a^2*d^2 - a*b*d*e + a*c*e^2) + (b^2*d - 2*a*c*d - b*c*e)*a
rctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*d^2 - a*b*d*e + a*c*e^2)*sqrt(
-b^2 + 4*a*c))
```

3.64.9 Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 966, normalized size of antiderivative = 6.48

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \frac{d^2\ln\left(d+e\,x\right)}{a\,d^2\,e-b\,d\,e^2+c\,e^3}$$
$$-\frac{\ln\left(a\,b^2\,d^4-2\,c^3\,e^4-4\,a^2\,c\,d^4+b^3\,d^3\,e+c^2\,e^4\,x\,\sqrt{b^2-4\,a\,c}+10\,a\,c^2\,d^2\,e^2-4\,b^2\,c\,d^2\,e^2-b^3\,d^2\,e^2\,x+a\,b\right.}{}$$
$$+\frac{\ln\left(2\,c^3\,e^4-a\,b^2\,d^4+4\,a^2\,c\,d^4-b^3\,d^3\,e+c^2\,e^4\,x\,\sqrt{b^2-4\,a\,c}-10\,a\,c^2\,d^2\,e^2+4\,b^2\,c\,d^2\,e^2+b^3\,d^2\,e^2\,x+a\,b\right.}{}$$

input
```
int(1/((d + e*x)*(a + b/x + c/x^2)),x)
```

output
```
(d^2*log(d + e*x))/(c*e^3 + a*d^2*e - b*d*e^2) - (log(a*b^2*d^4 - 2*c^3*e^
4 - 4*a^2*c*d^4 + b^3*d^3*e + c^2*e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a*c^2*d^2
*e^2 - 4*b^2*c*d^2*e^2 - b^3*d^2*e^2*x + a*b*d^4*(b^2 - 4*a*c)^(1/2) + 3*b
*c^2*d*e^3 - b*c^2*e^4*x + b^2*d^3*e*(b^2 - 4*a*c)^(1/2) + 3*c^2*d*e^3*(b^
2 - 4*a*c)^(1/2) + 2*a^2*d^4*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*d^3*e*x + 6*a
*c^2*d*e^3*x - 10*a^2*c*d^3*e*x - 2*b*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) - 3*a*
b*c*d^3*e + b^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) - 5*a*c*d^3*e*(b^2 - 4*a*c)^
(1/2) - a*b*d^3*e*x*(b^2 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*x - 5*a*c*d^2*e^2*
x*(b^2 - 4*a*c)^(1/2))*(e*((b^2*c)/2 - 2*a*c^2 + (b*c*(b^2 - 4*a*c)^(1/2))
/2) - (b^3*d)/2 - (b^2*d*(b^2 - 4*a*c)^(1/2))/2 + a*c*d*(b^2 - 4*a*c)^(1/2
) + 2*a*b*c*d))/(4*a^3*c*d^2 - a^2*b^2*d^2 + 4*a^2*c^2*e^2 + a*b^3*d*e - a
*b^2*c*e^2 - 4*a^2*b*c*d*e) + (log(2*c^3*e^4 - a*b^2*d^4 + 4*a^2*c*d^4 - b
^3*d^3*e + c^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2 + 4*b^2*c*d^2*
e^2 + b^3*d^2*e^2*x + a*b*d^4*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d*e^3 + b*c^2*
e^4*x + b^2*d^3*e*(b^2 - 4*a*c)^(1/2) + 3*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) +
2*a^2*d^4*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*d^3*e*x - 6*a*c^2*d*e^3*x + 10*a
^2*c*d^3*e*x - 2*b*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d^3*e + b^2*d^2
*e^2*x*(b^2 - 4*a*c)^(1/2) - 5*a*c*d^3*e*(b^2 - 4*a*c)^(1/2) - a*b*d^3*e*x
*(b^2 - 4*a*c)^(1/2) - a*b*c*d^2*e^2*x - 5*a*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/
2))*((b^3*d)/2 + e*(2*a*c^2 - (b^2*c)/2 + (b*c*(b^2 - 4*a*c)^(1/2))/2) ...
```

3.65 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$

3.65.1 Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx = \frac{(bd-2ce)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} - \frac{d\log(d+ex)}{ad^2-e(bd-ce)} + \frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-e(bd-ce)\right)}$$

output
```
-d*ln(e*x+d)/(a*d^2-e*(b*d-c*e))+1/2*d*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))
+(b*d-2*c*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))/(-4
*a*c+b^2)^(1/2)
```

3.65.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx = \frac{2(bd-2ce)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,d(2\log(d+ex)-\log(c+x(b+ax)))}{2\sqrt{-b^2+4ac}\,(-ad^2+e(bd-ce))}$$

input
```
Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

output
```
(2*(b*d - 2*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*
c]*d*(2*Log[d + e*x] - Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d
^2) + e*(b*d - c*e)))
```

3.65.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \text{1893}$$

$$\int \frac{x}{(d+ex)\left(ax^2+bx+c\right)}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{adx+ce}{\left(ax^2+bx+c\right)\left(ad^2-e(bd-ce)\right)}+\frac{de}{(d+ex)\left(e(bd-ce)-ad^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bd-2ce)\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}-\frac{d\log(d+ex)}{ad^2-bde+ce^2}$$

input
```
Int[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

output
```
((b*d - 2*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*
(a*d^2 - e*(b*d - c*e))) - (d*Log[d + e*x])/(a*d^2 - b*d*e + c*e^2) + (d*L
og[c + b*x + a*x^2])/(2*(a*d^2 - b*d*e + c*e^2))
```

3.65.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.65.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

method	result
default	$\frac{\frac{d\ln\left(a\,x^{2}+bx+c\right)}{2}+\frac{2\left(-\frac{bd}{2}+ec\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a\,d^{2}-bde+c\,e^{2}}-\frac{d\ln\left(ex+d\right)}{a\,d^{2}-bde+c\,e^{2}}$
risch	$-\frac{d\ln\left(ex+d\right)}{a\,d^{2}-bde+c\,e^{2}}+\left(\sum_{\_R=\text{RootOf}\left(\left(4a^{2}c\,d^{2}-a\,b^{2}d^{2}-4abcde+4a\,c^{2}e^{2}+b^{3}de-b^{2}c\,e^{2}\right)\_Z^{2}+\left(-4acd+b^{2}d\right)\_Z+c\right)}\_R\ln\left(\left(\left(-2\right.\right.\right.$

input
```
int(1/(a+c/x^2+b/x)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2-b*d*e+c*e^2)*(1/2*d*ln(a*x^2+b*x+c)+2*(-1/2*b*d+e*c)/(4*a*c-b^2)^
(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-d/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$$
$$= \left[\frac{(b^2-4\,ac)d\log\left(ax^2+bx+c\right)-2\left(b^2-4\,ac\right)d\log\left(ex+d\right)-\sqrt{b^2-4\,ac}(bd-2\,ce)\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2}{ax^2}\right.}{2\left((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2\right)}\right.$$

input
```
integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="fricas")
```

output
```
[1/2*((b^2 - 4*a*c)*d*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d
) - sqrt(b^2 - 4*a*c)*(b*d - 2*c*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c
 - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d
^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), 1/2*((b^2 - 4*a*c)*d*lo
g(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)
*(b*d - 2*c*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*
b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

3.65.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/x/(e*x+d),x)
```

output
```
Timed out
```

3.65.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx = -\frac{de\log\left(|ex+d|\right)}{ad^2e-bde^2+ce^3}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)} - \frac{(bd-2\,ce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(ad^2-bde+ce^2)\sqrt{-b^2+4\,ac}}$$

input
```
integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="giac")
```

output
```
-d*e*log(abs(e*x + d))/(a*d^2*e - b*d*e^2 + c*e^3) + 1/2*d*log(a*x^2 + b*x
 + c)/(a*d^2 - b*d*e + c*e^2) - (b*d - 2*c*e)*arctan((2*a*x + b)/sqrt(-b^2
 + 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

3.65.9 Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 801, normalized size of antiderivative = 6.46

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$$

$$= \frac{\ln\left(a\,e\,x-\frac{\left(d\left(\frac{b\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\sqrt{b^2-4\,a\,c}\right)\left(x\,(d\,a^2\,e+b\,a\,e^2)+\frac{\left(d\left(\frac{b\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\sqrt{b^2-4\,a\,c}\right)\left(x\left(2\,a^3\,d^2\,e-2\,a^2\,b\,d\cdots\right.\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e\cdots}}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2\cdots}\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\cdots}$$

$$-\frac{\ln\left(\frac{\left(d\left(2\,a\,c+\frac{b\sqrt{b^2-4\,a\,c}}{2}-\frac{b^2}{2}\right)-c\,e\sqrt{b^2-4\,a\,c}\right)\left(x\,(d\,a^2\,e+b\,a\,e^2)-\frac{\left(d\left(2\,a\,c+\frac{b\sqrt{b^2-4\,a\,c}}{2}-\frac{b^2}{2}\right)-c\,e\sqrt{b^2-4\,a\,c}\right)\left(x\left(2\,a^3\,d^2\,e-2\,a^2\,b\,d\,e^2-6\,c\,a\cdots\right.\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\cdots}}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2\cdots}\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\cdots}$$

$$-\frac{d\ln(d+e\,x)}{a\,d^2-b\,d\,e+c\,e^2}$$

input
```
int(1/(x*(d + e*x)*(a + b/x + c/x^2)),x)
```

output
```
(log(a*e*x - ((d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2/2) - c*e*(b^2 -
4*a*c)^(1/2))*(x*(a*b*e^2 + a^2*d*e) + ((d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*
a*c + b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(2*a*b^2*e^3 - 6*a^2*c*e^3 + 2*
a^3*d^2*e - 2*a^2*b*d*e^2) + a*b*c*e^3 + a*b^2*d*e^2 + a^2*b*d^2*e - 8*a^2
*c*d*e^2))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e +
4*a*b*c*d*e) + a*c*e^2 + a*b*d*e))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2
+ b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e))*(d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*
c + b^2/2) - c*e*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*
e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (log(((d*(2*a*c + (b*(b^2 - 4*a
*c)^(1/2))/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(a*b*e^2 + a^2*d*e) -
((d*(2*a*c + (b*(b^2 - 4*a*c)^(1/2))/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))
*(x*(2*a*b^2*e^3 - 6*a^2*c*e^3 + 2*a^3*d^2*e - 2*a^2*b*d*e^2) + a*b*c*e^3
+ a*b^2*d*e^2 + a^2*b*d^2*e - 8*a^2*c*d*e^2))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4
*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) + a*c*e^2 + a*b*d*e))/(a*b
^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) +
a*e*x)*(d*(2*a*c + (b*(b^2 - 4*a*c)^(1/2))/2 - b^2/2) - c*e*(b^2 - 4*a*c)^
(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a
*b*c*d*e) - (d*log(d + e*x))/(a*d^2 + c*e^2 - b*d*e)
```

3.66 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx$

3.66.1 Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = -\frac{(2ad-be)\text{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}$$

output
```
e*ln(e*x+d)/(a*d^2-b*d*e+c*e^2)-1/2*e*ln(a*x^2+b*x+c)/(a*d^2-b*d*e+c*e^2)-
(2*a*d-b*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))/(-4*
a*c+b^2)^(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = \frac{(-4ad+2be)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}e(-2\log(d+ex)+\log(c+x(b+ax)))}{2\sqrt{-b^2+4ac}\,(-ad^2+e(bd-ce))}$$

input
```
Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

output
```
((-4*a*d + 2*b*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a
*c]*e*(-2*Log[d + e*x] + Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a
*d^2) + e*(b*d - c*e)))
```

3.66.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1893, 1144, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \text{1893}$$

$$\int \frac{1}{(d+ex)(ax^2+bx+c)}dx$$

$$\Big\downarrow \text{1144}$$

$$\frac{\int \frac{ad-be-aex}{ax^2+bx+c}dx}{ad^2-bde+ce^2}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}(2ad-be)\int \frac{1}{ax^2+bx+c}dx-\frac{1}{2}e\int \frac{b+2ax}{ax^2+bx+c}dx}{ad^2-bde+ce^2}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-(2ad-be)\int \frac{1}{b^2-(b+2ax)^2-4ac}d(b+2ax)-\frac{1}{2}e\int \frac{b+2ax}{ax^2+bx+c}dx}{ad^2-bde+ce^2}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{1}{2}e\int \frac{b+2ax}{ax^2+bx+c}dx-\frac{(2ad-be)\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{ad^2-bde+ce^2}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\frac{(2ad-be)\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{1}{2}e\log\left(ax^2+bx+c\right)}{ad^2-bde+ce^2}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

input
```
Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

output
```
(e*Log[d + e*x])/(a*d^2 - b*d*e + c*e^2) + (-(((2*a*d - b*e)*ArcTanh[(b +
2*a*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]) - (e*Log[c + b*x + a*x^2])/2
)/(a*d^2 - b*d*e + c*e^2)
```

3.66.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

3.66.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

method	result
default	$\frac{-\frac{e\ln\left(a\,x^{2}+bx+c\right)}{2}+\frac{2\left(da-\frac{be}{2}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a\,d^{2}-bde+c\,e^{2}}+\frac{e\ln\left(ex+d\right)}{a\,d^{2}-bde+c\,e^{2}}$
risch	$\frac{e\ln\left(ex+d\right)}{a\,d^{2}-bde+c\,e^{2}}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{2}c\,d^{2}-a\,b^{2}d^{2}-4abcde+4a\,c^{2}e^{2}+b^{3}de-b^{2}c\,e^{2}\right)\_Z^{2}+\left(4ace-b^{2}e\right)\_Z+a\right)}\_R\ln\left(\left(\left(-2a^{2}d\right.\right.\right.\right.$

input
```
int(1/(a+c/x^2+b/x)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2-b*d*e+c*e^2)*(-1/2*e*ln(a*x^2+b*x+c)+2*(d*a-1/2*b*e)/(4*a*c-b^2)^
(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+e*ln(e*x+d)/(a*d^2-b*d*e+c*e^2)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.48

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx$$
$$=\left[-\frac{(b^2-4\,ac)e\log\left(ax^2+bx+c\right)-2\,(b^2-4\,ac)e\log\left(ex+d\right)+\sqrt{b^2-4\,ac}(2\,ad-be)\log\left(\frac{2\,a^2x^2+2\,abx+b^2\cdots}{a\cdots}\right.}{2\,((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2)}\right.$$
$$-\frac{(b^2-4\,ac)e\log\left(ax^2+bx+c\right)-2\,(b^2-4\,ac)e\log\left(ex+d\right)+2\,\sqrt{-b^2+4\,ac}(2\,ad-be)\arctan\left(-\frac{\sqrt{-b^2\cdots}}{\cdots}\right.}{2\,((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2)}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 4*a*c)*e*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x +
d) + sqrt(b^2 - 4*a*c)*(2*a*d - b*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*
c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*
d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), -1/2*((b^2 - 4*a*c)*e*
log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x + d) + 2*sqrt(-b^2 + 4*a*
c)*(2*a*d - b*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((
a*b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d),x)
```

output
```
Timed out
```

3.66.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = \frac{e^2\log\left(|ex+d|\right)}{ad^2e-bde^2+ce^3} - \frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)} + \frac{(2\,ad-be)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(ad^2-bde+ce^2)\sqrt{-b^2+4\,ac}}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
e^2*log(abs(e*x + d))/(a*d^2*e - b*d*e^2 + c*e^3) - 1/2*e*log(a*x^2 + b*x
+ c)/(a*d^2 - b*d*e + c*e^2) + (2*a*d - b*e)*arctan((2*a*x + b)/sqrt(-b^2
+ 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

3.66.9 Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 521, normalized size of antiderivative = 4.24

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx$$
$$= \frac{\ln\left(3\,a^2\,e^2\,x+a\,b\,e^2+a^2\,d\,e-\frac{a\,e\,\left(\frac{b^2\,e}{2}-2\,a\,c\,e+a\,d\,\sqrt{b^2-4\,a\,c}-\frac{b\,e\,\sqrt{b^2-4\,a\,c}}{2}\right)\,\left(2\,x\,a^2\,d^2+a\,b\,d^2-2\,x\,a\,b\,d\,e-8\,c\,a\,d\,e-6\,c\,x\,a\,e^2+b^2\,d\right.}{\left(4\,a\,c-b^2\right)\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e}$$
$$-\frac{\ln\left(3\,a^2\,e^2\,x+a\,b\,e^2+a^2\,d\,e-\frac{a\,e\,\left(\frac{b^2\,e}{2}-2\,a\,c\,e-a\,d\,\sqrt{b^2-4\,a\,c}+\frac{b\,e\,\sqrt{b^2-4\,a\,c}}{2}\right)\,\left(2\,x\,a^2\,d^2+a\,b\,d^2-2\,x\,a\,b\,d\,e-8\,c\,a\,d\,e-6\,c\,x\,a\,e^2+b\right.}{\left(4\,a\,c-b^2\right)\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d}$$
$$+\frac{e\,\ln\left(d+e\,x\right)}{a\,d^2-b\,d\,e+c\,e^2}$$

input
```
int(1/(x^2*(d + e*x)*(a + b/x + c/x^2)),x)
```

output
```
(log(3*a^2*e^2*x + a*b*e^2 + a^2*d*e - (a*e*((b^2*e)/2 - 2*a*c*e + a*d*(b^
2 - 4*a*c)^(1/2) - (b*e*(b^2 - 4*a*c)^(1/2))/2)*(2*a^2*d^2*x + 2*b^2*e^2*x
 + a*b*d^2 + b*c*e^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e - 2*a*b*d*e*x))/(
(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(e*(2*a*c + (b*(b^2 - 4*a*c)^(1/2)
)/2 - b^2/2) - a*d*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^
2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (log(3*a^2*e^2*x + a*b*e^2 +
a^2*d*e - (a*e*((b^2*e)/2 - 2*a*c*e - a*d*(b^2 - 4*a*c)^(1/2) + (b*e*(b^2
- 4*a*c)^(1/2))/2)*(2*a^2*d^2*x + 2*b^2*e^2*x + a*b*d^2 + b*c*e^2 + b^2*d*
e - 6*a*c*e^2*x - 8*a*c*d*e - 2*a*b*d*e*x))/((4*a*c - b^2)*(a*d^2 + c*e^2
- b*d*e)))*(e*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2/2) - a*d*(b^2 - 4*a
*c)^(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e +
 4*a*b*c*d*e) + (e*log(d + e*x))/(a*d^2 + c*e^2 - b*d*e)
```

3.67 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx$

3.67.1 Optimal result

Integrand size = 25, antiderivative size = 158

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \frac{(abd-b^2e+2ace)\,\text{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{\log(x)}{cd} - \frac{e^2\log(d+ex)}{d\,(ad^2-bde+ce^2)} - \frac{(ad-be)\log\left(c+bx+ax^2\right)}{2c\,(ad^2-e(bd-ce))}$$

output
```
ln(x)/c/d-e^2*ln(e*x+d)/d/(a*d^2-b*d*e+c*e^2)-1/2*(a*d-b*e)*ln(a*x^2+b*x+c
)/c/(a*d^2-e*(b*d-c*e))+(a*b*d+2*a*c*e-b^2*e)*arctanh((2*a*x+b)/(-4*a*c+b^
2)^(1/2))/c/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = -\frac{2d(abd-b^2e+2ace)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}(-2(ad^2+e(-bd+ce))\log(x)+2ce^2\log(d+e}{2c\sqrt{-b^2+4ac}d\,(ad^2+e(-bd+ce))}$$

input
```
Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

output
```
-1/2*(2*d*(a*b*d - b^2*e + 2*a*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]]
 + Sqrt[-b^2 + 4*a*c]*(-2*(a*d^2 + e*(-(b*d) + c*e))*Log[x] + 2*c*e^2*Log[
d + e*x] + d*(a*d - b*e)*Log[c + x*(b + a*x)]))/(c*Sqrt[-b^2 + 4*a*c]*d*(a
*d^2 + e*(-(b*d) + c*e)))
```

3.67.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(d+ex)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \text{ 1893}$$

$$\int \frac{1}{x(d+ex)\left(ax^2+bx+c\right)}dx$$

$$\Big\downarrow \text{ 1200}$$

$$\int \left(\frac{-a(bd+ce)-ax(ad-be)+b^2e}{c\left(ax^2+bx+c\right)\left(ad^2-e(bd-ce)\right)}+\frac{e^3}{d(d+ex)\left(e(bd-ce)-ad^2\right)}+\frac{1}{cdx}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(abd+2ace+b^2(-e)\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)}{cd}$$

input
```
Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

output
```
((a*b*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt
[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + Log[x]/(c*d) - (e^2*Log[d + e*x])
/(d*(a*d^2 - e*(b*d - c*e))) - ((a*d - b*e)*Log[c + b*x + a*x^2])/(2*c*(a*
d^2 - e*(b*d - c*e)))
```

3.67.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.67.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{\ln(x)}{cd}+\frac{\frac{\left(-d\,a^{2}+abe\right)\ln\left(a\,x^{2}+bx+c\right)}{2a}+\frac{2\left(-dab-ace+b^{2}e-\frac{\left(-d\,a^{2}+abe\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{(a\,d^{2}-bde+c\,e^{2})c}-\frac{e^{2}\ln(ex+d)}{d(a\,d^{2}-bde+c\,e^{2})}$	160
risch	Expression too large to display	19172

input
```
int(1/(a+c/x^2+b/x)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/c/d+1/(a*d^2-b*d*e+c*e^2)/c*(1/2*(-a^2*d+a*b*e)/a*ln(a*x^2+b*x+c)+2*
(-d*a*b-a*c*e+b^2*e-1/2*(-a^2*d+a*b*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2)))-e^2*ln(e*x+d)/d/(a*d^2-b*d*e+c*e^2)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 184.15 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.19

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx$$
$$= \left[-\frac{2\,(b^2c-4\,ac^2)e^2\log{(ex+d)}-(abd^2-(b^2-2\,ac)de)\sqrt{b^2-4\,ac}\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,ax+b)}{ax^2+bx+c}\right.}{2\,((ab^2c-4\,a^2c^2)d^3-}\right.$$
$$-\frac{2\,(b^2c-4\,ac^2)e^2\log{(ex+d)}-2\,(abd^2-(b^2-2\,ac)de)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,ax+b)}{b^2-4\,ac}\right)+((ab}{2\,((ab^2c-4\,a^2c^2)d^3-(b^3c}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
[-1/2*(2*(b^2*c - 4*a*c^2)*e^2*log(e*x + d) - (a*b*d^2 - (b^2 - 2*a*c)*d*e
)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*
a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((a*b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a
*b*c)*d*e)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a*b*
c)*d*e + (b^2*c - 4*a*c^2)*e^2)*log(x))/((a*b^2*c - 4*a^2*c^2)*d^3 - (b^3*
c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2), -1/2*(2*(b^2*c - 4*a*c^
2)*e^2*log(e*x + d) - 2*(a*b*d^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*a
rctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((a*b^2 - 4*a^2*c)*
d^2 - (b^3 - 4*a*b*c)*d*e)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^2
 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)*log(x))/((a*b^2*c - 4*a^2*
c^2)*d^3 - (b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2)]
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d),x)
```

output
```
Timed out
```

3.67.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = -\frac{e^3\log(|ex+d|)}{ad^3e-bd^2e^2+cde^3} - \frac{(ad-be)\log(ax^2+bx+c)}{2\,(acd^2-bcde+c^2e^2)} - \frac{(abd-b^2e+2\,ace)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(acd^2-bcde+c^2e^2)\sqrt{-b^2+4\,ac}} + \frac{\log(|x|)}{cd}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
-e^3*log(abs(e*x + d))/(a*d^3*e - b*d^2*e^2 + c*d*e^3) - 1/2*(a*d - b*e)*l
og(a*x^2 + b*x + c)/(a*c*d^2 - b*c*d*e + c^2*e^2) - (a*b*d - b^2*e + 2*a*c
*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - b*c*d*e + c^2*e^2)*
sqrt(-b^2 + 4*a*c)) + log(abs(x))/(c*d)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 11.09 (sec) , antiderivative size = 2399, normalized size of antiderivative = 15.18

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*(d + e*x)*(a + b/x + c/x^2)),x)
```

output
```
(log(b^3*c^3*e^5 - 6*a^4*c^2*d^5 + 2*a^3*b^2*c*d^5 + 8*a^2*c^4*d*e^4 - b^4
*c^2*d*e^4 - 2*b^5*c*d^2*e^3 + 2*a^3*b^3*d^5*x + 8*a^2*c^4*e^5*x + b^4*c^2
*e^5*x - 2*b^6*d^2*e^3*x + b^2*c^3*e^5*(b^2 - 4*a*c)^(1/2) + 18*a^3*c^3*d^
3*e^2 - 4*a*b*c^4*e^5 - 4*a*c^4*e^5*(b^2 - 4*a*c)^(1/2) - 5*a^2*c^3*d^2*e^
3*(b^2 - 4*a*c)^(1/2) - 7*a^4*b*c*d^5*x - b^5*c*d*e^4*x - 27*a^2*b^2*c^2*d
^3*e^2 + 2*a^3*b*c*d^5*(b^2 - 4*a*c)^(1/2) - 3*a^4*c*d^5*x*(b^2 - 4*a*c)^(
1/2) + 2*a*b^2*c^3*d*e^4 + 6*a*b^4*c*d^3*e^2 - 6*a^2*b^3*c*d^4*e + 21*a^3*
b*c^2*d^4*e - 6*a*b^2*c^3*e^5*x + 6*a*b^5*d^3*e^2*x - 6*a^2*b^4*d^4*e*x -
14*a^4*c^2*d^4*e*x + 7*a^3*c^2*d^4*e*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d*e^4*(
b^2 - 4*a*c)^(1/2) - 2*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^3*b^2*d^5*x
*(b^2 - 4*a*c)^(1/2) + b^3*c^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*x
*(b^2 - 4*a*c)^(1/2) + 13*a*b^3*c^2*d^2*e^3 - 21*a^2*b*c^3*d^2*e^3 + 10*a^
3*c^3*d^2*e^3*x + 6*a*b^3*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^2*c*d^4*
e*(b^2 - 4*a*c)^(1/2) + 6*a*b^4*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^3*
d^4*e*x*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^3*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 32*a
^2*b^3*c*d^3*e^2*x + 35*a^3*b*c^2*d^3*e^2*x + 7*a*b^2*c^2*d^2*e^3*(b^2 - 4
*a*c)^(1/2) - 13*a^2*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 9*a^3*c^2*d^3*e^2
*x*(b^2 - 4*a*c)^(1/2) - 27*a^2*b^2*c^2*d^2*e^3*x + 4*a*b*c^3*d*e^4*(b^2 -
 4*a*c)^(1/2) - 4*a*b*c^3*e^5*x*(b^2 - 4*a*c)^(1/2) - b^4*c*d*e^4*x*(b^2 -
 4*a*c)^(1/2) + 5*a*b^3*c^2*d*e^4*x + 14*a*b^4*c*d^2*e^3*x - 4*a^2*b*c^...
```

3.68 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx$

3.68.1 Optimal result

Integrand size = 25, antiderivative size = 193

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = & -\frac{1}{cdx}+\frac{(2a^2cd+b^3e-ab(bd+3ce))\,\text{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}\\ & -\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\,(ad^2-e(bd-ce))}\\ & +\frac{(abd-b^2e+ace)\log\left(c+bx+ax^2\right)}{2c^2\,(ad^2-e(bd-ce))}\end{aligned}$$

output
```
-1/c/d/x-(b*d+c*e)*ln(x)/c^2/d^2+e^3*ln(e*x+d)/d^2/(a*d^2-e*(b*d-c*e))+1/2
*(a*b*d+a*c*e-b^2*e)*ln(a*x^2+b*x+c)/c^2/(a*d^2-e*(b*d-c*e))+(2*a^2*c*d+b^
3*e-a*b*(b*d+3*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*d^2-e*(b
*d-c*e))/(-4*a*c+b^2)^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = & -\frac{1}{cdx}+\frac{(2a^2cd+b^3e-ab(bd+3ce))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}\,(-ad^2+e(bd-ce))}\\ & -\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{ad^4+d^2e(-bd+ce)}\\ & +\frac{(abd-b^2e+ace)\log(c+x(b+ax))}{2c^2\,(ad^2+e(-bd+ce))}\end{aligned}$$

input
```
Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

output
```
-(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTan[(b + 2*a*x)
/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e)))
- ((b*d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(a*d^4 + d^2*e*(-(b*
d) + c*e)) + ((a*b*d - b^2*e + a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2
+ e*(-(b*d) + c*e)))
```

3.68.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(d+ex)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \; 1893$$

$$\int \frac{1}{x^2(d+ex)\left(ax^2+bx+c\right)}dx$$

$$\Big\downarrow \; 1200$$

$$\int \left(\frac{-a^2cd+ax\left(abd+ace+b^2(-e)\right)+ab(bd+2ce)+b^3(-e)}{c^2\left(ax^2+bx+c\right)\left(ad^2-e(bd-ce)\right)}+\frac{e^4}{d^2(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{-bd-ce}{c^2d^2x}+\frac{}{cd\cdots}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(2a^2cd-ab(bd+3ce)+b^3e\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd+ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}-\frac{\log(x)(bd+ce)}{c^2d^2}-\frac{1}{cdx}$$

input
```
Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

output
```
-(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTanh[(b + 2*a*x
)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - ((
b*d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c
*e))) + ((a*b*d - b^2*e + a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(
b*d - c*e)))
```

3.68.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.68.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.07

method	result
default	$-\frac{1}{cdx}+\frac{(-bd-ec)\ln(x)}{c^2d^2}+\frac{\frac{\left(a^2bd+a^2ce-a\,b^2e\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(-a^2cd+a\,b^2d+2abce-b^3e-\frac{\left(a^2bd+a^2ce-a\,b^2e\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right.}{\sqrt{4ac-b^2}}}{(a\,d^2-bde+c\,e^2)c^2}$
risch	Expression too large to display

input
```
int(1/(a+c/x^2+b/x)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/c/d/x+1/c^2/d^2*(-b*d-c*e)*ln(x)+1/(a*d^2-b*d*e+c*e^2)/c^2*(1/2*(a^2*b*
d+a^2*c*e-a*b^2*e)/a*ln(a*x^2+b*x+c)+2*(-a^2*c*d+a*b^2*d+2*a*b*c*e-b^3*e-1
/2*(a^2*b*d+a^2*c*e-a*b^2*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*
c-b^2)^(1/2)))+e^3/d^2/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

3.68.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d),x)
```

output
```
Timed out
```

3.68.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = \frac{e^4\log\left(|ex+d|\right)}{ad^4e-bd^3e^2+cd^2e^3}+\frac{(abd-b^2e+ace)\log\left(ax^2+bx+c\right)}{2\left(ac^2d^2-bc^2de+c^3e^2\right)} + \frac{(ab^2d-2\,a^2cd-b^3e+3\,abce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(ac^2d^2-bc^2de+c^3e^2)\sqrt{-b^2+4\,ac}} - \frac{(bd+ce)\log\left(|x|\right)}{c^2d^2}-\frac{1}{cdx}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="giac")
```

output
```
e^4*log(abs(e*x + d))/(a*d^4*e - b*d^3*e^2 + c*d^2*e^3) + 1/2*(a*b*d - b^2
*e + a*c*e)*log(a*x^2 + b*x + c)/(a*c^2*d^2 - b*c^2*d*e + c^3*e^2) + (a*b^
2*d - 2*a^2*c*d - b^3*e + 3*a*b*c*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c)
)/((a*c^2*d^2 - b*c^2*d*e + c^3*e^2)*sqrt(-b^2 + 4*a*c)) - (b*d + c*e)*log
(abs(x))/(c^2*d^2) - 1/(c*d*x)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 22.98 (sec) , antiderivative size = 2388, normalized size of antiderivative = 12.37

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^4*(d + e*x)*(a + b/x + c/x^2)),x)
```

output

```
(e^3*log(d + e*x))/(a*d^4 + c*d^2*e^2 - b*d^3*e) + (log((a^4*e^4*x)/(c^2*d
^2) - (((a*e*x*(a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 + 2*a^3*c*d^2*e^2 - 4*a*
b^2*c*e^4 + 2*a^2*b*c*d*e^3))/(c^2*d^2) - (((a*e*(a^2*b^2*d^4 - 4*a*c^3*e^
4 - a^3*c*d^4 + b^2*c^2*e^4 + b^4*d^2*e^2 + 4*a^2*c^2*d^2*e^2 - 2*a*b^3*d^
3*e + b^3*c*d*e^3 - 4*a*b*c^2*d*e^3 + 5*a^2*b*c*d^3*e - 5*a*b^2*c*d^2*e^2)
)/(c*d) + (a*e*x*(2*a^3*b*d^4 + 2*b^3*c*e^4 + 2*b^4*d*e^3 - 2*a*b^3*d^2*e^
2 - 2*a^2*b^2*d^3*e + 12*a^2*c^2*d*e^3 - 8*a*b*c^2*e^4 + a^3*c*d^3*e - 11*
a*b^2*c*d*e^3 + 8*a^2*b*c*d^2*e^2))/(c*d) + (a*e*(b^4*e + b^3*e*(b^2 - 4*a
*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e - a*b^2*d*(b
^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c
)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*
x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^
3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*
e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a
^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2
 - b*d*e)))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4
*a^2*b*c*d - 5*a*b^2*c*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 -
4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2
 + c*e^2 - b*d*e)) + (a*e*(b*d + c*e)*(a^3*d^3 + b^3*e^3 - 3*a*b*c*e^3))/(
c^2*d^2))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + ...
```

3.68. $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx$

3.69 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx$

3.69.1 Optimal result

Integrand size = 25, antiderivative size = 252

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx &= -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}\\ &\quad-\frac{\left(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}\\ &\quad+\frac{\left(b^2d^2+bcde-c(ad^2-ce^2)\right)\log(x)}{c^3d^3}\\ &\quad-\frac{e^4\log(d+ex)}{d^3\,(ad^2-e(bd-ce))}\\ &\quad+\frac{\left(a^2cd+b^3e-ab(bd+2ce)\right)\log\left(c+bx+ax^2\right)}{2c^3\,(ad^2-e(bd-ce))}\end{aligned}$$

output
```
-1/2/c/d/x^2+(b*d+c*e)/c^2/d^2/x+(b^2*d^2+b*c*d*e-c*(a*d^2-c*e^2))*ln(x)/c
^3/d^3-e^4*ln(e*x+d)/d^3/(a*d^2-e*(b*d-c*e))+1/2*(a^2*c*d+b^3*e-a*b*(b*d+2
*c*e))*ln(a*x^2+b*x+c)/c^3/(a*d^2-e*(b*d-c*e))-(b^4*e+a^2*c*(3*b*d+2*c*e)-
a*b^2*(b*d+4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(a*d^2-e*(b*d
-c*e))/(-4*a*c+b^2)^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x} - \frac{(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^3\sqrt{-b^2+4ac}\,(-ad^2+e(bd-ce))} + \frac{(b^2d^2+bcde+c(-ad^2+ce^2))\log(x)}{c^3d^3} - \frac{e^4\log(d+ex)}{ad^5+d^3e(-bd+ce)} + \frac{(a^2cd+b^3e-ab(bd+2ce))\log(c+x(b+ax))}{2c^3\,(ad^2+e(-bd+ce))}$$

input
```
Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

output
```
-1/2*1/(c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*
e) - a*b^2*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^3*Sqr
t[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e + c*(-(a
*d^2) + c*e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(a*d^5 + d^3*e*(-(b
*d) + c*e)) + ((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + x*(b + a*x)])
/(2*c^3*(a*d^2 + e*(-(b*d) + c*e)))
```

3.69.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5(d+ex)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \text{1893}$$

$$\int \frac{1}{x^3(d+ex)\left(ax^2+bx+c\right)}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{ax(a^2cd - ab(bd + 2ce) + b^3e) + a^2c(2bd + ce) - ab^2(bd + 3ce) + b^4e}{c^3(ax^2 + bx + c)(ad^2 - e(bd - ce))} + \frac{-c(ad^2 - ce^2) + b^2d^2 + bcde}{c^3d^3x} + \frac{}{d^3(c}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)(a^2c(3bd + 2ce) - ab^2(bd + 4ce) + b^4e)}{c^3\sqrt{b^2-4ac}(ad^2 - e(bd - ce))} + \frac{(a^2cd - ab(bd + 2ce) + b^3e)\log(ax^2 + bx + c)}{2c^3(ad^2 - e(bd - ce))} + \frac{\log(x)(-c(ad^2 - ce^2) + b^2d^2 + bcde)}{c^3d^3} - \frac{e^4\log(d + ex)}{d^3(ad^2 - e(bd - ce))} + \frac{bd + ce}{c^2d^2x} - \frac{1}{2cdx^2}$$

input
```
Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

output
```
-1/2*1/(c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*
e) - a*b^2*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqr
t[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e - c*(a*d^2 -
 c*e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e
))) + ((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + b*x + a*x^2])/(2*c^3*
(a*d^2 - e*(b*d - c*e)))
```

3.69.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.69.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.10

method	result
default	$-\frac{1}{2cd\,x^2}-\frac{-bd-ec}{x\,c^2d^2}+\frac{\left(-d^2ac+b^2d^2+bcde+e^2c^2\right)\ln\left(x\right)}{d^3c^3}+\frac{\frac{\left(a^3cd-a^2b^2d-2a^2bce+a\,b^3e\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(2a^2bcd+a^2c^2e-a\,b^3d-\right.}{}}{\left(a\,d^2-bde+c\right.}$
risch	Expression too large to display

input
```
int(1/(a+c/x^2+b/x)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c/d/x^2-(-b*d-c*e)/x/c^2/d^2+1/d^3/c^3*(-a*c*d^2+b^2*d^2+b*c*d*e+c^2*
e^2)*ln(x)+1/(a*d^2-b*d*e+c*e^2)/c^3*(1/2*(a^3*c*d-a^2*b^2*d-2*a^2*b*c*e+a
*b^3*e)/a*ln(a*x^2+b*x+c)+2*(2*a^2*b*c*d+a^2*c^2*e-a*b^3*d-3*a*b^2*c*e+b^4
*e-1/2*(a^3*c*d-a^2*b^2*d-2*a^2*b*c*e+a*b^3*e)*b/a)/(4*a*c-b^2)^(1/2)*arct
an((2*a*x+b)/(4*a*c-b^2)^(1/2)))-e^4/d^3/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

3.69.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d),x)
```

output
```
Timed out
```

3.69.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx &= -\frac{e^5\log(|ex+d|)}{ad^5e-bd^4e^2+cd^3e^3} - \frac{(ab^2d-a^2cd-b^3e+2\,abce)\log(ax^2+bx+c)}{2\,(ac^3d^2-bc^3de+c^4e^2)} \\ &\quad - \frac{(ab^3d-3\,a^2bcd-b^4e+4\,ab^2ce-2\,a^2c^2e)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(ac^3d^2-bc^3de+c^4e^2)\sqrt{-b^2+4\,ac}} \\ &\quad + \frac{(b^2d^2-acd^2+bcde+c^2e^2)\log(|x|)}{c^3d^3} - \frac{c^2d^2-2\,(bcd^2+c^2de)x}{2\,c^3d^3x^2}\end{aligned}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="giac")
```

output
```
-e^5*log(abs(e*x + d))/(a*d^5*e - b*d^4*e^2 + c*d^3*e^3) - 1/2*(a*b^2*d -
a^2*c*d - b^3*e + 2*a*b*c*e)*log(a*x^2 + b*x + c)/(a*c^3*d^2 - b*c^3*d*e +
 c^4*e^2) - (a*b^3*d - 3*a^2*b*c*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ar
ctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c^3*d^2 - b*c^3*d*e + c^4*e^2)*sq
rt(-b^2 + 4*a*c)) + (b^2*d^2 - a*c*d^2 + b*c*d*e + c^2*e^2)*log(abs(x))/(c
^3*d^3) - 1/2*(c^2*d^2 - 2*(b*c*d^2 + c^2*d*e)*x)/(c^3*d^3*x^2)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 28.67 (sec) , antiderivative size = 3530, normalized size of antiderivative = 14.01

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^5*(d + e*x)*(a + b/x + c/x^2)),x)
```

output
```
(log((a^4*e^4*(b^2*d^2 + c^2*e^2 - a*c*d^2 + b*c*d*e))/(c^4*d^4) - (((((a*
e*(a^2*b^3*d^5 - 4*a*c^4*e^5 + b^2*c^3*e^5 + b^5*d^3*e^2 - 3*a^3*c^2*d^4*e
 + b^3*c^2*d*e^4 + b^4*c*d^2*e^3 + 4*a^2*c^3*d^2*e^3 - 2*a^3*b*c*d^5 - 2*a
*b^4*d^4*e - 4*a*b*c^3*d*e^4 - 6*a*b^3*c*d^3*e^2 + 7*a^2*b^2*c*d^4*e - 5*a
*b^2*c^2*d^2*e^3 + 8*a^2*b*c^2*d^3*e^2))/(c^2*d^2) + (a*e*x*(2*a^3*b^2*d^5
 - 3*a^4*c*d^5 + 2*b^3*c^2*e^5 + 2*b^5*d^2*e^3 - 2*a*b^4*d^3*e^2 - 2*a^2*b
^3*d^4*e + 8*a^2*c^3*d*e^4 - 8*a^3*c^2*d^3*e^2 - 8*a*b*c^3*e^5 + b^4*c*d*e
^4 + 4*a^3*b*c*d^4*e - 6*a*b^2*c^2*d*e^4 - 12*a*b^3*c*d^2*e^3 + 16*a^2*b*c
^2*d^2*e^3 + 10*a^2*b^2*c*d^3*e^2))/(c^2*d^2) - (a*e*(b^4*e*(b^2 - 4*a*c)^
(1/2) - b^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c
)^(1/2) - 5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2)
+ 3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(4*a^
2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*
e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*
a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*
c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x
 - 6*a*b^2*c*d^2*e^2*x))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b
^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a
*b^3*d*(b^2 - 4*a*c)^(1/2) - 5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(
b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2...
```

3.70 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

3.70.1 Optimal result

Integrand size = 25, antiderivative size = 343

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = -\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)}$$
$$+\frac{\left(b^5d^2-2b^4cde+8ab^2c^2de-4a^2c^3de+abc^2(5ad^2-3ce^2)-b^3c(5ad^2-ce^2)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$
$$+\frac{d^4(3ad^2-e(4bd-5ce))\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)^2}$$
$$+\frac{\left(b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)-b^2c(3ad^2-ce^2)\right)\log\left(c+bx+ax^2\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}$$

output
```
-(2*a*d+b*e)*x/a^2/e^3+1/2*x^2/a/e^2+d^5/e^4/(a*d^2-e*(b*d-c*e))/(e*x+d)+d
^4*(3*a*d^2-e*(4*b*d-5*c*e))*ln(e*x+d)/e^4/(a*d^2-e*(b*d-c*e))^2+1/2*(b^4*
d^2-2*b^3*c*d*e+4*a*b*c^2*d*e+a*c^2*(a*d^2-c*e^2)-b^2*c*(3*a*d^2-c*e^2))*l
n(a*x^2+b*x+c)/a^3/(a*d^2-e*(b*d-c*e))^2+(b^5*d^2-2*b^4*c*d*e+8*a*b^2*c^2*
d*e-4*a^2*c^3*d*e+a*b*c^2*(5*a*d^2-3*c*e^2)-b^3*c*(5*a*d^2-c*e^2))*arctanh
((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2
)
```

3.70.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = -\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2+e(-bd+ce)\right)(d+ex)} - \frac{\left(b^5d^2-2b^4cde+8ab^2c^2de-4a^2c^3de+abc^2(5ad^2-3ce^2)+b^3c(-5ad^2+ce^2)\right)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^3\sqrt{-b^2+4ac}\left(ad^2+e(-bd+ce)\right)^2} + \frac{\left(3ad^6+d^4e(-4bd+5ce)\right)\log(d+ex)}{e^4\left(ad^2+e(-bd+ce)\right)^2} + \frac{\left(b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)+b^2c(-3ad^2+ce^2)\right)\log(c+x(b+ax))}{2a^3\left(ad^2+e(-bd+ce)\right)^2}$$

input
```
Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

output
```
-(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 + e*(-(b*
d) + c*e))*(d + e*x)) - ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*
c^3*d*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) + b^3*c*(-5*a*d^2 + c*e^2))*ArcTan[(
b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d)
 + c*e))^2) + ((3*a*d^6 + d^4*e*(-4*b*d + 5*c*e))*Log[d + e*x])/(e^4*(a*d^
2 + e*(-(b*d) + c*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2
*(a*d^2 - c*e^2) + b^2*c*(-3*a*d^2 + c*e^2))*Log[c + x*(b + a*x)])/(2*a^3*
(a*d^2 + e*(-(b*d) + c*e))^2)
```

3.70.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)^2\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \text{ 1893}$$

$$\int \frac{x^5}{(d+ex)^2(ax^2+bx+c)}dx$$

$$\Big\downarrow 1200$$

$$\int \left(\frac{c(bd-ce)\left(-2acd+b^2d-bce\right)+x\left(-b^2c\left(3ad^2-ce^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)}{a^2\left(ax^2+bx+c\right)\left(ad^2-e(bd-ce)\right)^2}+\frac{-2ac}{a^2}\right.$$

$$\Big\downarrow 2009$$

$$\frac{\left(-b^2c\left(3ad^2-ce^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}-\frac{x(2ad+be)}{a^2e^3}+\frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(-4a^2c^3de-b^3c\left(5ad^2-ce^2\right)+8ab^2c^2de+abc^2\left(5ad^2-3ce^2\right)+b^5d^2-2b^4cde\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{d^5}{e^4(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{d^4\log(d+ex)\left(3ad^2-e(4bd-5ce)\right)}{e^4\left(ad^2-e(bd-ce)\right)^2}+\frac{x^2}{2ae^2}$$

input
```
Int[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

output
```
-(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 - e*(b*d
- c*e))*(d + e*x)) + ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^3
*d*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) - b^3*c*(5*a*d^2 - c*e^2))*ArcTanh[(b +
 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))
^2) + (d^4*(3*a*d^2 - e*(4*b*d - 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 - e*(b*
d - c*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^2 - c*
e^2) - b^2*c*(3*a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a^3*(a*d^2 - e*(b
*d - c*e))^2)
```

3.70.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.70.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.05

method	result
default	$-\frac{-\frac{1}{2}ae\,x^{2}+2adx+bex}{e^{3}a^{2}}+\frac{\frac{\left(a^{2}c^{2}d^{2}-3a\,b^{2}c\,d^{2}+4ab\,c^{2}de-a\,c^{3}e^{2}+b^{4}d^{2}-2b^{3}cde+b^{2}c^{2}e^{2}\right)\ln\left(a\,x^{2}+bx+c\right)}{2a}+\frac{2\left(-2b\,c^{2}d^{2}a+2a\,c^{3}de+b^{3}c\,d^{2}-2b\right.}{}}{(a\,d^{2}-bde+c\,e^{2}}$
risch	Expression too large to display

input
```
int(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/e^3/a^2*(-1/2*a*e*x^2+2*a*d*x+b*e*x)+1/(a*d^2-b*d*e+c*e^2)^2/a^2*(1/2*(
a^2*c^2*d^2-3*a*b^2*c*d^2+4*a*b*c^2*d*e-a*c^3*e^2+b^4*d^2-2*b^3*c*d*e+b^2*
c^2*e^2)/a*ln(a*x^2+b*x+c)+2*(-2*b*c^2*d^2*a+2*a*c^3*d*e+b^3*c*d^2-2*b^2*c
^2*d*e+b*c^3*e^2-1/2*(a^2*c^2*d^2-3*a*b^2*c*d^2+4*a*b*c^2*d*e-a*c^3*e^2+b^
4*d^2-2*b^3*c*d*e+b^2*c^2*e^2)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*
a*c-b^2)^(1/2)))+1/e^4*d^4*(3*a*d^2-4*b*d*e+5*c*e^2)/(a*d^2-b*d*e+c*e^2)^2
*ln(e*x+d)+1/e^4*d^5/(a*d^2-b*d*e+c*e^2)/(e*x+d)
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1341 vs. 2(335) = 670.

Time = 64.45 (sec) , antiderivative size = 2703, normalized size of antiderivative = 7.88

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a^4*b^2 - 4*a^5*c)*d^7 - 2*(a^3*b^3 - 4*a^4*b*c)*d^6*e + 2*(a^3*b
^2*c - 4*a^4*c^2)*d^5*e^2 + ((a^4*b^2 - 4*a^5*c)*d^4*e^3 - 2*(a^3*b^3 - 4*
a^4*b*c)*d^3*e^4 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^5 - 2*(a^2*b^
3*c - 4*a^3*b*c^2)*d*e^6 + (a^2*b^2*c^2 - 4*a^3*c^3)*e^7)*x^3 - (3*(a^4*b^
2 - 4*a^5*c)*d^5*e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d^4*e^3 - (a^2*b^4 - 10*a^3
*b^2*c + 24*a^4*c^2)*d^3*e^4 + 2*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e
^5 - (4*a*b^4*c - 19*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^6 + 2*(a*b^3*c^2 - 4*a^
2*b*c^3)*e^7)*x^2 - ((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3*e^4 - 2*(b^4*c -
4*a*b^2*c^2 + 2*a^2*c^3)*d^2*e^5 + (b^3*c^2 - 3*a*b*c^3)*d*e^6 + ((b^5 - 5
*a*b^3*c + 5*a^2*b*c^2)*d^2*e^5 - 2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^
6 + (b^3*c^2 - 3*a*b*c^3)*e^7)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b
*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*(
2*(a^4*b^2 - 4*a^5*c)*d^6*e - 3*(a^3*b^3 - 4*a^4*b*c)*d^5*e^2 + 4*(a^3*b^2
*c - 4*a^4*c^2)*d^4*e^3 + (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d^3*e^4 - 2*
(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^5 + (a*b^3*c^2 - 4*a^2*b*c^3)*
d*e^6)*x + ((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^4 - 2*(b^
5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d^2*e^5 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*
c^4)*d*e^6 + ((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^5 - 2*(
b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e^6 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*
c^4)*e^7)*x)*log(a*x^2 + b*x + c) + 2*(3*(a^4*b^2 - 4*a^5*c)*d^7 - 4*(a...
```

3.70.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

output
```
Timed out
```

3.70.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.68

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \frac{d^5e^4}{(ad^2e^8-bde^9+ce^{10})(ex+d)}$$
$$+\frac{(b^4d^2-3\,ab^2cd^2+a^2c^2d^2-2\,b^3cde+4\,abc^2de+b^2c^2e^2-ac^3e^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{b}{(ex}\right.}{2\,(a^5d^4-2\,a^4bd^3e+a^3b^2d^2e^2+2\,a^4cd^2e^2-2\,a^3bcde^3+a^3c^2e^4)}$$
$$+\frac{(b^5d^2e^2-5\,ab^3cd^2e^2+5\,a^2bc^2d^2e^2-2\,b^4cde^3+8\,ab^2c^2de^3-4\,a^2c^3de^3+b^3c^2e^4-3\,abc^3e^4)\arctan\left(-\frac{2}{}\right.}{(a^5d^4-2\,a^4bd^3e+a^3b^2d^2e^2+2\,a^4cd^2e^2-2\,a^3bcde^3+a^3c^2e^4)\sqrt{-b^2+4\,ace^2}}$$
$$+\frac{\left(a^2-\frac{2\,(3\,a^2de+abe^2)}{(ex+d)e}\right)(ex+d)^2}{2\,a^3e^4}-\frac{(3\,a^2d^2+2\,abde+b^2e^2-ace^2)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{a^3e^4}$$

input
```
integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

output
```
d^5*e^4/((a*d^2*e^8 - b*d*e^9 + c*e^10)*(e*x + d)) + 1/2*(b^4*d^2 - 3*a*b^
2*c*d^2 + a^2*c^2*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + b^2*c^2*e^2 - a*c^3*
e^2)*log(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/
(e*x + d)^2 - c*e^2/(e*x + d)^2)/(a^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^
2 + 2*a^4*c*d^2*e^2 - 2*a^3*b*c*d*e^3 + a^3*c^2*e^4) + (b^5*d^2*e^2 - 5*a*
b^3*c*d^2*e^2 + 5*a^2*b*c^2*d^2*e^2 - 2*b^4*c*d*e^3 + 8*a*b^2*c^2*d*e^3 -
4*a^2*c^3*d*e^3 + b^3*c^2*e^4 - 3*a*b*c^3*e^4)*arctan(-(2*a*d - 2*a*d^2/(e
*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/(sqrt(-b^2 + 4*a*c)
*e))/((a^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 + 2*a^4*c*d^2*e^2 - 2*a^3
*b*c*d*e^3 + a^3*c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2) + 1/2*(a^2 - 2*(3*a^2*d*
e + a*b*e^2)/((e*x + d)*e))*(e*x + d)^2/(a^3*e^4) - (3*a^2*d^2 + 2*a*b*d*e
 + b^2*e^2 - a*c*e^2)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/(a^3*e^4)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 3503, normalized size of antiderivative = 10.21

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

output
```
(log(d + e*x)*(3*a*d^6 + 5*c*d^4*e^2 - 4*b*d^5*e))/(c^2*e^8 + a^2*d^4*e^4
+ b^2*d^2*e^6 - 2*b*c*d*e^7 - 2*a*b*d^3*e^5 + 2*a*c*d^2*e^6) - (log(12*a^5
*c*d^8 - 2*a*c^5*e^8 - 3*a^4*b^2*d^8 + b^2*c^4*e^8 + b^6*d^4*e^4 + 4*a^3*b
^3*d^7*e - 4*b^3*c^3*d*e^7 - 4*b^5*c*d^3*e^5 + b^5*d^4*e^4*(b^2 - 4*a*c)^(
1/2) + 12*a^2*c^4*d^2*e^6 - 22*a^3*c^3*d^4*e^4 + 8*a^4*c^2*d^6*e^2 + 6*b^4
*c^2*d^2*e^6 - 3*a^4*b*d^8*(b^2 - 4*a*c)^(1/2) + b*c^4*e^8*(b^2 - 4*a*c)^(
1/2) - 6*a^5*d^8*x*(b^2 - 4*a*c)^(1/2) + 12*a*b*c^4*d*e^7 - 16*a^4*b*c*d^7
*e - 4*a^2*c^3*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 20*a^3*c^2*d^5*e^3*(b^2 - 4*a
*c)^(1/2) + 6*b^3*c^2*d^2*e^6*(b^2 - 4*a*c)^(1/2) + a*b*c^4*e^8*x + 24*a^5
*c*d^7*e*x + 14*a^2*b^2*c^2*d^4*e^4 + 4*a*c^4*d*e^7*(b^2 - 4*a*c)^(1/2) +
12*a^4*c*d^7*e*(b^2 - 4*a*c)^(1/2) + a*c^4*e^8*x*(b^2 - 4*a*c)^(1/2) - 6*a
*b^4*c*d^4*e^4 + a*b^5*d^4*e^4*x - 6*a^4*b^2*d^7*e*x + 8*a^2*c^4*d*e^7*x +
 4*a^3*b^2*d^7*e*(b^2 - 4*a*c)^(1/2) - 4*b^2*c^3*d*e^7*(b^2 - 4*a*c)^(1/2)
 - 4*b^4*c*d^3*e^5*(b^2 - 4*a*c)^(1/2) - 24*a*b^2*c^3*d^2*e^6 + 20*a*b^3*c
^2*d^3*e^5 - 20*a^2*b*c^3*d^3*e^5 - 4*a^2*b^3*c*d^5*e^3 + 16*a^3*b*c^2*d^5
*e^3 - 2*a^3*b^2*c*d^6*e^2 - 4*a^2*b^4*d^5*e^3*x + 11*a^3*b^3*d^6*e^2*x -
8*a^3*c^3*d^3*e^5*x + 40*a^4*c^2*d^5*e^3*x - 12*a*b*c^3*d^2*e^6*(b^2 - 4*a
*c)^(1/2) - 4*a*b^3*c*d^4*e^4*(b^2 - 4*a*c)^(1/2) - 24*a^3*b*c*d^6*e^2*(b^
2 - 4*a*c)^(1/2) + a*b^4*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 4*a^4*c*d^6*e^2*x
*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c^2*d^2*e^6*x - 18*a^2*b*c^3*d^2*e^6*x -...
```

3.70. $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

3.71 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

3.71.1 Optimal result

Integrand size = 25, antiderivative size = 274

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{\left(b^4d^2-2b^3cde+6abc^2de+2ac^2(ad^2-ce^2)-b^2c(4ad^2-ce^2)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} - \frac{d^3(2ad^2-e(3bd-4ce))\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)^2} - \frac{(bd-ce)\left(b^2d-2acd-bce\right)\log\left(c+bx+ax^2\right)}{2a^2\left(ad^2-e(bd-ce)\right)^2}$$

output
```
x/a/e^2-d^4/e^3/(a*d^2-e*(b*d-c*e))/(e*x+d)-d^3*(2*a*d^2-e*(3*b*d-4*c*e))*
ln(e*x+d)/e^3/(a*d^2-e*(b*d-c*e))^2-1/2*(b*d-c*e)*(-2*a*c*d+b^2*d-b*c*e)*l
n(a*x^2+b*x+c)/a^2/(a*d^2-e*(b*d-c*e))^2-(b^4*d^2-2*b^3*c*d*e+6*a*b*c^2*d*
e+2*a*c^2*(a*d^2-c*e^2)-b^2*c*(4*a*d^2-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b
^2)^(1/2))/a^2/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$$
$$= \frac{x}{ae^2} - \frac{d^4}{e^3(ad^2+e(-bd+ce))(d+ex)} + \frac{(b^4d^2-2b^3cde+6abc^2de+2ac^2(ad^2-ce^2)+b^2c(-4ad^2+ce^2))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^2\sqrt{-b^2+4ac}\,(ad^2+e(-bd+ce))^2} - \frac{(2ad^5+d^3e(-3bd+4ce))\log(d+ex)}{e^3(ad^2+e(-bd+ce))^2} + \frac{(bd-ce)(-b^2d+2acd+bce)\log(c+x(b+ax))}{2a^2(ad^2+e(-bd+ce))^2}$$

input
```
Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

output
```
x/(a*e^2) - d^4/(e^3*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((b^4*d^2 - 2
*b^3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) + b^2*c*(-4*a*d^2 + c
*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(a*
d^2 + e*(-(b*d) + c*e))^2) - ((2*a*d^5 + d^3*e*(-3*b*d + 4*c*e))*Log[d + e
*x])/(e^3*(a*d^2 + e*(-(b*d) + c*e))^2) + ((b*d - c*e)*(-(b^2*d) + 2*a*c*d
 + b*c*e)*Log[c + x*(b + a*x)])/(2*a^2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

3.71.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)^2\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}\,dx$$
$$\big\downarrow \text{ 1893}$$
$$\int \frac{x^4}{(d+ex)^2(ax^2+bx+c)}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{-c(-c(ad^2-ce^2)+b^2d^2-2bcde)-x(bd-ce)\left(-2acd+b^2d-bce\right)}{a\left(ax^2+bx+c\right)\left(ad^2-e(bd-ce)\right)^2}+\frac{d^4}{e^2(d+ex)^2\left(ad^2-e(bd-ce)\right)}+\frac{}{e^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(-b^2c\left(4ad^2-ce^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{(bd-ce)\left(-2acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)^2}-\frac{d^4}{e^3(d+ex)\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)\left(2ad^2-e(3bd-4ce)\right)}{e^3\left(ad^2-e(bd-ce)\right)^2}+\frac{x}{ae^2}$$

input
```
Int[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

output
```
x/(a*e^2) - d^4/(e^3*(a*d^2 - e*(b*d - c*e))*(d + e*x)) - ((b^4*d^2 - 2*b^
3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) - b^2*c*(4*a*d^2 - c*e^2
))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 -
 e*(b*d - c*e))^2) - (d^3*(2*a*d^2 - e*(3*b*d - 4*c*e))*Log[d + e*x])/(e^3
*(a*d^2 - e*(b*d - c*e))^2) - ((b*d - c*e)*(b^2*d - 2*a*c*d - b*c*e)*Log[c
 + b*x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e))^2)
```

3.71.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.71.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.06

method	result
default	$\frac{x}{a\,e^{2}}+\frac{\frac{\left(2b\,d^{2}ca-2a\,c^{2}de-b^{3}d^{2}+2b^{2}cde-b\,c^{2}e^{2}\right)\ln\left(a\,x^{2}+bx+c\right)}{2a}+\frac{2\left(a\,c^{2}d^{2}-b^{2}c\,d^{2}+2ed\,c^{2}b-c^{3}e^{2}-\frac{\left(2b\,d^{2}ca-2a\,c^{2}de-b^{3}d^{2}+2b^{2}cde-b\,c^{2}e^{2}\right)}{2a}\right.}{\sqrt{4ac-b^{2}}}}{\left(a\,d^{2}-bde+c\,e^{2}\right)^{2}a}$
risch	Expression too large to display

input
```
int(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
x/a/e^2+1/(a*d^2-b*d*e+c*e^2)^2/a*(1/2*(2*a*b*c*d^2-2*a*c^2*d*e-b^3*d^2+2*
b^2*c*d*e-b*c^2*e^2)/a*ln(a*x^2+b*x+c)+2*(a*c^2*d^2-b^2*c*d^2+2*e*d*c^2*b-
c^3*e^2-1/2*(2*a*b*c*d^2-2*a*c^2*d*e-b^3*d^2+2*b^2*c*d*e-b*c^2*e^2)*b/a)/(
4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-1/e^3*d^4/(a*d^2-b*d
*e+c*e^2)/(e*x+d)-1/e^3*d^3*(2*a*d^2-3*b*d*e+4*c*e^2)/(a*d^2-b*d*e+c*e^2)^
2*ln(e*x+d)
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. 2(268) = 536.

Time = 26.47 (sec) , antiderivative size = 2139, normalized size of antiderivative = 7.81

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 2*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 2*(a^2*
b^2*c - 4*a^3*c^2)*d^4*e^2 - 2*((a^3*b^2 - 4*a^4*c)*d^4*e^2 - 2*(a^2*b^3 -
 4*a^3*b*c)*d^3*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^4 - 2*(a*b^3
*c - 4*a^2*b*c^2)*d*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*e^6)*x^2 + ((b^4 - 4*a*b
^2*c + 2*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 3*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 2*a
*c^3)*d*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 3*a*b*c^
2)*d*e^5 + (b^2*c^2 - 2*a*c^3)*e^6)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 +
2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c))
- 2*((a^3*b^2 - 4*a^4*c)*d^5*e - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + (a*b^4
- 2*a^2*b^2*c - 8*a^3*c^2)*d^3*e^3 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^4 + (
a*b^2*c^2 - 4*a^2*c^3)*d*e^5)*x + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^3*e^3
 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2*e^4 + (b^3*c^2 - 4*a*b*c^3)*d*e
^5 + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^2*e^4 - 2*(b^4*c - 5*a*b^2*c^2 + 4
*a^2*c^3)*d*e^5 + (b^3*c^2 - 4*a*b*c^3)*e^6)*x)*log(a*x^2 + b*x + c) + 2*(
2*(a^3*b^2 - 4*a^4*c)*d^6 - 3*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 4*(a^2*b^2*c -
 4*a^3*c^2)*d^4*e^2 + (2*(a^3*b^2 - 4*a^4*c)*d^5*e - 3*(a^2*b^3 - 4*a^3*b*
c)*d^4*e^2 + 4*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^3)*x)*log(e*x + d))/((a^4*b^2
 - 4*a^5*c)*d^5*e^3 - 2*(a^3*b^3 - 4*a^4*b*c)*d^4*e^4 + (a^2*b^4 - 2*a^3*b
^2*c - 8*a^4*c^2)*d^3*e^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^6 + (a^2*b^2
*c^2 - 4*a^3*c^3)*d*e^7 + ((a^4*b^2 - 4*a^5*c)*d^4*e^4 - 2*(a^3*b^3 - 4...
```

3.71.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**2/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

output
```
Timed out
```

3.71.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = -\frac{d^4e^3}{(ad^2e^6-bde^7+ce^8)(ex+d)}$$
$$-\frac{(b^3d^2-2\,abcd^2-2\,b^2cde+2\,ac^2de+bc^2e^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{2\,(a^4d^4-2\,a^3bd^3e+a^2b^2d^2e^2+2\,a^3cd^2e^2-2\,a^2bcde^3+a^2c^2e^4)}$$
$$-\frac{(b^4d^2e^2-4\,ab^2cd^2e^2+2\,a^2c^2d^2e^2-2\,b^3cde^3+6\,abc^2de^3+b^2c^2e^4-2\,ac^3e^4)\arctan\left(-\frac{2\,ad-\frac{2\,ad^2}{ex+d}-be+\frac{2\,bde}{ex+d}}{\sqrt{-b^2+4\,ace}}\right.}{(a^4d^4-2\,a^3bd^3e+a^2b^2d^2e^2+2\,a^3cd^2e^2-2\,a^2bcde^3+a^2c^2e^4)\sqrt{-b^2+4\,ac}e^2}$$
$$+\frac{ex+d}{ae^3}+\frac{(2\,ad+be)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{a^2e^3}$$

input
```
integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-d^4*e^3/((a*d^2*e^6 - b*d*e^7 + c*e^8)*(e*x + d)) - 1/2*(b^3*d^2 - 2*a*b*
c*d^2 - 2*b^2*c*d*e + 2*a*c^2*d*e + b*c^2*e^2)*log(-a + 2*a*d/(e*x + d) -
a*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/(e*x + d)^2 - c*e^2/(e*x + d)^2)
/(a^4*d^4 - 2*a^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e^2 - 2*a^2*b*c*
d*e^3 + a^2*c^2*e^4) - (b^4*d^2*e^2 - 4*a*b^2*c*d^2*e^2 + 2*a^2*c^2*d^2*e^
2 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3 + b^2*c^2*e^4 - 2*a*c^3*e^4)*arctan(-(
2*a*d - 2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/(
sqrt(-b^2 + 4*a*c)*e))/((a^4*d^4 - 2*a^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3
*c*d^2*e^2 - 2*a^2*b*c*d*e^3 + a^2*c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2) + (e*x
 + d)/(a*e^3) + (2*a*d + b*e)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/(a^2*
e^3)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 11.20 (sec) , antiderivative size = 2495, normalized size of antiderivative = 9.11

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx=\text{Too large to display}$$

input
```
int(x^2/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

output
```
x/(a*e^2) - (log(d + e*x)*(2*a*d^5 + 4*c*d^3*e^2 - 3*b*d^4*e))/(c^2*e^7 +
a^2*d^4*e^3 + b^2*d^2*e^5 - 2*b*c*d*e^6 - 2*a*b*d^3*e^4 + 2*a*c*d^2*e^5) +
 (log(8*a^4*c*d^7 + b*c^4*e^7 + c^4*e^7*(b^2 - 4*a*c)^(1/2) - 2*a^3*b^2*d^
7 + b^5*d^4*e^3 + 3*a^2*b^3*d^6*e - 4*b^2*c^3*d*e^6 - 4*b^4*c*d^3*e^4 + b^
4*d^4*e^3*(b^2 - 4*a*c)^(1/2) - 24*a^2*c^3*d^3*e^4 + 8*a^3*c^2*d^5*e^2 + 6
*b^3*c^2*d^2*e^5 + 8*a*c^4*d*e^6 + 2*a*c^4*e^7*x - 2*a^3*b*d^7*(b^2 - 4*a*
c)^(1/2) - 4*a^4*d^7*x*(b^2 - 4*a*c)^(1/2) - 12*a^3*b*c*d^6*e + 17*a^2*c^2
*d^4*e^3*(b^2 - 4*a*c)^(1/2) + 6*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) + 16*
a^4*c*d^6*e*x + 8*a^3*c*d^6*e*(b^2 - 4*a*c)^(1/2) - 4*b*c^3*d*e^6*(b^2 - 4
*a*c)^(1/2) - 18*a*b*c^3*d^2*e^5 - 8*a*b^3*c*d^4*e^3 - 2*a*b^4*d^4*e^3*x -
 4*a^3*b^2*d^6*e*x + 3*a^2*b^2*d^6*e*(b^2 - 4*a*c)^(1/2) - 6*a*c^3*d^2*e^5
*(b^2 - 4*a*c)^(1/2) - 4*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) + 20*a*b^2*c^2*
d^3*e^4 + 17*a^2*b*c^2*d^4*e^3 - 2*a^2*b^2*c*d^5*e^2 + 8*a^2*b^3*d^5*e^2*x
 - 12*a^2*c^3*d^2*e^5*x + 34*a^3*c^2*d^4*e^3*x + 4*a*b*c^2*d^3*e^4*(b^2 -
4*a*c)^(1/2) - 18*a^2*b*c*d^5*e^2*(b^2 - 4*a*c)^(1/2) + 4*a*b^3*d^4*e^3*x*
(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*
d^2*e^5*x - 4*a^2*b*c^2*d^3*e^4*x - 8*a^2*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2
) - 4*a*b*c^3*d*e^6*x + 12*a^2*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a^3*
b*d^6*e*x*(b^2 - 4*a*c)^(1/2) - 4*a*c^3*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 32*a
^3*b*c*d^5*e^2*x + 6*a*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 8*a*b^2*c*...
```

3.71. $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

3.72 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

3.72.1 Optimal result

Integrand size = 23, antiderivative size = 246

$$\begin{aligned}\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx &= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} \\ &\quad+\frac{\left(b^3d^2-2b^2cde+4ac^2de-bc(3ad^2-ce^2)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} \\ &\quad+\frac{d^2(ad^2-e(2bd-3ce))\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(b^2d^2-2bcde-c(ad^2-ce^2)\right)\log\left(c+bx+ax^2\right)}{2a\left(ad^2-e(bd-ce)\right)^2}\end{aligned}$$

output
```
d^3/e^2/(a*d^2-e*(b*d-c*e))/(e*x+d)+d^2*(a*d^2-e*(2*b*d-3*c*e))*ln(e*x+d)/
e^2/(a*d^2-e*(b*d-c*e))^2+1/2*(b^2*d^2-2*b*c*d*e-c*(a*d^2-c*e^2))*ln(a*x^2
+b*x+c)/a/(a*d^2-e*(b*d-c*e))^2+(b^3*d^2-2*b^2*c*d*e+4*a*c^2*d*e-b*c*(3*a*
d^2-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a/(a*d^2-e*(b*d-c*e))^2/
(-4*a*c+b^2)^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.84

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \frac{\frac{2d^3(ad^2+e(-bd+ce))}{e^2(d+ex)} - \frac{2(b^3d^2-2b^2cde+4ac^2de+bc(-3ad^2+ce^2))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a\sqrt{-b^2+4ac}} + \frac{2(ad^4+d^2e(-2bd+3ce))\log(d+ex)}{e^2} + \frac{(b^2d^2-2b\ldots}{\ldots}}{2\left(ad^2+e(-bd+ce)\right)^2}$$

input
```
Integrate[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

output
```
((2*d^3*(a*d^2 + e*(-(b*d) + c*e)))/(e^2*(d + e*x)) - (2*(b^3*d^2 - 2*b^2*
c*d*e + 4*a*c^2*d*e + b*c*(-3*a*d^2 + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2
 + 4*a*c]])/(a*Sqrt[-b^2 + 4*a*c]) + (2*(a*d^4 + d^2*e*(-2*b*d + 3*c*e))*L
og[d + e*x])/e^2 + ((b^2*d^2 - 2*b*c*d*e + c*(-(a*d^2) + c*e^2))*Log[c + x
*(b + a*x)])/a)/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

3.72.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)^2\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \text{1893}$$

$$\int \frac{x^3}{(d+ex)^2(ax^2+bx+c)}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{x\left(-c(ad^2-ce^2)+b^2d^2-2bcde\right)+cd(bd-2ce)}{(ax^2+bx+c)(ad^2-e(bd-ce))^2} + \frac{d^2(ad^2-e(2bd-3ce))}{e(d+ex)(ad^2-e(bd-ce))^2} + \frac{d^3}{e(d+ex)^2(e(bd-ce)-\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(-bc(3ad^2-ce^2)+4ac^2de+b^3d^2-2b^2cde\right)}{a\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}+\frac{(-c(ad^2-ce^2)+b^2d^2-2bcde)\log\left(ax^2+bx+c\right)}{2a\,(ad^2-e(bd-ce))^2}+\frac{d^2\log(d+ex)\left(ad^2-e(2bd-3ce)\right)}{e^2\,(ad^2-e(bd-ce))^2}+\frac{d^3}{e^2(d+ex)\,(ad^2-e(bd-ce))}$$

input
```
Int[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

output
```
d^3/(e^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*d^2 - 2*b^2*c*d*e + 4*
a*c^2*d*e - b*c*(3*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])
/(a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d^2*(a*d^2 - e*(2*b*d
 - 3*c*e))*Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2 - 2*b*
c*d*e - c*(a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d - c*
e))^2)
```

3.72.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.72.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{\left(-d^2ac+b^2d^2-2bcde+e^2c^2\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(bc\,d^2-2c^2de-\frac{\left(-d^2ac+b^2d^2-2bcde+e^2c^2\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{\left(a\,d^2-bde+c\,e^2\right)^2}+\frac{d^2\left(a\,d^2-2bde+3c\,e^2\right.}{\left(a\,d^2-bde+c\,e\right.}$
risch	Expression too large to display

input
```
int(x/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(-a*c*d^2+b^2*d^2-2*b*c*d*e+c^2*e^2)/a*ln(a*x
^2+b*x+c)+2*(b*c*d^2-2*c^2*d*e-1/2*(-a*c*d^2+b^2*d^2-2*b*c*d*e+c^2*e^2)*b/
a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+d^2*(a*d^2-2*b*d
*e+3*c*e^2)/(a*d^2-b*d*e+c*e^2)^2/e^2*ln(e*x+d)+d^3/e^2/(a*d^2-b*d*e+c*e^2
)/(e*x+d)
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 723 vs. 2(240) = 480.

Time = 11.05 (sec) , antiderivative size = 1465, normalized size of antiderivative = 5.96

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx=\text{Too large to display}$$

input
```
integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2*c
 - 4*a^2*c^2)*d^3*e^2 + (b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2*c
- 2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2*a
*c^2)*d*e^4)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c +
 sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((b^4 - 5*a*b^2*c + 4
*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*
e^4 + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4
 + (b^2*c^2 - 4*a*c^3)*e^5)*x)*log(a*x^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*
c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 3*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + (
(a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + 3*(a*b^2*c - 4
*a^2*c^2)*d^2*e^3)*x)*log(e*x + d))/((a^3*b^2 - 4*a^4*c)*d^5*e^2 - 2*(a^2*
b^3 - 4*a^3*b*c)*d^4*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^3*e^4 - 2*(
a*b^3*c - 4*a^2*b*c^2)*d^2*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*d*e^6 + ((a^3*b^2
 - 4*a^4*c)*d^4*e^3 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^4 + (a*b^4 - 2*a^2*b^2
*c - 8*a^3*c^2)*d^2*e^5 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^6 + (a*b^2*c^2 - 4
*a^2*c^3)*e^7)*x), 1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*
d^4*e + 2*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + 2*(b*c^2*d*e^4 + (b^3 - 3*a*b*c)
*d^3*e^2 - 2*(b^2*c - 2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*
e^3 - 2*(b^2*c - 2*a*c^2)*d*e^4)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3...
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(x/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

output
```
Timed out
```

3.72.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.71

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$$

$$= \frac{\frac{2\,d^3e^2}{(ad^2e^3-bde^4+ce^5)(ex+d)} + \frac{(b^2d^2e-acd^2e-2\,bcde^2+c^2e^3)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{a^3d^4-2\,a^2bd^3e+ab^2d^2e^2+2\,a^2cd^2e^2-2\,abcde^3+ac^2e^4} - \frac{2\,\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{ae} + \cdots}{2\,e}$$

input
```
integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/2*(2*d^3*e^2/((a*d^2*e^3 - b*d*e^4 + c*e^5)*(e*x + d)) + (b^2*d^2*e - a*
c*d^2*e - 2*b*c*d*e^2 + c^2*e^3)*log(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d
)^2 - b*e/(e*x + d) + b*d*e/(e*x + d)^2 - c*e^2/(e*x + d)^2)/(a^3*d^4 - 2*
a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4)
 - 2*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/(a*e) + 2*(b^3*d^2*e^3 - 3*a*b
*c*d^2*e^3 - 2*b^2*c*d*e^4 + 4*a*c^2*d*e^4 + b*c^2*e^5)*arctan(-(2*a*d - 2
*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/(sqrt(-b^2
 + 4*a*c)*e))/((a^3*d^4 - 2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2
- 2*a*b*c*d*e^3 + a*c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2))/e
```

3.72.9 Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 2037, normalized size of antiderivative = 8.28

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(x/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

output
```
(log(d + e*x)*(a*d^4 + 3*c*d^2*e^2 - 2*b*d^3*e))/(c^2*e^6 + a^2*d^4*e^2 +
b^2*d^2*e^4 - 2*b*c*d*e^5 - 2*a*b*d^3*e^3 + 2*a*c*d^2*e^4) - (log(a^2*b^2*
d^6 - 4*a^3*c*d^6 - 2*c^4*e^6 - b^4*d^4*e^2 + c^3*e^6*x*(b^2 - 4*a*c)^(1/2
) + 24*a*c^3*d^2*e^4 + 6*b^3*c*d^3*e^3 + 2*b^4*d^3*e^3*x - b^3*d^4*e^2*(b^
2 - 4*a*c)^(1/2) - 10*a^2*c^2*d^4*e^2 - 9*b^2*c^2*d^2*e^4 - 2*a*b^3*d^5*e
+ 4*b*c^3*d*e^5 - b*c^3*e^6*x + a^2*b*d^6*(b^2 - 4*a*c)^(1/2) + 4*c^3*d*e^
5*(b^2 - 4*a*c)^(1/2) + 2*a^3*d^6*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d^5*e
+ 8*a*c^3*d*e^5*x - 8*a^3*c*d^5*e*x - 2*a*b^2*d^5*e*(b^2 - 4*a*c)^(1/2) -
4*a^2*c*d^5*e*(b^2 - 4*a*c)^(1/2) - 20*a*b*c^2*d^3*e^3 + 6*a*b^2*c*d^4*e^2
 - 6*a*b^3*d^4*e^2*x + 2*a^2*b^2*d^5*e*x - 3*b^3*c*d^2*e^4*x - 16*a*c^2*d^
3*e^3*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*
d^3*e^3*(b^2 - 4*a*c)^(1/2) - 2*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) - 32*a^2
*c^2*d^3*e^3*x + 4*a*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 12*a*c^2*d^2*e^4*
x*(b^2 - 4*a*c)^(1/2) + 5*a^2*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^
2*e^4*x*(b^2 - 4*a*c)^(1/2) + 14*a*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2
*b*d^5*e*x*(b^2 - 4*a*c)^(1/2) + 6*a*b*c^2*d^2*e^4*x + 2*a*b^2*c*d^3*e^3*x
 + 23*a^2*b*c*d^4*e^2*x + 2*a*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2
- 4*a*c^3*e^2 + b^3*d^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d^2 + b^2*c^2*e^2
- 2*b^3*c*d*e - 5*a*b^2*c*d^2 + b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^2*
d*e - 3*a*b*c*d^2*(b^2 - 4*a*c)^(1/2) + 4*a*c^2*d*e*(b^2 - 4*a*c)^(1/2)...
```

3.73 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

3.73.1 Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(b^2d^2-2bcde-2c\left(ad^2-ce^2\right)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} + \frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} - \frac{d(bd-2ce)\log\left(c+bx+ax^2\right)}{2\left(ad^2-e(bd-ce)\right)^2}$$

output
```
-d^2/e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+d*(b*d-2*c*e)*ln(e*x+d)/(a*d^2-e*(b*d-c
*e))^2-1/2*d*(b*d-2*c*e)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2-(b^2*d^2-2*
b*c*d*e-2*c*(a*d^2-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*
(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \frac{-\frac{2d^2\left(ad^2+e(-bd+ce)\right)}{e(d+ex)} + \frac{2\left(b^2d^2-2bcde+2c\left(-ad^2+ce^2\right)\right)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + 2d(bd-2ce)\log(d+ex) - d(bd-2ce)\operatorname{lo}}{2\left(ad^2+e(-bd+ce)\right)^2}$$

input
```
Integrate[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

output
```
((-2*d^2*(a*d^2 + e*(-(b*d) + c*e)))/(e*(d + e*x)) + (2*(b^2*d^2 - 2*b*c*d
*e + 2*c*(-(a*d^2) + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] + 2*d*(b*d - 2*c*e)*Log[d + e*x] - d*(b*d - 2*c*e)*Log[c + x
*(b + a*x)])/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

3.73.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1775, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \text{ 1775}$$

$$\int \frac{x^2}{(d+ex)^2\left(ax^2+bx+c\right)}dx$$

$$\Big\downarrow \text{ 1200}$$

$$\int \left(\frac{-adx(bd-2ce)-c\left(ad^2-ce^2\right)}{\left(ax^2+bx+c\right)\left(ad^2-e(bd-ce)\right)^2}+\frac{d^2}{(d+ex)^2\left(ad^2-e(bd-ce)\right)}+\frac{de(bd-2ce)}{(d+ex)\left(ad^2-e(bd-ce)\right)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(-2c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{d^2}{e(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{d(bd-2ce)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}$$

input
```
Int[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

output
```
-(d^2/(e*(a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((b^2*d^2 - 2*b*c*d*e - 2*c
*(a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*
c]*(a*d^2 - e*(b*d - c*e))^2) + (d*(b*d - 2*c*e)*Log[d + e*x])/(a*d^2 - e*
(b*d - c*e))^2 - (d*(b*d - 2*c*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d
 - c*e))^2)
```

3.73.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1775
```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.73.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.97

method	result
default	$\frac{\frac{\left(-ab\,d^{2}+2acde\right)\ln\left(a\,x^{2}+bx+c\right)}{2a}+\frac{2\left(-d^{2}ac+e^{2}c^{2}-\frac{\left(-ab\,d^{2}+2acde\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{\left(a\,d^{2}-bde+c\,e^{2}\right)^{2}}-\frac{d^{2}}{e\left(a\,d^{2}-bde+c\,e^{2}\right)\left(ex+d\right)}+\frac{d(bd-2ec)\ln}{(a\,d^{2}-bde+}$
risch	Expression too large to display

input
```
int(1/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(-a*b*d^2+2*a*c*d*e)/a*ln(a*x^2+b*x+c)+2*(-d^
2*a*c+e^2*c^2-1/2*(-a*b*d^2+2*a*c*d*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2)))-d^2/e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+d*(b*d-2*c*e)/(
a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(188) = 376$.

Time = 3.64 (sec) , antiderivative size = 1120, normalized size of antiderivative = 5.77

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*
c^2)*d^2*e^2 + (2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c
*d*e^3 - 2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*sqrt(b^2 - 4*a*c)*log((2*a^
2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*
x + c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4
*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(a*x^2 + b*x + c) - 2*(
(b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2
*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^
5*e - 2*(a*b^3 - 4*a^2*b*c)*d^4*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^
3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 4*a*c^3)*d*e^5 + ((a^2*b^2
- 4*a^3*c)*d^4*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^3 + (b^4 - 2*a*b^2*c - 8*
a^2*c^2)*d^2*e^4 - 2*(b^3*c - 4*a*b*c^2)*d*e^5 + (b^2*c^2 - 4*a*c^3)*e^6)*
x), -1/2*(2*(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4
*a*c^2)*d^2*e^2 - 2*(2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (
2*b*c*d*e^3 - 2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*sqrt(-b^2 + 4*a*c)*arc
tan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d^3*
e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*
a*c^2)*d*e^3)*x)*log(a*x^2 + b*x + c) - 2*((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*
c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^
3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5*e - 2*(a*b^3 - 4*a^2*b*c)*...
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

output
```
Timed out
```

3.73.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.75

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx &= -\frac{d^2e}{(ad^2e^2-bde^3+ce^4)(ex+d)} \\ &\quad - \frac{(bd^2-2\,cde)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{2\,(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4)} \\ &\quad - \frac{(b^2d^2e^2-2\,acd^2e^2-2\,bcde^3+2\,c^2e^4)\arctan\left(-\frac{2\,ad-\frac{2\,ad^2}{ex+d}-be+\frac{2\,bde}{ex+d}-\frac{2\,ce^2}{ex+d}}{\sqrt{-b^2+4\,ac}e}\right)}{(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4)\sqrt{-b^2+4\,ac}e^2}\end{aligned}$$

input
```
integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-d^2*e/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(e*x + d)) - 1/2*(b*d^2 - 2*c*d*e)*l
og(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/(e*x +
 d)^2 - c*e^2/(e*x + d)^2)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^
2*e^2 - 2*b*c*d*e^3 + c^2*e^4) - (b^2*d^2*e^2 - 2*a*c*d^2*e^2 - 2*b*c*d*e^
3 + 2*c^2*e^4)*arctan(-(2*a*d - 2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d
) - 2*c*e^2/(e*x + d))/(sqrt(-b^2 + 4*a*c)*e))/((a^2*d^4 - 2*a*b*d^3*e + b
^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2
)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 11.25 (sec) , antiderivative size = 1585, normalized size of antiderivative = 8.17

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

output
```
(log(2*a*b^3*d^4 + b*c^3*e^4 - c^3*e^4*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d^
3*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2*e^2 + a^2*b^2*d^4*x + b^2*c^2*e^4*x - b^
4*d^2*e^2*x - 7*a^2*b*c*d^4 - 16*a*c^3*d*e^3 - 2*a^3*c*d^4*x - 2*a*c^3*e^4
*x + 2*a*b^2*d^4*(b^2 - 4*a*c)^(1/2) - a^2*c*d^4*(b^2 - 4*a*c)^(1/2) - 6*a
*b^2*c*d^3*e + 2*a*b^3*d^3*e*x + 2*b^3*c*d*e^3*x - 2*b*c^2*d*e^3*(b^2 - 4*
a*c)^(1/2) + 3*a^2*b*d^4*x*(b^2 - 4*a*c)^(1/2) - b*c^2*e^4*x*(b^2 - 4*a*c)
^(1/2) + 10*a*b*c^2*d^2*e^2 + 14*a*c^2*d^2*e^2*(b^2 - 4*a*c)^(1/2) + b^2*c
*d^2*e^2*(b^2 - 4*a*c)^(1/2) + b^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + 28*a^2*
c^2*d^2*e^2*x - 10*a*b*c*d^3*e*(b^2 - 4*a*c)^(1/2) - 12*a*b*c^2*d*e^3*x -
12*a^2*b*c*d^3*e*x - 2*a*b^2*d^3*e*x*(b^2 - 4*a*c)^(1/2) + 8*a*c^2*d*e^3*x
*(b^2 - 4*a*c)^(1/2) - 8*a^2*c*d^3*e*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d*e^3
*x*(b^2 - 4*a*c)^(1/2) + 2*a*b*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2))*(d^2*(b^3/
2 + (b^2*(b^2 - 4*a*c)^(1/2))/2) - c*(d^2*(2*a*b + a*(b^2 - 4*a*c)^(1/2))
+ d*(b^2*e + b*e*(b^2 - 4*a*c)^(1/2))) + c^2*(e^2*(b^2 - 4*a*c)^(1/2) + 4*
a*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*
e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3
- 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) - (log(2*a*b^3*d^4 + b*c^3*e^4 + c^
3*e^4*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d^3*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2
*e^2 + a^2*b^2*d^4*x + b^2*c^2*e^4*x - b^4*d^2*e^2*x - 7*a^2*b*c*d^4 - 16*
a*c^3*d*e^3 - 2*a^3*c*d^4*x - 2*a*c^3*e^4*x - 2*a*b^2*d^4*(b^2 - 4*a*c)...
```

3.74 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2} dx$

3.74.1 Optimal result

Integrand size = 25, antiderivative size = 183

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2} dx = \frac{d}{(ad^2-bde+ce^2)(d+ex)} + \frac{(bce^2+ad(bd-4ce))\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2} - \frac{(ad^2-ce^2)\log(d+ex)}{(ad^2-e(bd-ce))^2} + \frac{(ad^2-ce^2)\log(c+bx+ax^2)}{2(ad^2-e(bd-ce))^2}$$

output
```
d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-(a*d^2-c*e^2)*ln(e*x+d)/(a*d^2-e*(b*d-c*e))^
2+1/2*(a*d^2-c*e^2)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2+(b*c*e^2+a*d*(b*
d-4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))^2/(-4*
a*c+b^2)^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2} dx = \frac{\frac{2d(ad^2+e(-bd+ce))}{d+ex} - \frac{2(bce^2+ad(bd-4ce))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (-2ad^2+2ce^2)\log(d+ex) + (ad^2-ce^2)\log(c+x(\ldots}{2(ad^2+e(-bd+ce))^2}$$

input
```
Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

output
```
((2*d*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) - (2*(b*c*e^2 + a*d*(b*d - 4*c
*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-2*a*d^
2 + 2*c*e^2)*Log[d + e*x] + (a*d^2 - c*e^2)*Log[c + x*(b + a*x)])/(2*(a*d^
2 + e*(-(b*d) + c*e))^2)
```

3.74.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)^2 \left(a+\frac{b}{x}+\frac{c}{x^2}\right)} dx$$

$$\Big\downarrow \text{1893}$$

$$\int \frac{x}{(d+ex)^2 \left(ax^2+bx+c\right)}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{ce(2ad-be)+ax\left(ad^2-ce^2\right)}{\left(ax^2+bx+c\right)\left(ad^2-e(bd-ce)\right)^2}+\frac{e\left(ce^2-ad^2\right)}{(d+ex)\left(ad^2-e(bd-ce)\right)^2}+\frac{de}{(d+ex)^2\left(e(bd-ce)-ad^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(ad(bd-4ce)+bce^2\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(ad^2-ce^2\right)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}$$

input
```
Int[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

output
```
d/((a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b*c*e^2 + a*d*(b*d - 4*c*e))*Arc
Tanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d -
c*e))^2) - ((a*d^2 - c*e^2)*Log[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 + ((a*
d^2 - c*e^2)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)
```

3.74.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.74.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02

method	result
default	$\frac{\frac{\left(a^{2} d^{2}-e^{2} a c\right) \ln\left(a\,x^{2}+b x+c\right)}{2 a}+\frac{2\left(2 a c d e-b c\,e^{2}-\frac{\left(a^{2} d^{2}-e^{2} a c\right) b}{2 a}\right) \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\sqrt{4 a c-b^{2}}}}{\left(a\,d^{2}-b d e+c\,e^{2}\right)^{2}}+\frac{d}{\left(a\,d^{2}-b d e+c\,e^{2}\right)\left(e x+d\right)}-\frac{\left(a\,d^{2}-c\,e^{2}\right) \ln(ex+}{\left(a\,d^{2}-b d e+c\,e^{2}\right)^{2}}$
risch	$\frac{d}{\left(a\,d^{2}-b d e+c\,e^{2}\right)\left(e x+d\right)}-\frac{\ln\left(e x+d\right) a\,d^{2}}{a^{2} d^{4}-2 a b\,d^{3} e+2 a c\,d^{2} e^{2}+b^{2} d^{2} e^{2}-2 b c d\,e^{3}+c^{2} e^{4}}+\frac{\ln\left(e x+d\right) c\,e^{2}}{a^{2} d^{4}-2 a b\,d^{3} e+2 a c\,d^{2} e^{2}+b^{2} d^{2} e^{2}-2 b c d\,e^{3}+c^{2} e^{4}}+$

input
```
int(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(a^2*d^2-a*c*e^2)/a*ln(a*x^2+b*x+c)+2*(2*a*c*
d*e-b*c*e^2-1/2*(a^2*d^2-a*c*e^2)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/
(4*a*c-b^2)^(1/2)))+d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-(a*d^2-c*e^2)/(a*d^2-b*d
*e+c*e^2)^2*ln(e*x+d)
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(177) = 354$.

Time = 3.32 (sec) , antiderivative size = 1059, normalized size of antiderivative = 5.79

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2*c - 4*a*c
^2)*d*e^2 + (a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4*a*c*d*e^2
+ b*c*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + s
qrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((a*b^2 - 4*a^2*c)*d^3
- (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e
^3)*x)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)
*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d)
)/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*
c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^
3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + (b
^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c
^2 - 4*a*c^3)*e^5)*x), 1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^
2*e + 2*(b^2*c - 4*a*c^2)*d*e^2 + 2*(a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (
a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4
*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(a
*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a
*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d))/((a^2*b^2
- 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^
2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 ...
```

3.74.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/x/(e*x+d)**2,x)
```

output
```
Timed out
```

3.74.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx$$
$$=\frac{1}{2}e\left(\frac{(ad^2-ce^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{a^2d^4e-2\,abd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,bcde^4+c^2e^5}+\frac{2\,de}{(ad^2e^2-bde^3+ce^4)(ex+d)}+\right.$$

input
```
integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/2*e*((a*d^2 - c*e^2)*log(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e/
(e*x + d) + b*d*e/(e*x + d)^2 - c*e^2/(e*x + d)^2)/(a^2*d^4*e - 2*a*b*d^3*
e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*b*c*d*e^4 + c^2*e^5) + 2*d*e/((a*d^2
*e^2 - b*d*e^3 + c*e^4)*(e*x + d)) + 2*(a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)
*arctan(-(2*a*d - 2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e
*x + d))/(sqrt(-b^2 + 4*a*c)*e))/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2
*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2))
```

3.74.9 Mupad [B] (verification not implemented)

Time = 12.95 (sec) , antiderivative size = 1768, normalized size of antiderivative = 9.66

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

output
```
d/((d + e*x)*(a*d^2 + c*e^2 - b*d*e)) - (log(56*a^3*b^2*c*d^4 - 96*a^4*c^2
*d^4 - 96*a^2*c^4*e^4 - 8*b^4*c^2*e^4 - 8*a^2*b^4*d^4 + 56*a*b^2*c^3*e^4 -
 4*a^3*b^3*d^4*x + 320*a^3*c^3*d^2*e^2 + 8*a*d^3*e*(b^2 - 4*a*c)^(5/2) - 8
*c*d*e^3*(b^2 - 4*a*c)^(5/2) - 3*c*e^4*x*(b^2 - 4*a*c)^(5/2) - 8*b^5*c*e^4
*x + 8*a^2*b*d^4*(b^2 - 4*a*c)^(3/2) - 8*b*c^2*e^4*(b^2 - 4*a*c)^(3/2) + 1
2*a^3*d^4*x*(b^2 - 4*a*c)^(3/2) - 6*b*d*e^3*x*(b^2 - 4*a*c)^(5/2) + 16*a^4
*b*c*d^4*x - 112*a^2*b^2*c^2*d^2*e^2 - 8*a*b^2*d^3*e*(b^2 - 4*a*c)^(3/2) +
 8*b^2*c*d*e^3*(b^2 - 4*a*c)^(3/2) + 10*a*d^2*e^2*x*(b^2 - 4*a*c)^(5/2) -
5*b^2*c*e^4*x*(b^2 - 4*a*c)^(3/2) + 6*b^3*d*e^3*x*(b^2 - 4*a*c)^(3/2) + 16
*a*b^3*c^2*d*e^3 + 8*a*b^4*c*d^2*e^2 - 64*a^2*b*c^3*d*e^3 + 16*a^2*b^3*c*d
^3*e - 64*a^3*b*c^2*d^3*e + 60*a*b^3*c^2*e^4*x - 112*a^2*b*c^3*e^4*x + 4*a
*b^5*d^2*e^2*x - 8*a^2*b^4*d^3*e*x + 256*a^3*c^3*d*e^3*x - 256*a^4*c^2*d^3
*e*x - 6*a*b^2*d^2*e^2*x*(b^2 - 4*a*c)^(3/2) - 160*a^2*b^2*c^2*d*e^3*x - 5
6*a^2*b^3*c*d^2*e^2*x + 160*a^3*b*c^2*d^2*e^2*x + 24*a*b^4*c*d*e^3*x - 8*a
^2*b*d^3*e*x*(b^2 - 4*a*c)^(3/2) + 96*a^3*b^2*c*d^3*e*x)*(b^2*((a*d^2)/2 -
 (c*e^2)/2) - b*((a*d^2*(b^2 - 4*a*c)^(1/2))/2 + (c*e^2*(b^2 - 4*a*c)^(1/2
))/2) - 2*a^2*c*d^2 + 2*a*c^2*e^2 + 2*a*c*d*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3
*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2
*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3
*e + 2*a*b^2*c*d^2*e^2) - (log(8*a^2*b^4*d^4 + 96*a^4*c^2*d^4 + 96*a^2*...
```

3.75 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx$

3.75.1 Optimal result

Integrand size = 25, antiderivative size = 189

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = -\frac{e}{(ad^2-bde+ce^2)(d+ex)} - \frac{(2a^2d^2+b^2e^2-2ae(bd+ce))\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2} + \frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2} - \frac{e(2ad-be)\log(c+bx+ax^2)}{2(ad^2-e(bd-ce))^2}$$

output
```
-e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+e*(2*a*d-b*e)*ln(e*x+d)/(a*d^2-e*(b*d-c*e))
^2-1/2*e*(2*a*d-b*e)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2-(2*a^2*d^2+b^2*
e^2-2*a*e*(b*d+c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c
*e))^2/(-4*a*c+b^2)^(1/2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \frac{-\frac{2e(ad^2+e(-bd+ce))}{d+ex}+\frac{2(2a^2d^2+b^2e^2-2ae(bd+ce))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2e(-2ad+be)\log(d+ex)+e(-2ad+be)\,\mathrm{lo}}{2\left(ad^2+e(-bd+ce)\right)^2}$$

input
```
Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

output
```
((-2*e*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) + (2*(2*a^2*d^2 + b^2*e^2 - 2
*a*e*(b*d + c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c] - 2*e*(-2*a*d + b*e)*Log[d + e*x] + e*(-2*a*d + b*e)*Log[c + x*(b + a*x
)])/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

3.75.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1893, 1145, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)^2\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \text{1893}$$

$$\int \frac{1}{(d+ex)^2(ax^2+bx+c)}dx$$

$$\Big\downarrow \text{1145}$$

$$\frac{\int \frac{ad-be-aex}{(d+ex)(ax^2+bx+c)}dx}{ad^2-bde+ce^2}-\frac{e}{(d+ex)(ad^2-bde+ce^2)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\int \left(\frac{(2ad-be)e^2}{(ad^2-e(bd-ce))(d+ex)}+\frac{a^2d^2+b^2e^2-ae(2bd+ce)-ae(2ad-be)x}{(ad^2-e(bd-ce))(ax^2+bx+c)}\right)dx}{ad^2-bde+ce^2}-\frac{e}{(d+ex)(ad^2-bde+ce^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e(2ad-be)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)}+\frac{e(2ad-be)\log(d+ex)}{ad^2-e(bd-ce)}}{ad^2-bde+ce^2}-\frac{e}{(d+ex)\left(ad^2-bde+ce^2\right)}$$

input
```
Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

output
```
-(e/((a*d^2 - b*d*e + c*e^2)*(d + e*x))) + (-(((2*a^2*d^2 + b^2*e^2 - 2*a*
e*(b*d + c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*
(a*d^2 - e*(b*d - c*e)))) + (e*(2*a*d - b*e)*Log[d + e*x])/(a*d^2 - e*(b*d
 - c*e)) - (e*(2*a*d - b*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e
))))/(a*d^2 - b*d*e + c*e^2)
```

3.75.3.1 Defintions of rubi rules used

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.75.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.04

method	result
default	$\frac{\frac{\left(-2a^2de+ab\,e^2\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(a^2d^2-2abde-e^2ac+b^2e^2-\frac{\left(-2a^2de+ab\,e^2\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{\left(a\,d^2-bde+c\,e^2\right)^2}-\frac{e}{\left(a\,d^2-bde+c\,e^2\right)(ex+d)}+\frac{e(2}{(}$
risch	$-\frac{e}{\left(a\,d^2-bde+c\,e^2\right)(ex+d)}+\frac{2e\ln(ex+d)da}{a^2d^4-2ab\,d^3e+2ac\,d^2e^2+b^2d^2e^2-2bcd\,e^3+c^2e^4}-\frac{e^2\ln(ex+d)b}{a^2d^4-2ab\,d^3e+2ac\,d^2e^2+b^2d^2e^2-2bcd\,e^3+c^2e^4}$

input
```
int(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(-2*a^2*d*e+a*b*e^2)/a*ln(a*x^2+b*x+c)+2*(a^2
*d^2-2*a*b*d*e-e^2*a*c+b^2*e^2-1/2*(-2*a^2*d*e+a*b*e^2)*b/a)/(4*a*c-b^2)^(
1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+e*
(2*a*d-b*e)/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(183) = 366.

Time = 1.86 (sec) , antiderivative size = 1079, normalized size of antiderivative = 5.71

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e - 2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*
a*c^2)*e^3 + (2*a^2*d^3 - 2*a*b*d^2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e
 - 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 +
2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c))
+ (2*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c
)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a
^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 -
4*a*b*c)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2
*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2
)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*
b^3 - 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*
c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x), -1/2*(2*(a*b^2 - 4*a^2
*c)*d^2*e - 2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*a*c^2)*e^3 + 2*(2*a^2*d
^3 - 2*a*b*d^2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e - 2*a*b*d*e^2 + (b^2
 - 2*a*c)*e^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b
)/(b^2 - 4*a*c)) + (2*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2
*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(a*x^2 + b*x + c) -
2*(2*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c
)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 -
 2*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - ...
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d)**2,x)
```

output
```
Timed out
```

3.75.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.78

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx$$
$$= -\frac{e^3}{(ad^2e^2-bde^3+ce^4)(ex+d)} - \frac{(2\,ade-be^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{2\,(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4)} - \frac{(2\,a^2d^2e^2-2\,abde^3+b^2e^4-2\,ace^4)\arctan\left(-\frac{2\,ad-\frac{2\,ad^2}{ex+d}-be+\frac{2\,bde}{ex+d}-\frac{2\,ce^2}{ex+d}}{\sqrt{-b^2+4\,ace}}\right)}{(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4)\sqrt{-b^2+4\,ace^2}}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
-e^3/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(e*x + d)) - 1/2*(2*a*d*e - b*e^2)*log
(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/(e*x + d
)^2 - c*e^2/(e*x + d)^2)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*
e^2 - 2*b*c*d*e^3 + c^2*e^4) - (2*a^2*d^2*e^2 - 2*a*b*d*e^3 + b^2*e^4 - 2*
a*c*e^4)*arctan(-(2*a*d - 2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*
c*e^2/(e*x + d))/(sqrt(-b^2 + 4*a*c)*e))/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2
*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 12.75 (sec) , antiderivative size = 1782, normalized size of antiderivative = 9.43

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

output
```
(log(c*e^4*(b^2 - 4*a*c)^(5/2) - 8*b^5*c*e^4 - 8*b^6*e^4*x - 4*a^3*d^4*(b^
2 - 4*a*c)^(3/2) - 4*a^3*b^3*d^4 + 4*b^3*e^4*x*(b^2 - 4*a*c)^(3/2) + 60*a*
b^3*c^2*e^4 - 112*a^2*b*c^3*e^4 + 4*a*b^5*d^2*e^2 - 8*a^2*b^4*d^3*e + 256*
a^3*c^3*d*e^3 - 256*a^4*c^2*d^3*e - 8*a^4*b^2*d^4*x + 32*a^3*c^3*e^4*x + 1
0*b*d*e^3*(b^2 - 4*a*c)^(5/2) + 4*b*e^4*x*(b^2 - 4*a*c)^(5/2) + 16*a^4*b*c
*d^4 + 32*a^5*c*d^4*x - 14*a*d^2*e^2*(b^2 - 4*a*c)^(5/2) + 7*b^2*c*e^4*(b^
2 - 4*a*c)^(3/2) - 10*b^3*d*e^3*(b^2 - 4*a*c)^(3/2) - 8*a*d*e^3*x*(b^2 - 4
*a*c)^(5/2) + 24*a*b^4*c*d*e^3 + 64*a*b^4*c*e^4*x + 32*a*b^5*d*e^3*x - 8*a
^2*b*d^3*e*(b^2 - 4*a*c)^(3/2) - 32*a^3*d^3*e*x*(b^2 - 4*a*c)^(3/2) + 96*a
^3*b^2*c*d^3*e + 16*a^3*b^3*d^3*e*x + 18*a*b^2*d^2*e^2*(b^2 - 4*a*c)^(3/2)
 - 160*a^2*b^2*c^2*d*e^3 - 56*a^2*b^3*c*d^2*e^2 + 160*a^3*b*c^2*d^2*e^2 -
136*a^2*b^2*c^2*e^4*x - 40*a^2*b^4*d^2*e^2*x - 448*a^4*c^2*d^2*e^2*x + 48*
a^2*b*d^2*e^2*x*(b^2 - 4*a*c)^(3/2) + 272*a^3*b^2*c*d^2*e^2*x - 64*a^4*b*c
*d^3*e*x - 24*a*b^2*d*e^3*x*(b^2 - 4*a*c)^(3/2) - 240*a^2*b^3*c*d*e^3*x +
448*a^3*b*c^2*d*e^3*x)*(a*(e^2*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) + e*(b^2*d
- b*d*(b^2 - 4*a*c)^(1/2))) - e^2*(b^3/2 - (b^2*(b^2 - 4*a*c)^(1/2))/2) +
a^2*(d^2*(b^2 - 4*a*c)^(1/2) - 4*c*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2
*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e +
 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) -
(log(d + e*x)*(b*e^2 - 2*a*d*e))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a...
```

3.76 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx$

3.76.1 Optimal result

Integrand size = 25, antiderivative size = 248

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \frac{e^2}{d\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{\left(b^3e^2-abe(2bd+3ce)+a^2d(bd+4ce)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} + \frac{\log(x)}{cd^2} - \frac{e^2\left(3ad^2-e(2bd-ce)\right)\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2} - \frac{\left(a^2d^2+b^2e^2-ae(2bd+ce)\right)\log\left(c+bx+ax^2\right)}{2c\left(ad^2-e(bd-ce)\right)^2}$$

output
```
e^2/d/(a*d^2-b*d*e+c*e^2)/(e*x+d)+ln(x)/c/d^2-e^2*(3*a*d^2-e*(2*b*d-c*e))*
ln(e*x+d)/d^2/(a*d^2-e*(b*d-c*e))^2-1/2*(a^2*d^2+b^2*e^2-a*e*(2*b*d+c*e))*
ln(a*x^2+b*x+c)/c/(a*d^2-e*(b*d-c*e))^2+(b^3*e^2-a*b*e*(2*b*d+3*c*e)+a^2*d
*(b*d+4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c/(a*d^2-e*(b*d-c*e))^
2/(-4*a*c+b^2)^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.99

$$
\begin{aligned}
&\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right) x^3 (d+ex)^2}\,dx \\
&= \frac{e^2}{d\left(ad^2+e(-bd+ce)\right)(d+ex)} \\
&\quad - \frac{\left(b^3e^2-abe(2bd+3ce)+a^2d(bd+4ce)\right)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c\sqrt{-b^2+4ac}\left(ad^2+e(-bd+ce)\right)^2} \\
&\quad + \frac{\log(x)}{cd^2} - \frac{e^2\left(3ad^2+e(-2bd+ce)\right)\log(d+ex)}{\left(ad^3+de(-bd+ce)\right)^2} \\
&\quad + \frac{\left(-a^2d^2-b^2e^2+ae(2bd+ce)\right)\log(c+x(b+ax))}{2c\left(ad^2+e(-bd+ce)\right)^2}
\end{aligned}
$$

input
```
Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

output
```
e^2/(d*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) - ((b^3*e^2 - a*b*e*(2*b*d +
3*c*e) + a^2*d*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c*S
qrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) + Log[x]/(c*d^2) - (e^2*(3
*a*d^2 + e*(-2*b*d + c*e))*Log[d + e*x])/(a*d^3 + d*e*(-(b*d) + c*e))^2 +
((-(a^2*d^2) - b^2*e^2 + a*e*(2*b*d + c*e))*Log[c + x*(b + a*x)])/(2*c*(a*
d^2 + e*(-(b*d) + c*e))^2)
```

3.76.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$
\int \frac{1}{x^3(d+ex)^2\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx
$$

$$
\Big\downarrow \; 1893
$$

$$
\int \frac{1}{x(d+ex)^2\left(ax^2+bx+c\right)}dx
$$

$$\Big\downarrow \ 1200$$

$$\int \left(\frac{-ax\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)-\left((ad-be)\left(abd+2ace+b^2(-e)\right)\right)}{c\left(ax^2+bx+c\right)\left(ad^2-e(bd-ce)\right)^2}+\frac{e^3\left(e(2bd-ce)-3ad^2\right)}{d^2(d+ex)\left(ad^2-e(bd-ce)\right)^2}+\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)^2}+\frac{e^2}{d(d+ex)\left(ad^2-e(bd-ce)\right)}-\frac{e^2\log(d+ex)\left(3ad^2-e(2bd-ce)\right)}{d^2\left(ad^2-e(bd-ce)\right)^2}+\frac{\log(x)}{cd^2}$$

input
```
Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

output
```
e^2/(d*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*e^2 - a*b*e*(2*b*d + 3*c
*e) + a^2*d*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt
[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*d^2
- e*(2*b*d - c*e))*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e))^2) - ((a^2*d
^2 + b^2*e^2 - a*e*(2*b*d + c*e))*Log[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b
*d - c*e))^2)
```

3.76.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.76.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.13

method	result
default	$\frac{\ln(x)}{c\,d^2}+\frac{\frac{\left(-a^3d^2+2a^2bde+a^2c\,e^2-a\,b^2e^2\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(-a^2b\,d^2-2ecd\,a^2+2a\,b^2de+2ab\,e^2c-b^3e^2-\frac{\left(-a^3d^2+2a^2bde+a^2c\,e^2-a\,b^2e^2\right)b}{2a}\cdots\right.}{\sqrt{4ac-b^2}}}{\left(a\,d^2-bde+c\,e^2\right)^2c}$
risch	Expression too large to display

input
```
int(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/c/d^2+1/(a*d^2-b*d*e+c*e^2)^2/c*(1/2*(-a^3*d^2+2*a^2*b*d*e+a^2*c*e^2
-a*b^2*e^2)/a*ln(a*x^2+b*x+c)+2*(-a^2*b*d^2-2*e*c*d*a^2+2*a*b^2*d*e+2*a*b*
e^2*c-b^3*e^2-1/2*(-a^3*d^2+2*a^2*b*d*e+a^2*c*e^2-a*b^2*e^2)*b/a)/(4*a*c-b
^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+e^2/d/(a*d^2-b*d*e+c*e^2)/(
e*x+d)-e^2*(3*a*d^2-2*b*d*e+c*e^2)/d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)
```

3.76.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d)**2,x)
```

output
```
Timed out
```

3.76.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.62

$$\begin{aligned}
&\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx \\
&= \frac{e^5}{(ad^3e^3-bd^2e^4+cde^5)(ex+d)} \\
&\quad - \frac{(a^2d^2-2\,abde+b^2e^2-ace^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{2\,(a^2cd^4-2\,abcd^3e+b^2cd^2e^2+2\,ac^2d^2e^2-2\,bc^2de^3+c^3e^4)} \\
&\quad + \frac{(a^2bd^2e^2-2\,ab^2de^3+4\,a^2cde^3+b^3e^4-3\,abce^4)\arctan\left(-\frac{2\,ad-\frac{2\,ad^2}{ex+d}-be+\frac{2\,bde}{ex+d}-\frac{2\,ce^2}{ex+d}}{\sqrt{-b^2+4\,ace}}\right)}{(a^2cd^4-2\,abcd^3e+b^2cd^2e^2+2\,ac^2d^2e^2-2\,bc^2de^3+c^3e^4)\sqrt{-b^2+4\,ace}^2} \\
&\quad + \frac{\log\left(\left|-\frac{d}{ex+d}+1\right|\right)}{cd^2}
\end{aligned}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="giac")
```

output
```
e^5/((a*d^3*e^3 - b*d^2*e^4 + c*d*e^5)*(e*x + d)) - 1/2*(a^2*d^2 - 2*a*b*d
*e + b^2*e^2 - a*c*e^2)*log(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e
/(e*x + d) + b*d*e/(e*x + d)^2 - c*e^2/(e*x + d)^2)/(a^2*c*d^4 - 2*a*b*c*d
^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 + c^3*e^4) + (a^2*b
*d^2*e^2 - 2*a*b^2*d*e^3 + 4*a^2*c*d*e^3 + b^3*e^4 - 3*a*b*c*e^4)*arctan(-
(2*a*d - 2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/
(sqrt(-b^2 + 4*a*c)*e))/((a^2*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d^2*e^2 + 2*a*
c^2*d^2*e^2 - 2*b*c^2*d*e^3 + c^3*e^4)*sqrt(-b^2 + 4*a*c)*e^2) + log(abs(-
d/(e*x + d) + 1))/(c*d^2)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 27.77 (sec) , antiderivative size = 3510, normalized size of antiderivative = 14.15

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

output
```
(log((a^4*e^4)/(d*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^5*x)/(d^2*(a*d^2 + c
*e^2 - b*d*e)^2) - (((a*e^3*(3*a^3*b*d^4 + b^3*c*e^4 - b^4*d*e^3 + 5*a*b^3
*d^2*e^2 - 7*a^2*b^2*d^3*e + 8*a^2*c^2*d*e^3 - 3*a*b*c^2*e^4 + 9*a^3*c*d^3
*e - a*b^2*c*d*e^3 - 8*a^2*b*c*d^2*e^2))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) +
 (((a*e*(a^3*b*d^5 - 4*a*c^3*e^5 + b^2*c^2*e^5 - b^4*d^2*e^3 + 3*a*b^3*d^3
*e^2 - 3*a^2*b^2*d^4*e - 8*a^2*c^2*d^2*e^3 + 4*a^3*c*d^4*e - b^3*c*d*e^4 +
 4*a*b*c^2*d*e^4 + 6*a*b^2*c*d^2*e^3 - 9*a^2*b*c*d^3*e^2))/(a*d^3 - b*d^2*
e + c*d*e^2) + (a*e*x*(3*a^4*d^5 + 2*b^3*c*e^5 - 4*b^4*d*e^4 + 9*a*b^3*d^2
*e^3 + 4*a^2*c^2*d*e^4 + 19*a^3*c*d^3*e^2 - 3*a^2*b^2*d^3*e^2 - 8*a*b*c^2*
e^5 - 5*a^3*b*d^4*e + 15*a*b^2*c*d*e^4 - 36*a^2*b*c*d^2*e^3))/(a*d^3 - b*d
^2*e + c*d*e^2) - (a*e*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2 - 4*a*c)^(1/2
) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 + a^2*b*d^2*
(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*
a*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^
2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*
x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^
3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*
d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6
*a*b^2*c*d^2*e^2*x))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e
^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^...
```

3.77 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx$

3.77.1 Optimal result

Integrand size = 25, antiderivative size = 291

$$\begin{aligned}
&\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx \\
&= -\frac{1}{cd^2x}-\frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} \\
&\quad+\frac{\left(2a^3cd^2-b^4e^2+2ab^2e(bd+2ce)-a^2\left(b^2d^2+6bcde+2c^2e^2\right)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} \\
&\quad-\frac{(bd+2ce)\log(x)}{c^2d^3}+\frac{e^3\left(4ad^2-e(3bd-2ce)\right)\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)^2} \\
&\quad+\frac{(ad-be)\left(abd-b^2e+2ace\right)\log\left(c+bx+ax^2\right)}{2c^2\left(ad^2-e(bd-ce)\right)^2}
\end{aligned}$$

output
```
-1/c/d^2/x-e^3/d^2/(a*d^2-e*(b*d-c*e))/(e*x+d)-(b*d+2*c*e)*ln(x)/c^2/d^3+e
^3*(4*a*d^2-e*(3*b*d-2*c*e))*ln(e*x+d)/d^3/(a*d^2-e*(b*d-c*e))^2+1/2*(a*d-
b*e)*(a*b*d+2*a*c*e-b^2*e)*ln(a*x^2+b*x+c)/c^2/(a*d^2-e*(b*d-c*e))^2+(2*a^
3*c*d^2-b^4*e^2+2*a*b^2*e*(b*d+2*c*e)-a^2*(b^2*d^2+6*b*c*d*e+2*c^2*e^2))*a
rctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2
)^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx$$
$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\left(ad^2+e(-bd+ce)\right)(d+ex)} + \frac{\left(-2a^3cd^2+b^4e^2-2ab^2e(bd+2ce)+a^2(b^2d^2+6bcde+2c^2e^2)\right)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}\left(ad^2+e(-bd+ce)\right)^2} - \frac{(bd+2ce)\log(x)}{c^2d^3} + \frac{e^3(4ad^2+e(-3bd+2ce))\log(d+ex)}{d^3\left(ad^2+e(-bd+ce)\right)^2} + \frac{(ad-be)\left(abd-b^2e+2ace\right)\log(c+x(b+ax))}{2c^2\left(ad^2+e(-bd+ce)\right)^2}$$

input
```
Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

output
```
-(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((-2*a^3
*c*d^2 + b^4*e^2 - 2*a*b^2*e*(b*d + 2*c*e) + a^2*(b^2*d^2 + 6*b*c*d*e + 2*
c^2*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*
(a*d^2 + e*(-(b*d) + c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4
*a*d^2 + e*(-3*b*d + 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 + e*(-(b*d) + c*e))
^2) + ((a*d - b*e)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*
(a*d^2 + e*(-(b*d) + c*e))^2)
```

3.77.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(d+ex)^2\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$
$$\Big\downarrow \; 1893$$
$$\int \frac{1}{x^2(d+ex)^2\left(ax^2+bx+c\right)}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{-a^3cd^2 + a^2\left(b^2d^2 + 4bcde + c^2e^2\right) + ax(ad-be)\left(abd + 2ace + b^2(-e)\right) - ab^2e(2bd+3ce) + b^4e^2}{c^2\left(ax^2+bx+c\right)\left(ad^2 - e(bd-ce)\right)^2} + \frac{}{d^2(d+}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(2a^3cd^2 - a^2\left(b^2d^2 + 6bcde + 2c^2e^2\right) + 2ab^2e(bd+2ce) + b^4\left(-e^2\right)\right)}{c^2\sqrt{b^2-4ac}\left(ad^2 - e(bd-ce)\right)^2} + \frac{(ad-be)\left(abd + 2ace + b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2 - e(bd-ce)\right)^2} - \frac{e^3}{d^2(d+ex)\left(ad^2 - e(bd-ce)\right)} + \frac{e^3\log(d+ex)\left(4ad^2 - e(3bd-2ce)\right)}{d^3\left(ad^2 - e(bd-ce)\right)^2} - \frac{\log(x)(bd+2ce)}{c^2d^3} - \frac{1}{cd^2x}$$

input
```
Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

output
```
-(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((2*a^3*c*d
^2 - b^4*e^2 + 2*a*b^2*e*(b*d + 2*c*e) - a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^2*
e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^
2 - e*(b*d - c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d^2 -
 e*(3*b*d - 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))^2) + ((a*d
- b*e)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(
b*d - c*e))^2)
```

3.77.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.77.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.19

method	result
default	$-\frac{1}{c\,d^{2}x}+\frac{(-bd-2ec)\ln(x)}{c^{2}d^{3}}+\frac{\frac{\left(a^{3}b\,d^{2}+2a^{3}cde-2a^{2}b^{2}de-2a^{2}bc\,e^{2}+b^{3}e^{2}a\right)\ln\left(a\,x^{2}+bx+c\right)}{2a}+2\left(-a^{3}c\,d^{2}+b^{2}d^{2}a^{2}+4a^{2}bcde+a^{2}c^{2}e^{2}-2a\,b^{3}\right)}{(a\,d^{2}-bde+c\,e^{2})^{2}}$
risch	Expression too large to display

input
```
int(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c/d^2/x+(-b*d-2*c*e)/c^2/d^3*ln(x)+1/(a*d^2-b*d*e+c*e^2)^2/c^2*(1/2*(a^
3*b*d^2+2*a^3*c*d*e-2*a^2*b^2*d*e-2*a^2*b*c*e^2+a*b^3*e^2)/a*ln(a*x^2+b*x+
c)+2*(-a^3*c*d^2+b^2*d^2*a^2+4*a^2*b*c*d*e+a^2*c^2*e^2-2*a*b^3*d*e-3*a*b^2
*c*e^2+b^4*e^2-1/2*(a^3*b*d^2+2*a^3*c*d*e-2*a^2*b^2*d*e-2*a^2*b*c*e^2+a*b^
3*e^2)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-e^3/d^2
/(a*d^2-b*d*e+c*e^2)/(e*x+d)+e^3*(4*a*d^2-3*b*d*e+2*c*e^2)/d^3/(a*d^2-b*d*
e+c*e^2)^2*ln(e*x+d)
```

3.77.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d)**2,x)
```

output
```
Timed out
```

3.77.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.69

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx &= -\frac{e^7}{(ad^4e^4-bd^3e^5+cd^2e^6)(ex+d)} \\ &+ \frac{(a^2bd^2-2\,ab^2de+2\,a^2cde+b^3e^2-2\,abce^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{2\,(a^2c^2d^4-2\,abc^2d^3e+b^2c^2d^2e^2+2\,ac^3d^2e^2-2\,bc^3de^3+c^4e^4)} \\ &- \frac{(a^2b^2d^2e^2-2\,a^3cd^2e^2-2\,ab^3de^3+6\,a^2bcde^3+b^4e^4-4\,ab^2ce^4+2\,a^2c^2e^4)\arctan\left(-\frac{2\,ad-\frac{2\,ad^2}{ex+d}-be+\frac{2\,bde}{ex+d}-}{\sqrt{-b^2+4\,ace}}\right.}{(a^2c^2d^4-2\,abc^2d^3e+b^2c^2d^2e^2+2\,ac^3d^2e^2-2\,bc^3de^3+c^4e^4)\sqrt{-b^2+4\,ac}e^2} \\ &+ \frac{e}{cd^3\left(\frac{d}{ex+d}-1\right)} - \frac{(bde+2\,ce^2)\log\left(\left|-\frac{d}{ex+d}+1\right|\right)}{c^2d^3e}\end{aligned}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="giac")
```

output
```
-e^7/((a*d^4*e^4 - b*d^3*e^5 + c*d^2*e^6)*(e*x + d)) + 1/2*(a^2*b*d^2 - 2*
a*b^2*d*e + 2*a^2*c*d*e + b^3*e^2 - 2*a*b*c*e^2)*log(-a + 2*a*d/(e*x + d)
- a*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/(e*x + d)^2 - c*e^2/(e*x + d)^
2)/(a^2*c^2*d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2*
b*c^3*d*e^3 + c^4*e^4) - (a^2*b^2*d^2*e^2 - 2*a^3*c*d^2*e^2 - 2*a*b^3*d*e^
3 + 6*a^2*b*c*d*e^3 + b^4*e^4 - 4*a*b^2*c*e^4 + 2*a^2*c^2*e^4)*arctan(-(2*
a*d - 2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/(sq
rt(-b^2 + 4*a*c)*e))/((a^2*c^2*d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2
*a*c^3*d^2*e^2 - 2*b*c^3*d*e^3 + c^4*e^4)*sqrt(-b^2 + 4*a*c)*e^2) + e/(c*d
^3*(d/(e*x + d) - 1)) - (b*d*e + 2*c*e^2)*log(abs(-d/(e*x + d) + 1))/(c^2*
d^3*e)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 34.25 (sec) , antiderivative size = 4948, normalized size of antiderivative = 17.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^4*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

output
```
(log(d + e*x)*(2*c*e^5 + 4*a*d^2*e^3 - 3*b*d*e^4))/(a^2*d^7 + b^2*d^5*e^2
+ c^2*d^3*e^4 - 2*a*b*d^6*e + 2*a*c*d^5*e^2 - 2*b*c*d^4*e^3) - (1/(c*d) +
(x*(2*c*e^3 + a*d^2*e - b*d*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)))/(d*x +
e*x^2) - (log((((a*e*(a^5*b*d^8 + 4*b^3*c^3*e^8 + b^6*d^3*e^5 - 2*a*b^5*d^
4*e^4 - 2*a^4*b^2*d^7*e + 16*a^2*c^4*d*e^7 - 4*b^4*c^2*d*e^7 - b^5*c*d^2*e
^6 + a^2*b^4*d^5*e^3 + a^3*b^3*d^6*e^2 + 16*a^3*c^3*d^3*e^5 + a^4*c^2*d^5*
e^3 - 12*a*b*c^4*e^8 + 2*a^5*c*d^7*e - 16*a^2*b^2*c^2*d^3*e^5 + 4*a*b^2*c^
3*d*e^7 - 2*a^4*b*c*d^6*e^2 + 13*a*b^3*c^2*d^2*e^6 - 20*a^2*b*c^3*d^2*e^6
+ a^2*b^3*c*d^4*e^4 + 8*a^3*b*c^2*d^4*e^4))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*
e)^2) - (((a*e*(a^4*c*d^6 + 8*a*c^4*e^6 - a^3*b^2*d^6 - 2*b^2*c^3*e^6 + b^
5*d^3*e^3 - 3*a*b^4*d^4*e^2 + 3*a^2*b^3*d^5*e + b^3*c^2*d*e^5 + b^4*c*d^2*
e^4 + 8*a^2*c^3*d^2*e^4 - 7*a^3*c^2*d^4*e^2 - 4*a*b*c^3*d*e^5 - 7*a^3*b*c*
d^5*e - 7*a*b^3*c*d^3*e^3 - 6*a*b^2*c^2*d^2*e^4 + 12*a^2*b*c^2*d^3*e^3 + 1
2*a^2*b^2*c*d^4*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(b^5*e^2 + b^
4*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b
^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*a^
3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2
) + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^
2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b
^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^...
```

3.77. $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx$

3.78 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx$

3.78.1 Optimal result

Integrand size = 25, antiderivative size = 372

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx &= -\frac{1}{2cd^2x^2}+\frac{bd+2ce}{c^2d^3x}+\frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)}\\ &+\frac{\left(b^5e^2-a^3cd(3bd+4ce)-ab^3e(2bd+5ce)+a^2b(b^2d^2+8bcde+5c^2e^2)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}\\ &+\frac{\left(b^2d^2+2bcde-c(ad^2-3ce^2)\right)\log(x)}{c^3d^4}-\frac{e^4\left(5ad^2-e(4bd-3ce)\right)\log(d+ex)}{d^4\left(ad^2-e(bd-ce)\right)^2}\\ &+\frac{\left(a^3cd^2-b^4e^2+ab^2e(2bd+3ce)-a^2(b^2d^2+4bcde+c^2e^2)\right)\log\left(c+bx+ax^2\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}\end{aligned}$$

output
```
-1/2/c/d^2/x^2+(b*d+2*c*e)/c^2/d^3/x+e^4/d^3/(a*d^2-e*(b*d-c*e))/(e*x+d)+(
b^2*d^2+2*b*c*d*e-c*(a*d^2-3*c*e^2))*ln(x)/c^3/d^4-e^4*(5*a*d^2-e*(4*b*d-3
*c*e))*ln(e*x+d)/d^4/(a*d^2-e*(b*d-c*e))^2+1/2*(a^3*c*d^2-b^4*e^2+a*b^2*e*
(2*b*d+3*c*e)-a^2*(b^2*d^2+4*b*c*d*e+c^2*e^2))*ln(a*x^2+b*x+c)/c^3/(a*d^2-
e*(b*d-c*e))^2+(b^5*e^2-a^3*c*d*(3*b*d+4*c*e)-a*b^3*e*(2*b*d+5*c*e)+a^2*b*
(b^2*d^2+8*b*c*d*e+5*c^2*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(
a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx=-\frac{1}{2cd^2x^2}+\frac{bd+2ce}{c^2d^3x}+\frac{e^4}{d^3\left(ad^2+e(-bd+ce)\right)(d+ex)}+\frac{\left(-b^5e^2+a^3cd(3bd+4ce)+ab^3e(2bd+5ce)-a^2b(b^2d^2+8bcde+5c^2e^2)\right)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^3\sqrt{-b^2+4ac}\left(ad^2+e(-bd+ce)\right)^2}+\frac{\left(b^2d^2+2bcde+c(-ad^2+3ce^2)\right)\log(x)}{c^3d^4}-\frac{e^4(5ad^2+e(-4bd+3ce))\log(d+ex)}{d^4\left(ad^2+e(-bd+ce)\right)^2}-\frac{\left(-a^3cd^2+b^4e^2-ab^2e(2bd+3ce)+a^2(b^2d^2+4bcde+c^2e^2)\right)\log(c+x(b+ax))}{2c^3\left(ad^2+e(-bd+ce)\right)^2}$$

input
```
Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

output
```
-1/2*1/(c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 + e*(-(b*
d) + c*e))*(d + e*x)) + ((-(b^5*e^2) + a^3*c*d*(3*b*d + 4*c*e) + a*b^3*e*(
2*b*d + 5*c*e) - a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTan[(b + 2*a*
x)/Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))
^2) + ((b^2*d^2 + 2*b*c*d*e + c*(-(a*d^2) + 3*c*e^2))*Log[x])/(c^3*d^4) -
(e^4*(5*a*d^2 + e*(-4*b*d + 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 + e*(-(b*d)
+ c*e))^2) - ((-(a^3*c*d^2) + b^4*e^2 - a*b^2*e*(2*b*d + 3*c*e) + a^2*(b^2
*d^2 + 4*b*c*d*e + c^2*e^2))*Log[c + x*(b + a*x)])/(2*c^3*(a*d^2 + e*(-(b*
d) + c*e))^2)
```

3.78.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1893, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5(d+ex)^2\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}dx$$

$$\Big\downarrow \text{1893}$$

$$\int \frac{1}{x^3(d+ex)^2\left(ax^2+bx+c\right)}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{ax\left(a^3cd^2 - a^2\left(b^2d^2 + 4bcde + c^2e^2\right) + ab^2e(2bd + 3ce) + b^4\left(-e^2\right)\right) - \left(abd + ace + b^2(-e)\right)\left(-2a^2cd + ab^2d \right.}{c^3\left(ax^2 + bx + c\right)\left(ad^2 - e(bd - ce)\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(-a^3cd(3bd + 4ce) + a^2b\left(b^2d^2 + 8bcde + 5c^2e^2\right) - ab^3e(2bd + 5ce) + b^5e^2\right)}{c^3\sqrt{b^2 - 4ac}\left(ad^2 - e(bd - ce)\right)^2} + \frac{\left(a^3cd^2 - a^2\left(b^2d^2 + 4bcde + c^2e^2\right) + ab^2e(2bd + 3ce) + b^4\left(-e^2\right)\right)\log\left(ax^2 + bx + c\right)}{2c^3\left(ad^2 - e(bd - ce)\right)^2} + \frac{\log(x)\left(-c\left(ad^2 - 3ce^2\right) + b^2d^2 + 2bcde\right)}{c^3d^4} - \frac{e^4\log(d + ex)\left(5ad^2 - e(4bd - 3ce)\right)}{d^4\left(ad^2 - e(bd - ce)\right)^2} + \frac{e^4}{d^3(d + ex)\left(ad^2 - e(bd - ce)\right)} + \frac{bd + 2ce}{c^2d^3x} - \frac{1}{2cd^2x^2}$$

input
```
Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

output
```
-1/2*1/(c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 - e*(b*d
- c*e))*(d + e*x)) + ((b^5*e^2 - a^3*c*d*(3*b*d + 4*c*e) - a*b^3*e*(2*b*d
+ 5*c*e) + a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTanh[(b + 2*a*x)/Sq
rt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + ((b^
2*d^2 + 2*b*c*d*e - c*(a*d^2 - 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*(5*a*d^2
 - e*(4*b*d - 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 - e*(b*d - c*e))^2) + ((a^
3*c*d^2 - b^4*e^2 + a*b^2*e*(2*b*d + 3*c*e) - a^2*(b^2*d^2 + 4*b*c*d*e + c
^2*e^2))*Log[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e))^2)
```

3.78.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1893
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.78.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.22

method	result
default	$-\frac{1}{2c\,d^2x^2}-\frac{-bd-2ec}{x\,c^2d^3}+\frac{\left(-d^2ac+b^2d^2+2bcde+3e^2c^2\right)\ln(x)}{d^4c^3}+\frac{\frac{\left(a^4c\,d^2-a^3b^2d^2-4a^3bcde-a^3c^2e^2+2a^2b^3de+3a^2b^2c\,e^2-a\,b^4e^2\right)\ln(}{2a}}{}$
risch	Expression too large to display

input
```
int(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c/d^2/x^2-(-b*d-2*c*e)/x/c^2/d^3+1/d^4/c^3*(-a*c*d^2+b^2*d^2+2*b*c*d*
e+3*c^2*e^2)*ln(x)+1/(a*d^2-b*d*e+c*e^2)^2/c^3*(1/2*(a^4*c*d^2-a^3*b^2*d^2
-4*a^3*b*c*d*e-a^3*c^2*e^2+2*a^2*b^3*d*e+3*a^2*b^2*c*e^2-a*b^4*e^2)/a*ln(a
*x^2+b*x+c)+2*(2*a^3*b*c*d^2+2*a^3*d*e*c^2-a^2*b^3*d^2-6*a^2*b^2*c*d*e-3*a
^2*c^2*e^2*b+2*a*b^4*d*e+4*e^2*a*c*b^3-b^5*e^2-1/2*(a^4*c*d^2-a^3*b^2*d^2-
4*a^3*b*c*d*e-a^3*c^2*e^2+2*a^2*b^3*d*e+3*a^2*b^2*c*e^2-a*b^4*e^2)*b/a)/(4
*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+e^4/d^3/(a*d^2-b*d*e+
c*e^2)/(e*x+d)-e^4*(5*a*d^2-4*b*d*e+3*c*e^2)/d^4/(a*d^2-b*d*e+c*e^2)^2*ln(
e*x+d)
```

3.78.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d)**2,x)
```

output
```
Timed out
```

3.78.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \frac{e^9}{(ad^5e^5-bd^4e^6+cd^3e^7)(ex+d)}$$
$$-\frac{(a^2b^2d^2-a^3cd^2-2\,ab^3de+4\,a^2bcde+b^4e^2-3\,ab^2ce^2+a^2c^2e^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{b}{(ex}\right.}{2\,(a^2c^3d^4-2\,abc^3d^3e+b^2c^3d^2e^2+2\,ac^4d^2e^2-2\,bc^4de^3+c^5e^4)}$$
$$+\frac{(a^2b^3d^2e^2-3\,a^3bcd^2e^2-2\,ab^4de^3+8\,a^2b^2cde^3-4\,a^3c^2de^3+b^5e^4-5\,ab^3ce^4+5\,a^2bc^2e^4)\arctan\left(-\frac{2\,ad}{}\right.}{(a^2c^3d^4-2\,abc^3d^3e+b^2c^3d^2e^2+2\,ac^4d^2e^2-2\,bc^4de^3+c^5e^4)\sqrt{-b^2+4\,ac}e^2}$$
$$+\frac{(b^2d^2e-acd^2e+2\,bcde^2+3\,c^2e^3)\log\left(\left|-\frac{d}{ex+d}+1\right|\right)}{c^3d^4e}+\frac{2\,bcde+5\,c^2e^2-\frac{2\,(bcd^2e^2+3\,c^2de^3)}{(ex+d)e}}{2\,c^3d^4\left(\frac{d}{ex+d}-1\right)^2}$$

input
```
integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="giac")
```

output
```
e^9/((a*d^5*e^5 - b*d^4*e^6 + c*d^3*e^7)*(e*x + d)) - 1/2*(a^2*b^2*d^2 - a
^3*c*d^2 - 2*a*b^3*d*e + 4*a^2*b*c*d*e + b^4*e^2 - 3*a*b^2*c*e^2 + a^2*c^2
*e^2)*log(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e
/(e*x + d)^2 - c*e^2/(e*x + d)^2)/(a^2*c^3*d^4 - 2*a*b*c^3*d^3*e + b^2*c^3
*d^2*e^2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e^3 + c^5*e^4) + (a^2*b^3*d^2*e^2 -
 3*a^3*b*c*d^2*e^2 - 2*a*b^4*d*e^3 + 8*a^2*b^2*c*d*e^3 - 4*a^3*c^2*d*e^3 +
 b^5*e^4 - 5*a*b^3*c*e^4 + 5*a^2*b*c^2*e^4)*arctan(-(2*a*d - 2*a*d^2/(e*x
+ d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/(sqrt(-b^2 + 4*a*c)*e)
)/((a^2*c^3*d^4 - 2*a*b*c^3*d^3*e + b^2*c^3*d^2*e^2 + 2*a*c^4*d^2*e^2 - 2*
b*c^4*d*e^3 + c^5*e^4)*sqrt(-b^2 + 4*a*c)*e^2) + (b^2*d^2*e - a*c*d^2*e +
2*b*c*d*e^2 + 3*c^2*e^3)*log(abs(-d/(e*x + d) + 1))/(c^3*d^4*e) + 1/2*(2*b
*c*d*e + 5*c^2*e^2 - 2*(b*c*d^2*e^2 + 3*c^2*d*e^3)/((e*x + d)*e))/(c^3*d^4
*(d/(e*x + d) - 1)^2)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 47.17 (sec) , antiderivative size = 7144, normalized size of antiderivative = 19.20

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^5*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

output
```
((x*(2*b*d + 3*c*e))/(2*c^2*d^2) - 1/(2*c*d) + (x^2*(3*c^2*e^4 - b^2*d^2*e
^2 + a*b*d^3*e - b*c*d*e^3 + 2*a*c*d^2*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d
*e)))/(d*x^2 + e*x^3) - (log(d + e*x)*(3*c*e^6 + 5*a*d^2*e^4 - 4*b*d*e^5))
/(a^2*d^8 + b^2*d^6*e^2 + c^2*d^4*e^4 - 2*a*b*d^7*e + 2*a*c*d^6*e^2 - 2*b*
c*d^5*e^3) + (log((((27*a^2*b*c^6*e^11 - 9*a*b^3*c^5*e^11 - a*b^8*d^5*e^6
- a^6*b^3*d^10*e - 36*a^3*c^6*d*e^10 + 2*a^2*b^7*d^6*e^5 - a^3*b^6*d^7*e^4
 - a^4*b^5*d^8*e^3 + 2*a^5*b^4*d^9*e^2 - 36*a^4*c^5*d^3*e^8 + 4*a^5*c^4*d^
5*e^6 + 3*a^6*c^3*d^7*e^4 + a^7*b*c*d^10*e - 39*a^2*b^3*c^4*d^2*e^9 - 15*a
^2*b^4*c^3*d^3*e^8 + 7*a^2*b^5*c^2*d^4*e^7 + 53*a^3*b^2*c^4*d^3*e^8 + 7*a^
3*b^3*c^3*d^4*e^7 - 33*a^3*b^4*c^2*d^5*e^6 + 20*a^4*b^2*c^3*d^5*e^6 + 33*a
^4*b^3*c^2*d^6*e^5 - 9*a^5*b^2*c^2*d^7*e^4 + 6*a*b^4*c^4*d*e^10 - 2*a*b^7*
c*d^4*e^7 + 5*a*b^5*c^3*d^2*e^9 + a*b^6*c^2*d^3*e^8 + 12*a^2*b^6*c*d^5*e^6
 + 51*a^3*b*c^5*d^2*e^9 - 16*a^3*b^5*c*d^6*e^5 - 27*a^4*b*c^4*d^4*e^7 + 6*
a^4*b^4*c*d^7*e^4 - 19*a^5*b*c^3*d^6*e^5 + 3*a^5*b^3*c*d^8*e^3 - a^6*b*c^2
*d^8*e^3 - 4*a^6*b^2*c*d^9*e^2)/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2) + (((a
*e*(12*a*c^5*e^7 - a^3*b^3*d^7 - 3*b^2*c^4*e^7 + b^6*d^4*e^3 - 3*a*b^5*d^5
*e^2 + 3*a^2*b^4*d^6*e + 4*a^4*c^2*d^6*e + b^3*c^3*d*e^6 + b^5*c*d^3*e^4 +
 8*a^2*c^4*d^2*e^5 - 8*a^3*c^3*d^4*e^3 + b^4*c^2*d^2*e^5 + 2*a^4*b*c*d^7 -
 4*a*b*c^4*d*e^6 + 18*a^2*b^2*c^2*d^4*e^3 - 8*a*b^4*c*d^4*e^3 - 10*a^3*b^2
*c*d^6*e - 6*a*b^2*c^3*d^2*e^5 - 7*a*b^3*c^2*d^3*e^4 + 12*a^2*b*c^3*d^3...
```

3.78. $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx$

3.79 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx$

3.79.1 Optimal result

Integrand size = 29, antiderivative size = 981

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx =$$

$$-\frac{2(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2(3b^2d^2-29bcde+50c^2e^2))\sqrt{a}}{3465a^4e^4}$$

$$+\frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex}$$

$$+\frac{2(233a^3d^3+48b^3e^3+abe^2(67bd-157ce)+4a^2de(18bd-37ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{3465a^3e^4}$$

$$-\frac{2(29a^2d^2+8b^2e^2+ae(19bd-18ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{693a^2e^4}$$

$$+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{7/2}}{99ae^4}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(128a^5d^5+128b^5e^5-4a^4d^3e(14bd-27ce)-8ab^3e^4(7bd+87ce)-a^2be^3(37b^2d^2-258bc}{34}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(ad^2-e(bd-ce))\,(128a^4d^4-64b^4e^4-4ab^2e^3(7bd-69ce)+4a^3d^2e(2bd+3ce)-3a^2e^2}{3}$$

output
```
2/3465*(233*a^3*d^3+48*b^3*e^3+a*b*e^2*(67*b*d-157*c*e)+4*a^2*d*e*(18*b*d-
37*c*e))*x*(e*x+d)^(3/2)*(a+c/x^2+b/x)^(1/2)/a^3/e^4-2/693*(29*a^2*d^2+8*b
^2*e^2+a*e*(19*b*d-18*c*e))*x*(e*x+d)^(5/2)*(a+c/x^2+b/x)^(1/2)/a^2/e^4+2/
99*(a*d+b*e)*x*(e*x+d)^(7/2)*(a+c/x^2+b/x)^(1/2)/a/e^4-2/3465*(187*a^4*d^4
+64*b^4*e^4+4*a*b^2*e^3*(7*b*d-69*c*e)-4*a^3*d^2*e*(2*b*d+3*c*e)+3*a^2*e^2
*(3*b^2*d^2-29*b*c*d*e+50*c^2*e^2))*x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/a^
4/e^4+2/11*x^5*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+1/3465*(128*a^5*d^5+128*b
^5*e^5-4*a^4*d^3*e*(14*b*d-27*c*e)-8*a*b^3*e^4*(7*b*d+87*c*e)-a^2*b*e^3*(3
7*b^2*d^2-258*b*c*d*e-771*c^2*e^2)-a^3*d*e^2*(37*b^2*d^2-135*b*c*d*e+156*c
^2*e^2))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^
(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a
*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a^5/e^5/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*
(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3465*(a*d^2-e*(b*d-c*e))*(128*a^4*d^4-64*
b^4*e^4-4*a*b^2*e^3*(7*b*d-69*c*e)+4*a^3*d^2*e*(2*b*d+3*c*e)-3*a^2*e^2*(3*
b^2*d^2-29*b*c*d*e+50*c^2*e^2))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b
+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/
2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)/a^5/e^5/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

3.79.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.47 (sec) , antiderivative size = 10904, normalized size of antiderivative = 11.12

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]
```

output
```
Result too large to show
```

3.79.3 Rubi [A] (verified)

Time = 3.10 (sec) , antiderivative size = 1021, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.517, Rules used = {1897, 1272, 25, 2184, 27, 2184, 27, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}dx$$

$$\Big\downarrow \text{ 1897}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\int x^3\sqrt{d+ex}\sqrt{ax^2+bx+c}dx}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{ 1272}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2}{11}x^4\sqrt{d+ex}\sqrt{ax^2+bx+c}-\frac{1}{11}\int -\frac{x^3\left((ad+be)x^2+2(bd+ce)x+3cd\right)}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{11}\int \frac{x^3\left((ad+be)x^2+2(bd+ce)x+3cd\right)}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+\frac{2}{11}x^4\sqrt{d+ex}\sqrt{ax^2+bx+c}\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\Bigg(\frac{1}{11}\Bigg(\frac{2\int -\frac{e^4\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)x^4+e^3\left(33a^2d^3+2ae(29bd-10ce)d+be^2(25bd+7ce)\right)x^3+3de^2(ad+be)\left(5ad^2+e(9bd+7ce)\right)}{2\sqrt{d+ex}\sqrt{ax^2+bx+c}}}{9ae^5}}{\sqrt{ax^2+b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\Bigg(\frac{1}{11}\Bigg(\frac{2(d+ex)^{7/2}\sqrt{ax^2+bx+c}(ad+be)}{9ae^4}-\frac{\int \frac{e^4\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)x^4+e^3\left(33a^2d^3+2ae(29bd-10ce)d+be^2(25bd+7ce)\right)}{\sqrt{d+}}}{}}{\sqrt{ax^2+bx}}$$

$$\Big\downarrow \text{ 2184}$$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{11}\left(\frac{2(d+ex)^{7/2}\sqrt{ax^2+bx+c}(ad+be)}{9ae^4}-\frac{2\int -\frac{\left(233a^3d^3+4a^2e(18bd-37ce)d+48b^3e^3+abe^2(67bd-157ce)\right)x^3e^7+2\left(107a^3d^4+2a^2e(\cdots\right.}{\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 27

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{11}\left(\frac{2(d+ex)^{7/2}\sqrt{ax^2+bx+c}(ad+be)}{9ae^4}-\frac{\frac{2e(d+ex)^{5/2}\sqrt{ax^2+bx+c}\left(29a^2d^2+ae(19bd-18ce)+8b^2e^2\right)}{7a}-\frac{\int \frac{\left(233a^3d^3+4a^2e(18bd-37c\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 2184

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{11}\left(\frac{2(d+ex)^{7/2}\sqrt{ax^2+bx+c}(ad+be)}{9ae^4}-\frac{\frac{2e(d+ex)^{5/2}\sqrt{ax^2+bx+c}\left(29a^2d^2+ae(19bd-18ce)+8b^2e^2\right)}{7a}-\frac{2\int -\frac{3\left(\left(187a^4d^4-4a^3e(2bd\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 27

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{11}\left(\frac{2(d+ex)^{7/2}\sqrt{ax^2+bx+c}(ad+be)}{9ae^4}-\frac{\frac{2e(d+ex)^{5/2}\sqrt{ax^2+bx+c}\left(29a^2d^2+ae(19bd-18ce)+8b^2e^2\right)}{7a}-\frac{\frac{2e^5(d+ex)^{3/2}\sqrt{ax^2+bx+c}(\cdots}{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 2184

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{11}\left(\frac{2(d+ex)^{7/2}\sqrt{ax^2+bx+c}(ad+be)}{9ae^4}-\frac{\frac{2e(d+ex)^{5/2}\sqrt{ax^2+bx+c}\left(29a^2d^2+ae(19bd-18ce)+8b^2e^2\right)}{7a}-\frac{\frac{2e^5(d+ex)^{3/2}\sqrt{ax^2+bx+c}(\cdots}{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 27

3.79. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^4\sqrt{d+ex}\,dx$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{11}\left(\frac{2(d+ex)^{7/2}\sqrt{ax^2+bx+c}(ad+be)}{9ae^4}-\frac{\frac{2e(d+ex)^{5/2}\sqrt{ax^2+bx+c}\left(29a^2d^2+ae(19bd-18ce)+8b^2e^2\right)}{7a}-\frac{2e^5(d+ex)^{3/2}\sqrt{ax^2+bx+c}\left(\right.}{}}{}\right.\right.$$

$\downarrow$ 1269

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{11}\left(\frac{2(d+ex)^{7/2}\sqrt{ax^2+bx+c}(ad+be)}{9ae^4}-\frac{\frac{2e(d+ex)^{5/2}\sqrt{ax^2+bx+c}\left(29a^2d^2+ae(19bd-18ce)+8b^2e^2\right)}{7a}-\frac{2e^5(d+ex)^{3/2}\sqrt{ax^2+bx+c}\left(\right.}{}}{}\right.\right.$$

$\downarrow$ 1172

3.79. $\int\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx$

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{2}{11}\sqrt{d+ex}\sqrt{ax^2+bx+c}x^4+\frac{1}{11}\left(\frac{2(ad+be)(d+ex)^{7/2}\sqrt{ax^2+bx+c}}{9ae^4}-\frac{2e\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)(d+ex)^{5/2}}{7a}\right.\right.$$

$\downarrow$ 321

3.79. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx$

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{2}{11}\sqrt{d+ex}\sqrt{ax^2+bx+c}x^4+\frac{1}{11}\left(\frac{2(ad+be)(d+ex)^{7/2}\sqrt{ax^2+bx+c}}{9ae^4}-\frac{2e\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)(d+ex)^{5/2}}{7a}\right.\right.$$

$\downarrow$ 327

3.79. $\int\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx$

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{2}{11}\sqrt{d+ex}\sqrt{ax^2+bx+c}x^4+\frac{1}{11}\left(\frac{2(ad+be)(d+ex)^{7/2}\sqrt{ax^2+bx+c}}{9ae^4}-\frac{2e\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)(d+ex)^{5/2}}{7a}\right.\right.$$

input
```
Int[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]
```

3.79. $\int\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx$

output
```
(Sqrt[a + c/x^2 + b/x]*x*((2*x^4*Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2])/11 +
 ((2*(a*d + b*e)*(d + e*x)^(7/2)*Sqrt[c + b*x + a*x^2])/(9*a*e^4) - ((2*e*
(29*a^2*d^2 + 8*b^2*e^2 + a*e*(19*b*d - 18*c*e))*(d + e*x)^(5/2)*Sqrt[c +
b*x + a*x^2])/(7*a) - ((2*e^5*(233*a^3*d^3 + 48*b^3*e^3 + a*b*e^2*(67*b*d
- 157*c*e) + 4*a^2*d*e*(18*b*d - 37*c*e))*(d + e*x)^(3/2)*Sqrt[c + b*x + a
*x^2])/(5*a) - (3*((2*e^8*(187*a^4*d^4 + 64*b^4*e^4 + 4*a*b^2*e^3*(7*b*d -
 69*c*e) - 4*a^3*d^2*e*(2*b*d + 3*c*e) + 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e
 + 50*c^2*e^2))*Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2])/(3*a) - (e^8*((Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(128*a^5*d^5 + 128*b^5*e^5 - 4*a^4*d^3*e*(14*b*d - 27*
c*e) - 8*a*b^3*e^4*(7*b*d + 87*c*e) - a^2*b*e^3*(37*b^2*d^2 - 258*b*c*d*e
- 771*c^2*e^2) - a^3*d*e^2*(37*b^2*d^2 - 135*b*c*d*e + 156*c^2*e^2))*Sqrt[
d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*e*Sqrt[(a*(d + e*x)
)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[c + b*x + a*x^2]) - (2*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(a*d^2 - b*d*e + c*e^2)*(128*a^4*d^4 + 8*a^3*b*d^3*e -
9*a^2*b^2*d^2*e^2 + 12*a^3*c*d^2*e^2 - 28*a*b^3*d*e^3 + 87*a^2*b*c*d*e^3 -
 64*b^4*e^4 + 276*a*b^2*c*e^4 - 150*a^2*c^2*e^4)*Sqrt[(a*(d + e*x))/(2*a*d
 - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*...
```

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1272
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Simp[1/(e*(2*m + 5))   Int[((d +
 e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
 + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

rule 1897
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p])   Int[x^(m
- 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
 d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
  !IntegerQ[q] && PosQ[n]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.79. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx$

3.79.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5003 vs. 2(899) = 1798.

Time = 2.70 (sec) , antiderivative size = 5004, normalized size of antiderivative = 5.10

method	result	size
risch	Expression too large to display	5004
default	Expression too large to display	11938

input
```
int(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.79.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 920, normalized size of antiderivative = 0.94

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx =$$
$$-\frac{2\left((128\,a^6d^6-120\,a^5bd^5e-3\,(11\,a^4b^2-68\,a^5c)d^4e^2-(20\,a^3b^3-87\,a^4bc)d^3e^3-3\,(11\,a^2b^4-53\,a^3b^2c\right.}{}$$

input
```
integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
-2/10395*((128*a^6*d^6 - 120*a^5*b*d^5*e - 3*(11*a^4*b^2 - 68*a^5*c)*d^4*e
^2 - (20*a^3*b^3 - 87*a^4*b*c)*d^3*e^3 - 3*(11*a^2*b^4 - 53*a^3*b^2*c + 34
*a^4*c^2)*d^2*e^4 - 3*(40*a*b^5 - 246*a^2*b^3*c + 329*a^3*b*c^2)*d*e^5 + (
128*b^6 - 888*a*b^4*c + 1599*a^2*b^2*c^2 - 450*a^3*c^3)*e^6)*sqrt(a*e)*wei
erstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4
/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*
b*c)*e^3)/(a^3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e)) + 3*(128*a^6*d^5*e -
 56*a^5*b*d^4*e^2 - (37*a^4*b^2 - 108*a^5*c)*d^3*e^3 - (37*a^3*b^3 - 135*a
^4*b*c)*d^2*e^4 - 2*(28*a^2*b^4 - 129*a^3*b^2*c + 78*a^4*c^2)*d*e^5 + (128
*a*b^5 - 696*a^2*b^3*c + 771*a^3*b*c^2)*e^6)*sqrt(a*e)*weierstrassZeta(4/3
*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d^3 - 3*a
^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*e^3),
 weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2)
, -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 -
9*a*b*c)*e^3)/(a^3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e))) - 3*(315*a^6*e^
6*x^5 + 35*(a^6*d*e^5 + a^5*b*e^6)*x^4 - 10*(4*a^6*d^2*e^4 - a^5*b*d*e^5 +
 (4*a^4*b^2 - 9*a^5*c)*e^6)*x^3 + (48*a^6*d^3*e^3 - 13*a^5*b*d^2*e^4 - (13
*a^4*b^2 - 32*a^5*c)*d*e^5 + (48*a^3*b^3 - 157*a^4*b*c)*e^6)*x^2 - 2*(32*a
^6*d^4*e^2 - 10*a^5*b*d^3*e^3 - (9*a^4*b^2 - 23*a^5*c)*d^2*e^4 - 5*(2*a^3*
b^3 - 7*a^4*b*c)*d*e^5 + (32*a^2*b^4 - 138*a^3*b^2*c + 75*a^4*c^2)*e^6)...
```

3.79.6 Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx = \int x^4\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
integrate(x**4*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

output
```
Integral(x**4*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

3.79.7 Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^4\,dx$$

input
```
integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4, x)
```

3.79.8 Giac [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^4\,dx$$

input
```
integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4, x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx=\int x^4\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
int(x^4*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
```

output
```
int(x^4*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.80 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx$

3.80.1 Optimal result

Integrand size = 29, antiderivative size = 778

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx$$

$$= \frac{2(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{315a^3e^3}$$

$$+\frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}$$

$$-\frac{4(8a^2d^2+3b^2e^2+ae(4bd-7ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{315a^2e^3}$$

$$+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{63ae^3}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(8a^4d^4+8b^4e^4-a^3d^2e(4bd-9ce)-4ab^2e^3(bd+9ce)-3a^2e^2(b^2d^2-5bcde-7c^2e^2))\sqrt{\cdots}}{315a^4e^4\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c\cdots}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(16a^3d^3+6a^2cde^2-8b^3e^3-3abe^2(bd-9ce))(ad^2-e(bd-ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a\cdots}{2ad-(b\cdots}}}{315a^4e^4\sqrt{d+ex}(c+bx\cdots}$$

output
```
-4/315*(8*a^2*d^2+3*b^2*e^2+a*e*(4*b*d-7*c*e))*x*(e*x+d)^(3/2)*(a+c/x^2+b/
x)^(1/2)/a^2/e^3+2/63*(a*d+b*e)*x*(e*x+d)^(5/2)*(a+c/x^2+b/x)^(1/2)/a/e^3+
2/315*(19*a^3*d^3-6*a^2*c*d*e^2+8*b^3*e^3+3*a*b*e^2*(b*d-9*c*e))*x*(a+c/x^
2+b/x)^(1/2)*(e*x+d)^(1/2)/a^3/e^3+2/9*x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/
2)-2/315*(8*a^4*d^4+8*b^4*e^4-a^3*d^2*e*(4*b*d-9*c*e)-4*a*b^2*e^3*(b*d+9*c
*e)-3*a^2*e^2*(b^2*d^2-5*b*c*d*e-7*c^2*e^2))*x*EllipticE(1/2*((b+2*a*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2
)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c
/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a^4/e^
4/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/315*(
16*a^3*d^3+6*a^2*c*d*e^2-8*b^3*e^3-3*a*b*e^2*(b*d-9*c*e))*(a*d^2-e*(b*d-c*
e))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
)*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c
+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a^4/e^4/(a
*x^2+b*x+c)/(e*x+d)^(1/2)
```

3.80.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.88 (sec) , antiderivative size = 7531, normalized size of antiderivative = 9.68

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
```

output
```
Result too large to show
```

3.80.3 Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 799, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {1897, 1272, 25, 2184, 27, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

$$\Big\downarrow \; 1897$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\int x^2\sqrt{d+ex}\sqrt{ax^2+bx+c}dx}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 1272$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2}{9}x^3\sqrt{d+ex}\sqrt{ax^2+bx+c}-\frac{1}{9}\int -\frac{x^2\left((ad+be)x^2+2(bd+ce)x+3cd\right)}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 25$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{9}\int \frac{x^2\left((ad+be)x^2+2(bd+ce)x+3cd\right)}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+\frac{2}{9}x^3\sqrt{d+ex}\sqrt{ax^2+bx+c}\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 2184$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{9}\left(\frac{2\int -\frac{2e^3\left(8a^2d^2+3b^2e^2+ae(4bd-7ce)\right)x^3+e^2\left(11a^2d^3+8ae(3bd-2ce)d+be^2(13bd+5ce)\right)x^2+2de(ad+be)\left(ad^2+e(4bd+5ce)\right)x+d^2e(a}{2\sqrt{d+ex}\sqrt{ax^2+bx+c}}}{7ae^4}\right.\right.}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{9}\left(\frac{2(d+ex)^{5/2}\sqrt{ax^2+bx+c}(ad+be)}{7ae^3}-\frac{\int \frac{2e^3\left(8a^2d^2+3b^2e^2+ae(4bd-7ce)\right)x^3+e^2\left(11a^2d^3+8ae(3bd-2ce)d+be^2(13bd+5ce)\right)x^2+2a}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}}{7ae^4}\right.\right.}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 2184$$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{9}\left(\frac{2(d+ex)^{5/2}\sqrt{ax^2+bx+c}(ad+be)}{7ae^3}-\frac{2\int -\frac{3\left(19a^3d^3-6a^2ce^2d+8b^3e^3+3abe^2(bd-9ce)\right)x^2e^5+d\left(a^2(11bd+23ce)d^2+6b^2e^2(bd+3c\right.}{}}{}\right.\right.$$

$$\Big\downarrow \; 27$$

3.80. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\Bigg(\frac{1}{9}\Bigg(\frac{2(d+ex)^{5/2}\sqrt{ax^2+bx+c}(ad+be)}{7ae^3}-\frac{4e(d+ex)^{3/2}\sqrt{ax^2+bx+c}\left(8a^2d^2+ae(4bd-7ce)+3b^2e^2\right)}{5a}-\int\frac{3\left(19a^3d^3-6a^2ce^2d+8b^3e^3+3a\right.}{}$$

$\downarrow$ 2184

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\Bigg(\frac{1}{9}\Bigg(\frac{2(d+ex)^{5/2}\sqrt{ax^2+bx+c}(ad+be)}{7ae^3}-\frac{4e(d+ex)^{3/2}\sqrt{ax^2+bx+c}\left(8a^2d^2+ae(4bd-7ce)+3b^2e^2\right)}{5a}-\frac{2\int-\frac{3e^6\left(4a^3(2bd-ce)d^3-3a^2e(\right.}{}}{}$$

$\downarrow$ 27

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\Bigg(\frac{1}{9}\Bigg(\frac{2(d+ex)^{5/2}\sqrt{ax^2+bx+c}(ad+be)}{7ae^3}-\frac{4e(d+ex)^{3/2}\sqrt{ax^2+bx+c}\left(8a^2d^2+ae(4bd-7ce)+3b^2e^2\right)}{5a}-\frac{2e^4\sqrt{d+ex}\sqrt{ax^2+bx+c}\left(19a^3d^3-\right.}{}$$

$\downarrow$ 1269

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\Bigg(\frac{1}{9}\Bigg(\frac{2(d+ex)^{5/2}\sqrt{ax^2+bx+c}(ad+be)}{7ae^3}-\frac{4e(d+ex)^{3/2}\sqrt{ax^2+bx+c}\left(8a^2d^2+ae(4bd-7ce)+3b^2e^2\right)}{5a}-\frac{2e^4\sqrt{d+ex}\sqrt{ax^2+bx+c}\left(19a^3d^3-\right.}{}$$

$\downarrow$ 1172

3.80. $\int\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^3\sqrt{d+ex}\,dx$

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{2}{9}\sqrt{d+ex}\sqrt{ax^2+bx+c}x^3+\frac{1}{9}\left(\frac{2(ad+be)(d+ex)^{5/2}\sqrt{ax^2+bx+c}}{7ae^3}-\frac{\frac{4e\left(8a^2d^2+3b^2e^2+ae(4bd-7ce)\right)(d+ex)^{3/2}\sqrt{ax^2}}{5a}}{}\right.\right.$$

$\downarrow$ 321

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{2}{9}\sqrt{d+ex}\sqrt{ax^2+bx+c}x^3+\frac{1}{9}\left(\frac{2(ad+be)(d+ex)^{5/2}\sqrt{ax^2+bx+c}}{7ae^3}-\frac{\frac{4e\left(8a^2d^2+3b^2e^2+ae(4bd-7ce)\right)(d+ex)^{3/2}\sqrt{ax^2}}{5a}}{}\right.\right.$$

$\downarrow$ 327

3.80. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx$

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{2}{9}\sqrt{d+ex}\sqrt{ax^2+bx+c}x^3+\frac{1}{9}\left(\frac{2(ad+be)(d+ex)^{5/2}\sqrt{ax^2+bx+c}}{7ae^3}-\frac{4e\left(8a^2d^2+3b^2e^2+ae(4bd-7ce)\right)(d+ex)^{3/2}\sqrt{ax^2}}{5a}\right.\right.$$

input
```
Int[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
```

output
```
(Sqrt[a + c/x^2 + b/x]*x*((2*x^3*Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2])/9 +
((2*(a*d + b*e)*(d + e*x)^(5/2)*Sqrt[c + b*x + a*x^2])/(7*a*e^3) - ((4*e*(
8*a^2*d^2 + 3*b^2*e^2 + a*e*(4*b*d - 7*c*e))*(d + e*x)^(3/2)*Sqrt[c + b*x
+ a*x^2])/(5*a) - ((2*e^4*(19*a^3*d^3 - 6*a^2*c*d*e^2 + 8*b^3*e^3 + 3*a*b*
e^2*(b*d - 9*c*e))*Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2])/a - (e^4*((2*Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(8*a^4*d^4 + 8*b^4*e^4 - a^3*d^2*e*(4*b*d - 9*c*e) -
4*a*b^2*e^3*(b*d + 9*c*e) - 3*a^2*e^2*(b^2*d^2 - 5*b*c*d*e - 7*c^2*e^2))*S
qrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*e*Sqrt[(a*(d +
e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[c + b*x + a*x^2]) - (2*Sqr
t[2]*Sqrt[b^2 - 4*a*c]*(a*d^2 - b*d*e + c*e^2)*(16*a^3*d^3 - 3*a*b^2*d*e^2
 + 6*a^2*c*d*e^2 - 8*b^3*e^3 + 27*a*b*c*e^3)*Sqrt[(a*(d + e*x))/(2*a*d - (
b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*El
lipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqr
t[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*e
*Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2])))/a)/(5*a*e^3))/(7*a*e^4))/9))/Sqrt[
c + b*x + a*x^2]
```

3.80. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx$

3.80.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1272
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Simp[1/(e*(2*m + 5))   Int[((d +
 e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
 + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

rule 1897
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p])   Int[x^(m
- 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
 d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
  !IntegerQ[q] && PosQ[n]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.80.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3660 vs. $2(702) = 1404$.

Time = 2.06 (sec) , antiderivative size = 3661, normalized size of antiderivative = 4.71

method	result	size
risch	Expression too large to display	3661
default	Expression too large to display	9182

input
```
int(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(35*a^3*e^3*x^3+5*a^3*d*e^2*x^2+5*a^2*b*e^3*x^2-6*a^3*d^2*e*x+2*a^2*
b*d*e^2*x+14*a^2*c*e^3*x-6*a*b^2*e^3*x+8*a^3*d^3-3*a^2*b*d^2*e+8*a^2*c*d*e
^2-3*a*b^2*d*e^2-27*a*b*c*e^3+8*b^3*e^3)*(e*x+d)^(1/2)/a^3/e^3*((a*x^2+b*x
+c)/x^2)^(1/2)*x-1/315/a^3/e^3*(16*a^3*b*d^4*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1
/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b
+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+
1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)
/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e
*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2)
)/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+16*b^4*d*e^3*(1/e*d-1/
2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a
))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(
1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b
^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*Ellip
ticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(
b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+16
*b^3*c*e^4*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(
-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/
2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/
e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x...
```

3.80.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 734, normalized size of antiderivative = 0.94

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx$$
$$= \frac{2\left((16\,a^5d^5-16\,a^4bd^4e-5\,(a^3b^2-6\,a^4c)d^3e^2-(5\,a^2b^3-21\,a^3bc)d^2e^3-2\,(8\,ab^4-42\,a^2b^2c+33\,a^3c^2)de\right.}{}$$

input
```
integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/945*((16*a^5*d^5 - 16*a^4*b*d^4*e - 5*(a^3*b^2 - 6*a^4*c)*d^3*e^2 - (5*a
^2*b^3 - 21*a^3*b*c)*d^2*e^3 - 2*(8*a*b^4 - 42*a^2*b^2*c + 33*a^3*c^2)*d*e
^4 + (16*b^5 - 96*a*b^3*c + 123*a^2*b*c^2)*e^5)*sqrt(a*e)*weierstrassPInve
rse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d^
3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^
3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e)) + 6*(8*a^5*d^4*e - 4*a^4*b*d^3*e^
2 - 3*(a^3*b^2 - 3*a^4*c)*d^2*e^3 - (4*a^2*b^3 - 15*a^3*b*c)*d*e^4 + (8*a*
b^4 - 36*a^2*b^2*c + 21*a^3*c^2)*e^5)*sqrt(a*e)*weierstrassZeta(4/3*(a^2*d
^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^
2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*e^3), weiers
trassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27
*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c
)*e^3)/(a^3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e))) + 3*(35*a^5*e^5*x^4 +
5*(a^5*d*e^4 + a^4*b*e^5)*x^3 - 2*(3*a^5*d^2*e^3 - a^4*b*d*e^4 + (3*a^3*b^
2 - 7*a^4*c)*e^5)*x^2 + (8*a^5*d^3*e^2 - 3*a^4*b*d^2*e^3 - (3*a^3*b^2 - 8*
a^4*c)*d*e^4 + (8*a^2*b^3 - 27*a^3*b*c)*e^5)*x)*sqrt(e*x + d)*sqrt((a*x^2
+ b*x + c)/x^2))/(a^5*e^5)
```

3.80.6 Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx=\int x^3\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
integrate(x**3*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

output
```
Integral(x**3*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

3.80.7 Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^3\,dx$$

input
```
integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3, x)
```

3.80.8 Giac [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx = \int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^3\,dx$$

input
```
integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3, x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx = \int x^3\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
int(x^3*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
```

output
```
int(x^3*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.81 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$

3.81.1 Optimal result

Integrand size = 29, antiderivative size = 636

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$$
$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x)}{105a^2e^2}$$
$$+\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(c+bx+ax^2)}{7a}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(8a^3d^3+8b^3e^3-a^2de(5bd-16ce)-abe^2(5bd+29ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx\cdots}{b^2-\cdots}}}{105a^3e^3\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(8a^2d^2-4b^2e^2-ae(bd-10ce))(ad^2-e(bd-ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\cdots}}{105a^3e^3\sqrt{d+ex}(c+bx+ax^2)}$$

output
```
-2/105*x*(4*a^2*d^2+4*b^2*e^2-a*e*(2*b*d-5*c*e)-3*a*e*(a*d-4*b*e)*x)*(a+c/
x^2+b/x)^(1/2)*(e*x+d)^(1/2)/a^2/e^2+2/7*x*(a*x^2+b*x+c)*(a+c/x^2+b/x)^(1/
2)*(e*x+d)^(1/2)/a+1/105*(8*a^3*d^3+8*b^3*e^3-a^2*d*e*(5*b*d-16*c*e)-a*b*e
^2*(5*b*d+29*c*e))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)
^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1
/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a^3/e^3/(a*x^2+b*x+c)/(a*(e*x+d)
/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/105*(8*a^2*d^2-4*b^2*e^2-a*e*(b
*d-10*c*e))*(a*d^2-e*(b*d-c*e))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b
+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/
2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)/a^3/e^3/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

3.81.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.03 (sec) , antiderivative size = 1314, normalized size of antiderivative = 2.07

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$$
$$= x\sqrt{d+ex}\left(\frac{4(-2a^2d^2+abde-2b^2e^2+5ace^2)}{105a^2e^2}+\frac{2(ad+be)x}{35ae}+\frac{2x^2}{7}\right)\sqrt{a+\frac{c+bx}{x^2}}$$
$$+\frac{x(d+ex)^{3/2}\sqrt{a+\frac{c+bx}{x^2}}\left(4\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}(8a^3d^3+8b^3e^3+a^2de(-5bd+16ce)-abe^2(5bd+29ce\cdots\right.}{\cdots}$$

input
```
Integrate[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
```

output
```
x*Sqrt[d + e*x]*((4*(-2*a^2*d^2 + a*b*d*e - 2*b^2*e^2 + 5*a*c*e^2))/(105*a
^2*e^2) + (2*(a*d + b*e)*x)/(35*a*e) + (2*x^2)/7)*Sqrt[a + (c + b*x)/x^2]
+ (x*(d + e*x)^(3/2)*Sqrt[a + (c + b*x)/x^2]*(4*Sqrt[(a*d^2 + e*(-(b*d) +
c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(8*a^3*d^3 + 8*b^3*e^3 + a
^2*d*e*(-5*b*d + 16*c*e) - a*b*e^2*(5*b*d + 29*c*e))*(a*(-1 + d/(d + e*x))
^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x)) - (I*Sqrt[2]*(
2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(8*a^3*d^3 + 8*b^3*e^3 + a^2*d*e*(-
5*b*d + 16*c*e) - a*b*e^2*(5*b*d + 29*c*e))*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2]
- (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*
x)))/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2
] + (2*c*e^2)/(d + e*x) + 2*a*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e
*x)))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[
2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])
/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e +
 Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(8*b^3*e^3*(-(b*e)
 + Sqrt[(b^2 - 4*a*c)*e^2]) + a^3*(-4*c*d^2*e^2 + 8*d^3*Sqrt[(b^2 - 4*a*c)
*e^2]) + a*b*e^2*(13*b^2*d*e + 37*b*c*e^2 - 5*b*d*Sqrt[(b^2 - 4*a*c)*e^2]
- 29*c*e*Sqrt[(b^2 - 4*a*c)*e^2]) + a^2*e*(b^2*d^2*e - 4*c*e*(5*c*e^2 - 4*
d*Sqrt[(b^2 - 4*a*c)*e^2]) - b*d*(52*c*e^2 + 5*d*Sqrt[(b^2 - 4*a*c)*e^2]))
)*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(...
```

3.81.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1897, 1236, 27, 1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}dx$$

$$\Big\downarrow \text{ 1897}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\int x\sqrt{d+ex}\sqrt{ax^2+bx+c}dx}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2\int -\frac{(3bd+ce-(ad-4be)x)\sqrt{ax^2+bx+c}}{2\sqrt{d+ex}}dx}{7a}+\frac{2\sqrt{d+ex}\left(ax^2+bx+c\right)^{3/2}}{7a}\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2\sqrt{d+ex}\left(ax^2+bx+c\right)^{3/2}}{7a}-\frac{\int \frac{(3bd+ce-(ad-4be)x)\sqrt{ax^2+bx+c}}{\sqrt{d+ex}}dx}{7a}\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 1231$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2\sqrt{d+ex}\left(ax^2+bx+c\right)^{3/2}}{7a}-\frac{\frac{2\sqrt{d+ex}\sqrt{ax^2+bx+c}\left(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2\right)}{15ae^2}-\frac{2\int \frac{2a^2(2bd-ce)d^2+4b^2e^2(bd+ce)-}{}}{}}{7a}\right.}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2\sqrt{d+ex}\left(ax^2+bx+c\right)^{3/2}}{7a}-\frac{\frac{2\sqrt{d+ex}\sqrt{ax^2+bx+c}\left(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2\right)}{15ae^2}-\frac{\int \frac{2\left(a^2(2bd-ce)d^2+2b^2e^2(bd+ce)-\right.}{}}{}}{7a}\right.}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 1269$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2\sqrt{d+ex}\left(ax^2+bx+c\right)^{3/2}}{7a}-\frac{2\sqrt{d+ex}\sqrt{ax^2+bx+c}\left(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2\right)}{15ae^2}-\frac{\left(8a^3d^3-a^2de(5bd-16ce)-abe^2(5b\right.}{e}\right.}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 1172$$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2\sqrt{d+ex}\left(ax^2+bx+c\right)^{3/2}}{7a}-\frac{2\sqrt{d+ex}\sqrt{ax^2+bx+c}\left(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2\right)}{15ae^2}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{a\left(ax^2+bx-\right.}{b^2-4ac}}}{}\right.$$

$$\Big\downarrow \; 321$$

3.81. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2\sqrt{d+ex}\left(ax^2+bx+c\right)^{3/2}}{7a}-\frac{\frac{2\sqrt{d+ex}\sqrt{ax^2+bx+c}\left(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2\right)}{15ae^2}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{a\left(ax^2+bx-\right.}{b^2-4ac}}}{}}{}\right.$$

$\downarrow$ 327

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2\sqrt{d+ex}\left(ax^2+bx+c\right)^{3/2}}{7a}-\frac{\frac{2\sqrt{d+ex}\sqrt{ax^2+bx+c}\left(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2\right)}{15ae^2}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{a\left(ax^2+bx-\right.}{b^2-4ac}}}{}}{}\right.$$

input
```
Int[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
```

3.81. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$

output
```
(Sqrt[a + c/x^2 + b/x]*x*((2*Sqrt[d + e*x]*(c + b*x + a*x^2)^(3/2))/(7*a)
- ((2*Sqrt[d + e*x]*(4*a^2*d^2 + 4*b^2*e^2 - a*e*(2*b*d - 5*c*e) - 3*a*e*(
a*d - 4*b*e)*x)*Sqrt[c + b*x + a*x^2])/(15*a*e^2) - ((Sqrt[2]*Sqrt[b^2 - 4
*a*c]*(8*a^3*d^3 + 8*b^3*e^3 - a^2*d*e*(5*b*d - 16*c*e) - a*b*e^2*(5*b*d +
 29*c*e))*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2
]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*e*Sq
rt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[c + b*x + a*x^2
]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d^2 - b*d*e + c*e^2)*(8*a^2*d^2 - a*b
*d*e - 4*b^2*e^2 + 10*a*c*e^2)*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*e*Sqrt[d + e*x]
*Sqrt[c + b*x + a*x^2]))/(15*a*e^2))/(7*a)))/Sqrt[c + b*x + a*x^2]
```

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1897
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p])   Int[x^(m
- 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
 d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
  !IntegerQ[q] && PosQ[n]
```

3.81. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$

3.81.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2661 vs. 2(572) = 1144.

Time = 1.84 (sec) , antiderivative size = 2662, normalized size of antiderivative = 4.19

method	result	size
risch	Expression too large to display	2662
default	Expression too large to display	6302

input
```
int(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(-15*a^2*e^2*x^2-3*a^2*d*e*x-3*a*b*e^2*x+4*a^2*d^2-2*a*b*d*e-10*a*c
*e^2+4*b^2*e^2)*(e*x+d)^(1/2)/a^2/e^2*((a*x^2+b*x+c)/x^2)^(1/2)*x+1/105/e^
2/a^2*(8*a^2*b*d^3*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-
1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-
1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))
/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+
b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/
2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a
*c+b^2)^(1/2))/a))^(1/2))+8*b^3*d*e^2*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)
*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*
c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+
(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x
^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*
(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-
1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+8*b^2*c*e^3*(1/e*d-1/2*(b+(-4
*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)
*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)
)^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2
))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x
+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4...
```

3.81.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.94

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx =$$
$$-\frac{2\left((8\,a^4d^4-9\,a^3bd^3e-2\,(2\,a^2b^2-11\,a^3c)d^2e^2-(9\,ab^3-41\,a^2bc)de^3+(8\,b^4-41\,ab^2c+30\,a^2c^2)e^4)\sqrt{\right.}{}$$

input
```
integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*((8*a^4*d^4 - 9*a^3*b*d^3*e - 2*(2*a^2*b^2 - 11*a^3*c)*d^2*e^2 - (9
*a*b^3 - 41*a^2*b*c)*d*e^3 + (8*b^4 - 41*a*b^2*c + 30*a^2*c^2)*e^4)*sqrt(a
*e)*weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e
^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3
 - 9*a*b*c)*e^3)/(a^3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e)) + 3*(8*a^4*d^
3*e - 5*a^3*b*d^2*e^2 - (5*a^2*b^2 - 16*a^3*c)*d*e^3 + (8*a*b^3 - 29*a^2*b
*c)*e^4)*sqrt(a*e)*weierstrassZeta(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*
e^2)/(a^2*e^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e
^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*e^3), weierstrassPInverse(4/3*(a^2*d^2 -
a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e -
 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*e^3), 1/3*(3*a*e*
x + a*d + b*e)/(a*e))) - 3*(15*a^4*e^4*x^3 + 3*(a^4*d*e^3 + a^3*b*e^4)*x^2
 - 2*(2*a^4*d^2*e^2 - a^3*b*d*e^3 + (2*a^2*b^2 - 5*a^3*c)*e^4)*x)*sqrt(e*x
 + d)*sqrt((a*x^2 + b*x + c)/x^2))/(a^4*e^4)
```

3.81.6 Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx = \int x^2\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
integrate(x**2*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

output
```
Integral(x**2*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

3.81.7 Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^2\,dx$$

input
```
integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2, x)
```

3.81.8 Giac [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^2\,dx$$

input
```
integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2, x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx=\int x^2\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
int(x^2*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
```

output
```
int(x^2*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.82 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx$

3.82.1 Optimal result

Integrand size = 27, antiderivative size = 550

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{15ae}+\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(a^2d^2+b^2e^2-ae(bd+3ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\Bigg(\arcsin\Bigg(\frac{\sqrt{\frac{b+\sqrt{b^2-4}}{\sqrt{b^2-}}}}{\sqrt{2}}}{15a^2e^2\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\,\mathrm{EllipticF}}{15a^2e^2\sqrt{d+ex}\,(c+bx+ax^2)}$$

output
```
2/5*x*(e*x+d)^(3/2)*(a+c/x^2+b/x)^(1/2)/e-2/15*(2*a*d-b*e)*x*(a+c/x^2+b/x)
^(1/2)*(e*x+d)^(1/2)/a/e-2/15*(a^2*d^2+b^2*e^2-a*e*(b*d+3*c*e))*x*Elliptic
E(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*
e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*
a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c
+b^2))^(1/2)/a^2/e^2/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/
2))))^(1/2)+2/15*(2*a*d-b*e)*(a*d^2-e*(b*d-c*e))*x*EllipticF(1/2*((b+2*a*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^
(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*
(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a^2/e^2/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

3.82.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.19 (sec) , antiderivative size = 1051, normalized size of antiderivative = 1.91

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx = \frac{1}{15}x\sqrt{d+ex}\sqrt{a+\frac{c+bx}{x^2}}\left(\frac{2b}{a}+\frac{2d}{e}+6x\right.$$

$$-\frac{(d+ex)\left(\frac{4e^2\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}(a^2d^2+b^2e^2-ae(bd+3ce))(c+x(b+ax))}{(d+ex)^2}-\frac{i\sqrt{2}\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)(a^2d^2+b^2e^2-ae(bd+3}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]
```

3.82. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx$

output
```
(x*Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]*((2*b)/a + (2*d)/e + 6*x - ((d +
e*x)*((4*e^2*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4
*a*c)*e^2])]*(a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*(c + x*(b + a*x)))/(d
 + e*x)^2 - (I*Sqrt[2]*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(a^2*d^2 +
b^2*e^2 - a*e*(b*d + 3*c*e))*Sqrt[(-2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] +
2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*a*d - b*e + S
qrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e
^2] - 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*a*d +
 b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*S
qrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqr
t[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqr
t[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(b^2*e^2*(-(b*e) + Sqr
t[(b^2 - 4*a*c)*e^2]) + a^2*d*(-8*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e
*(2*b^2*d*e + 4*b*c*e^2 - b*d*Sqrt[(b^2 - 4*a*c)*e^2] - 3*c*e*Sqrt[(b^2 -
4*a*c)*e^2]))*Sqrt[(-2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*a*d*e*x + e*S
qrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*
c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*a*d*e*x
 + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*a*d + b*e + Sqrt[(b^
2 - 4*a*c)*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*
d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]],...
```

3.82.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1897, 1162, 1236, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}dx$$

$$\Big\downarrow \text{1897}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\int \sqrt{d+ex}\sqrt{ax^2+bx+c}dx}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{1162}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2(d+ex)^{3/2}\sqrt{ax^2+bx+c}}{5e}-\frac{\int \frac{\sqrt{d+ex}(bd-2ce+(2ad-be)x)}{\sqrt{ax^2+bx+c}}dx}{5e}\right)}{\sqrt{ax^2+bx+c}}$$

$\downarrow$ 1236

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2(d+ex)^{3/2}\sqrt{ax^2+bx+c}}{5e}-\frac{\frac{2\int \frac{ad(bd-8ce)+be(bd+ce)+2\left(a^2d^2+b^2e^2-ae(bd+3ce)\right)x}{2\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx}{3a}+\frac{2\sqrt{d+ex}\sqrt{ax^2+bx+c}(2ad-be)}{3a}}{5e}\right)}{\sqrt{ax^2+bx+c}}$$

$\downarrow$ 27

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2(d+ex)^{3/2}\sqrt{ax^2+bx+c}}{5e}-\frac{\frac{\int \frac{ad(bd-8ce)+be(bd+ce)+2\left(a^2d^2+b^2e^2-ae(bd+3ce)\right)x}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx}{3a}+\frac{2\sqrt{d+ex}\sqrt{ax^2+bx+c}(2ad-be)}{3a}}{5e}\right)}{\sqrt{ax^2+bx+c}}$$

$\downarrow$ 1269

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2(d+ex)^{3/2}\sqrt{ax^2+bx+c}}{5e}-\frac{\frac{\frac{2\left(a^2d^2-ae(bd+3ce)+b^2e^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{ax^2+bx+c}}dx}{e}-\frac{(2ad-be)\left(ad^2-e(bd-ce)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx}{e}}{3a}+\cdots}{5e}\right.}{\sqrt{ax^2+bx+c}}$$

$\downarrow$ 1172

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2(d+ex)^{3/2}\sqrt{ax^2+bx+c}}{5e}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\left(a^2d^2-ae(bd+3ce)+b^2e^2\right)\int \frac{\sqrt{\frac{e\left(b+2ax+\sqrt{b^2-4ac}\right)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2ax+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\sqrt{\frac{b+2\cdots}{\cdots}}}{ae\sqrt{ax^2+bx+c}\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}\cdots}{\cdots}\right.$$

$\downarrow$ 321

3.82. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2(d+ex)^{3/2}\sqrt{ax^2+bx+c}}{5e}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(a^2d^2-ae(bd+3ce)+b^2e^2\right)\int\frac{\sqrt{\frac{e\left(b+2ax+\sqrt{b^2-4ac}\right)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2ax+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\cdots}{ae\sqrt{ax^2+bx+c}\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}}{\cdots}\right.$$

$\downarrow$ 327

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2(d+ex)^{3/2}\sqrt{ax^2+bx+c}}{5e}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(a^2d^2-ae(bd+3ce)+b^2e^2\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\cdots\right.}{ae\sqrt{ax^2+bx+c}\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}}{\cdots}\right.$$

input
```
Int[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]
```

output
```
(Sqrt[a + c/x^2 + b/x]*x*((2*(d + e*x)^(3/2)*Sqrt[c + b*x + a*x^2])/(5*e)
- ((2*(2*a*d - b*e)*Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2])/(3*a) + ((2*Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*Sqrt[d + e*x]
*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*
c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*e*Sqrt[(a*(d + e*x))/(2*a*d
 - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[c + b*x + a*x^2]) - (2*Sqrt[2]*Sqrt[b^
2 - 4*a*c]*(2*a*d - b*e)*(a*d^2 - e*(b*d - c*e))*Sqrt[(a*(d + e*x))/(2*a*d
 - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(a*e*Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2]))/(3*a))/(5*e)))/Sqrt[c + b*x + a
*x^2]
```

3.82. $\int\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx$

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1897
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p])   Int[x^(m
- 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
 d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
  !IntegerQ[q] && PosQ[n]
```

3.82.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1710 vs. $2(486) = 972$.

Time = 1.67 (sec) , antiderivative size = 1711, normalized size of antiderivative = 3.11

method	result	size
risch	Expression too large to display	1711
default	Expression too large to display	4361

input
```
int(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*a*e*x+a*d+b*e)*(e*x+d)^(1/2)/a/e*((a*x^2+b*x+c)/x^2)^(1/2)*x-1/15/
a/e*(2*a*b*d^2*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*
(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*
d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/
(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*
x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/
a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b
^2)^(1/2))/a))^(1/2))+2*b^2*d*e*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1
/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)
^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*
c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d
*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4
*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-
1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+2*b*c*e^2*(1/e*d-1/2*(b+(-4*a*c+b^2
)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2
*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*
((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(
1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/
(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(
1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))-16*a*c*d*e*(1/e...
```

3.82.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.89

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx = \frac{2\left((2\,a^3d^3-3\,a^2bd^2e-3\,(ab^2-6\,a^2c)de^2+(2\,b^3-9\,abc)e^3)\sqrt{ae}\,\text{weierstrassPInverse}\left(\frac{4\,(a^2d^2-abde+(b^2-3\,ac)}{3\,a^2e^2}\right.\right.}{}$$

input
```
integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/45*((2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*
a*b*c)*e^3)*sqrt(a*e)*weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 -
3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2
*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a
*e)) + 6*(a^3*d^2*e - a^2*b*d*e^2 + (a*b^2 - 3*a^2*c)*e^3)*sqrt(a*e)*weier
strassZeta(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2
*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e
^3)/(a^3*e^3), weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*
e^2)/(a^2*e^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e
^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e))) +
 3*(3*a^3*e^3*x^2 + (a^3*d*e^2 + a^2*b*e^3)*x)*sqrt(e*x + d)*sqrt((a*x^2 +
 b*x + c)/x^2))/(a^3*e^3)
```

3.82.6 Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx=\int x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
integrate(x*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

output
```
Integral(x*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

3.82.7 Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\,dx$$

input
```
integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x, x)
```

3.82.8 Giac [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx = \int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\,dx$$

input
```
integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x, x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx = \int x\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
int(x*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
```

output
```
int(x*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.83 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx$

3.83.1 Optimal result

Integrand size = 26, antiderivative size = 955

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx=\frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4a\cdots}}{2ad-\left(b+\sqrt{b^2\cdots}\right.}\right.}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}d(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4\cdots}}{\sqrt{b^2-\cdots}}}}{\sqrt{2}}\cdots\right.\right.}{3ae\sqrt{d+ex}(c+bx+ax^2)}$$

$$+\frac{4\sqrt{2}\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac\cdots}}{\sqrt{b^2-4a\cdots}}}}{\sqrt{2}}\cdots\right.\right.}{3a\sqrt{d+ex}(c+bx+ax^2)}$$

$$-\frac{\sqrt{2}c\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\text{EllipticPi}\left(\cdots\right.}{\sqrt{a}(c+bx+ax^2)}$$

output
```
2/3*x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+1/3*(a*d+b*e)*x*EllipticE(1/2*((b+
2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+
b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(
1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/
2)/a/e/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/
3*d*(a*d+b*e)*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2
))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+
c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/
a/e/(a*x^2+b*x+c)/(e*x+d)^(1/2)+4/3*(b*d+c*e)*x*EllipticF(1/2*((b+2*a*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/
2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+
c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e
*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a/(a*x^2+b*x+c)/(e*x+d)^(1/2)-c*x*Elliptic
Pi(2^(1/2)*a^(1/2)*(e*x+d)^(1/2)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/
2*(2*a*d-b*e+e*(-4*a*c+b^2)^(1/2))/a/d,((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-
2*a*d/e+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(a+c/x^2+b/x)^(1/2)*(1-2*a*(e*
x+d)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a*d-e*(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*
x+c)/a^(1/2)
```

3.83.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.32 (sec) , antiderivative size = 1258, normalized size of antiderivative = 1.32

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \frac{2}{3}x\sqrt{d+ex}\sqrt{a+\frac{c+bx}{x^2}} + \frac{x(d+ex)^{3/2}\sqrt{a+\frac{c+bx}{x^2}}\left(\frac{4e^2(ad+be)\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{\left(b^2-4ac\right)e^2}}}(c+x(b+ax))}{(d+ex)^2} - \frac{i\sqrt{2}(ad+be)\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{\frac{-2ce^2+2a\cdots}{\left(2a\cdots\right.}}\cdots}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
```

output
```
(2*x*Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2])/3 + (x*(d + e*x)^(3/2)*Sqrt[a
+ (c + b*x)/x^2]*((4*e^2*(a*d + b*e)*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))/(d + e*x)^2 - (I*S
qrt[2]*(a*d + b*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*c*e^2
+ 2*a*d*e*x + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d -
 b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*
e*(-d + e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b
*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -
((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c
)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(b*e*(-(b*e) + Sqrt[(b^2 - 4*a*c)*e^
2]) + a*(3*b*d*e - 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*Sqrt[(-2*c*e^2 +
2*a*d*e*x + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b
*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*
(-d + e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d
*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((
-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*
e^2]))])/Sqrt[d + e*x] + ((6*I)*Sqrt[2]*a*c*e^2*Sqrt[(-2*c*e^2 + 2*a*d*e*x
 + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + S...
```

3.83.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 857, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1779, 1272, 25, 2154, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}dx$$

$$\Big\downarrow \text{1779}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\int \frac{\sqrt{d+ex}\sqrt{ax^2+bx+c}}{x}dx}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{1272}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2}{3}\sqrt{d+ex}\sqrt{ax^2+bx+c}-\frac{1}{3}\int -\frac{(ad+be)x^2+2(bd+ce)x+3cd}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 25$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{3}\int \frac{(ad+be)x^2+2(bd+ce)x+3cd}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+\frac{2}{3}\sqrt{d+ex}\sqrt{ax^2+bx+c}\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 2154$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{3}\left(3cd\int \frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+\int \frac{2bd+2ce+(ad+be)x}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx\right)+\frac{2}{3}\sqrt{d+ex}\sqrt{ax^2+bx+c}\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 1269$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{3}\left(-\frac{(ad^2-e(bd+2ce))\int \frac{1}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx}{e}+3cd\int \frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+\frac{(ad+be)\int \frac{\sqrt{d+ex}}{\sqrt{ax^2+bx+c}}dx}{e}\right)+\frac{2}{3}\sqrt{\right.}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \; 1172$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{3}\left(-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,(ad^2-e(bd+2ce))\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2ax+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2ax+\sqrt{b^2-4ac}\right)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}}}{ae\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.}{}$$

$$\Big\downarrow \; 321$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{3}\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,(ad+be)\int \frac{\sqrt{\frac{e\left(b+2ax+\sqrt{b^2-4ac}\right)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2ax+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ae\sqrt{ax^2+bx+c}\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}+3cd\int \frac{1}{x\sqrt{d+ex}\sqrt{ax^2\right.\right.}{}$$

$$\Big\downarrow \; 327$$

3.83. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{3}\left(3cd\int\frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\left(ad^2-e(bd+2ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\Bigg(}{ae\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.$$

$$\Big\downarrow \ 1279$$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{3}\left(\frac{3cd\sqrt{-\sqrt{b^2-4ac}+2ax+b}\sqrt{\sqrt{b^2-4ac}+2ax+b}\int\frac{1}{x\sqrt{b+2ax-\sqrt{b^2-4ac}}\sqrt{b+2ax+\sqrt{b^2-4ac}}\sqrt{d+ex}}dx}{\sqrt{ax^2+bx+c}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{a(}{}}}{}\right.\right.$$

$$\Big\downarrow \ 187$$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{3}\left(-\frac{6cd\sqrt{-\sqrt{b^2-4ac}+2ax+b}\sqrt{\sqrt{b^2-4ac}+2ax+b}\int-\frac{1}{ex\sqrt{b+\frac{2a(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2ad}{e}}\sqrt{b+\frac{2a(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2ad}{e}}}d\sqrt{d+}}{\sqrt{ax^2+bx+c}}\right.\right.$$

$$\Big\downarrow \ 413$$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{3}\left(-\frac{6cd\sqrt{-\sqrt{b^2-4ac}+2ax+b}\sqrt{\sqrt{b^2-4ac}+2ax+b}\sqrt{1-\frac{2a(d+ex)}{2ad-e\left(b-\sqrt{b^2-4ac}\right)}}\int-\frac{1}{ex\sqrt{b+\frac{2a(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2ad}{e}}\sqrt{1-\frac{2}{2ad-(}}}}{\sqrt{ax^2+bx+c}\sqrt{-\sqrt{b^2-4ac}+\frac{2a(d+ex)}{e}-\frac{2ad}{e}+b}}\right.\right.$$

$$\Big\downarrow \ 413$$

3.83. $\int\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{3}\left(-\frac{6cd\sqrt{-\sqrt{b^2-4ac}+2ax+b}\sqrt{\sqrt{b^2-4ac}+2ax+b}\sqrt{1-\frac{2a(d+ex)}{2ad-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\int -\frac{\dots}{ex\sqrt{1-\frac{2a(d\dots}{2ad-\left(b-\sqrt{\dots}\right.}}}}{\sqrt{ax^2+bx+c}\sqrt{-\sqrt{b^2-4ac}+\frac{2a(d+ex)}{e}-\frac{2ad}{e}+b}\sqrt{\sqrt{b^2-4ac}+\frac{2a(d+ex)}{e}-\frac{2ad}{e}+\dots}}\right.\right.$$

$\downarrow$ 412

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{1}{3}\left(\frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{d+ex}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{ax^2+bx+c}}-\frac{2\sqrt{2}\sqrt{b^2-4\dots}}{\dots}\right.\right.$$

input
```
Int[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
```

output
```
(Sqrt[a + c/x^2 + b/x]*x*((2*Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2])/3 + ((Sq
rt[2]*Sqrt[b^2 - 4*a*c]*(a*d + b*e)*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[
b^2 - 4*a*c])*e)])/(a*e*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[c + b*x + a*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*
d + 2*c*e))*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-
((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*e*Sqrt[d + e*x]*Sqrt[c + b*x + a*x
^2]) - (3*Sqrt[2]*c*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[b - Sqrt[
b^2 - 4*a*c] + 2*a*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*a*x]*Sqrt[1 - (2*a*(d
 + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*
a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a
*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - S
qrt[b^2 - 4*a*c])*e]], (2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*a*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(Sqrt[a]*Sqrt[c + b*x + a*x^2]*Sqrt[b - Sqrt[b^2 -
4*a*c] - (2*a*d)/e + (2*a*(d + e*x))/e]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*a*
d)/e + (2*a*(d + e*x))/e]))/3))/Sqrt[c + b*x + a*x^2]
```

3.83. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx$

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

3.83. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1272
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Simp[1/(e*(2*m + 5))   Int[((d +
 e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
 + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1779
```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_
)^(n_.))^(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + b/x^n + c/x^(2*
n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p])   Int[((d + e*x^n)^q*(
c + b*x^n + a*x^(2*n))^p)/x^(2*n*p), x], x] /; FreeQ[{a, b, c, d, e, n, p,
q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] &
& PosQ[n]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.83. $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx$

3.83.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3022 vs. 2(836) = 1672.

Time = 0.45 (sec) , antiderivative size = 3023, normalized size of antiderivative = 3.17

method	result	size
default	Expression too large to display	3023

input
```
int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((a*x^2+b*x+c)/x^2)^(1/2)*x*(e*x+d)^(1/2)*(2^(1/2)*(-(e*x+d)*a/(e*(-4*
a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*d*a-b
*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c
+b^2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2
)^(1/2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*a-b*e+e*
(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*d^2*e-2^(1/2)*(-(e*x+d)*a
/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/
(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e
*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4
*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*
a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e^2-2*2^(1/2)*(
-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(
1/2)-b)*e/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(
1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d
)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*d*a+
b*e)/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e^3+3*2
^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*
c+b^2)^(1/2)-b)*e/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*
c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticF(2^(1/2)*
(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(...
```

3.83.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.83.6 Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \int \sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

3.83.7 Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2), x)
```

3.83.8 Giac [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx=\int \sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
int((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
```

output
```
int((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.84 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx$

3.84.1 Optimal result

Integrand size = 29, antiderivative size = 929

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}$$

$$+\frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$

$$-\frac{3\sqrt{2}\sqrt{b^2-4ac}d\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\ \cdots}{\sqrt{d+ex}(c+bx+ax^2)}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}\cdots}{\sqrt{b^2-4a\cdots}}}}{\sqrt{2}}\cdots\right.\right.}{a\sqrt{d+ex}(c+bx+ax^2)\cdots}$$

$$-\frac{(bd+ce)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\text{Ellipti}\cdots}{\sqrt{2}\sqrt{a}d(c+bx+ax^2)\cdots}$$

output
```
-(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+3/2*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a
*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2
)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/(a*x^2+b*x+c)*2^(1/2
)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-3*d*x*EllipticF(1/2*(
(b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a
*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2
)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+
d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*x+c)/(e*x+d)^(1/2)+2*(
a*d+b*e)*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^
(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-
4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a/(a*
x^2+b*x+c)/(e*x+d)^(1/2)-1/2*(b*d+c*e)*x*EllipticPi(2^(1/2)*a^(1/2)*(e*x+d
)^(1/2)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(2*a*d-b*e+e*(-4*a*c+b^
2)^(1/2))/a/d,((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*d/e+(-4*a*c+b^2)^(1/2
)))^(1/2))*(a+c/x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/
2))))^(1/2)*(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e
*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/d/(a*x^2+b*x+c)*2^(1/2)/a^(1/2)
```

3.84.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.80 (sec) , antiderivative size = 1207, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = \frac{1}{4}\sqrt{d+ex}\sqrt{a+\frac{c+bx}{x^2}}\left(-4\right.$$

$$+\frac{x(d+ex)\left(\frac{12de^2\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{\left(b^2-4ac\right)e^2}}}(c+x(b+ax))}{(d+ex)^2}-\frac{3i\sqrt{2}d\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{\frac{-2ce^2+2adex+be(d-ex)+\sqrt{\left(b^2-4ac\right)e^2}(d}{\left(2ad-be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}}}{}\right.}{}$$

input
```
Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

3.84. $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx$

output
```
(Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]*(-4 + (x*(d + e*x)*((12*d*e^2*Sqrt[
(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c +
x*(b + a*x)))/(d + e*x)^2 - ((3*I)*Sqrt[2]*d*(2*a*d - b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*Sqrt[(-2*c*e^2 + 2*a*d*e*x + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)
*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt
[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x)
)/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSin
h[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)
*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d
 - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(4*a*d^2 -
 b*d*e - 2*c*e^2 + 3*d*Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*c*e^2 + 2*a*d*e*x
 + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt
[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x
) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)
*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^
2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d +
b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/
Sqrt[d + e*x] + ((2*I)*Sqrt[2]*e*(b*d + c*e)*Sqrt[(-2*c*e^2 + 2*a*d*e*x +
b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b
^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x...
```

3.84.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 843, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {1897, 1271, 2154, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}dx$$

$$\Big\downarrow \text{1897}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\int \frac{\sqrt{d+ex}\sqrt{ax^2+bx+c}}{x^2}dx}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{1271}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{2}\int \frac{3aex^2+2(ad+be)x+bd+ce}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx-\frac{\sqrt{d+ex}\sqrt{ax^2+bx+c}}{x}\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{2154}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{2}\left((bd+ce)\int\frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+\int\frac{2ad+2be+3aex}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx\right)-\frac{\sqrt{d+ex}\sqrt{ax^2+bx+c}}{x}\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{1269}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{2}\left(-\left((ad-2be)\int\frac{1}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx\right)+(bd+ce)\int\frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+3a\int\frac{\sqrt{d+ex}}{\sqrt{ax^2+bx+c}}dx\right)-\right.}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{1172}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{2}\left(-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}(ad-2be)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\int\frac{1}{\sqrt{1-\frac{b+2ax+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2ax+\sqrt{b^2-4ac}\right)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{b+\cdots}}{}}{a\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.}{}$$

$$\Big\downarrow \text{321}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{2}\left(\frac{3\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\int\frac{\sqrt{\frac{e\left(b+2ax+\sqrt{b^2-4ac}\right)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2ax+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{\sqrt{ax^2+bx+c}\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}+(bd+ce)\int\frac{1}{x\sqrt{d+ex}\sqrt{ax^2\cdots}}\right.\right.}{}$$

$$\Big\downarrow \text{327}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{2}\left((bd+ce)\int\frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}(ad-2be)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(a\cdots\right.}{a\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.}{}$$

$$\Big\downarrow \text{1279}$$

3.84. $\displaystyle\int\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{2}\left(\frac{\sqrt{-\sqrt{b^2-4ac}+2ax+b}\sqrt{\sqrt{b^2-4ac}+2ax+b}(bd+ce)\int\frac{1}{x\sqrt{b+2ax-\sqrt{b^2-4ac}}\sqrt{b+2ax+\sqrt{b^2-4ac}}\sqrt{d+ex}}dx}{\sqrt{ax^2+bx+c}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{}}{}\right.\right.$$

$\downarrow$ 187

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{2}\left(-\frac{2\sqrt{-\sqrt{b^2-4ac}+2ax+b}\sqrt{\sqrt{b^2-4ac}+2ax+b}(bd+ce)\int-\frac{1}{ex\sqrt{b+\frac{2a(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2ad}{e}}\sqrt{b+\frac{2a(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2ad}{e}}}d}{\sqrt{ax^2+bx+c}}\right.\right.$$

$\downarrow$ 413

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{2}\left(-\frac{2\sqrt{-\sqrt{b^2-4ac}+2ax+b}\sqrt{\sqrt{b^2-4ac}+2ax+b}(bd+ce)\sqrt{1-\frac{2a(d+ex)}{2ad-e\left(b-\sqrt{b^2-4ac}\right)}}\int-\frac{1}{ex\sqrt{b+\frac{2a(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2ad}{e}}\sqrt{1-}}}{\sqrt{ax^2+bx+c}\sqrt{-\sqrt{b^2-4ac}+\frac{2a(d+ex)}{e}-\frac{2ad}{e}+b}}\right.\right.$$

$\downarrow$ 413

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{2}\left(-\frac{2\sqrt{-\sqrt{b^2-4ac}+2ax+b}\sqrt{\sqrt{b^2-4ac}+2ax+b}(bd+ce)\sqrt{1-\frac{2a(d+ex)}{2ad-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\int-\frac{}{ex\sqrt{1-\frac{}{2ad-}}}}{\sqrt{ax^2+bx+c}\sqrt{-\sqrt{b^2-4ac}+\frac{2a(d+ex)}{e}-\frac{2ad}{e}+b}\sqrt{\sqrt{b^2-4ac}+\frac{2a(d+ex)}{e}-\frac{2a}{e}}}\right.\right.$$

$\downarrow$ 412

3.84. $\int\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx$

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{1}{2}\left(\frac{3\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{ax^2+bx+c}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}(ad-\right.\right.$$

input
```
Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

output
```
(Sqrt[a + c/x^2 + b/x]*x*(-((Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2])/x) + ((3
*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2
 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[c
+ b*x + a*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d - 2*b*e)*Sqrt[(a*(d +
e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b
^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4
*a*c])*e)])/(a*Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2]) - (Sqrt[2]*(b*d + c*e)
*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*a*
x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*a*x]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (
b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2
 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), Arc
Sin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e
]], (2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(Sqrt[a]*d*Sqrt[c + b*x + a*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*a*d)/
e + (2*a*(d + e*x))/e]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e + (2*a*(d +
e*x))/e]))/2))/Sqrt[c + b*x + a*x^2]
```

3.84.3.1 Defintions of rubi rules used

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.84. $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx$

rule 1271
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((d + e*x
)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f +
b*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[2*m] && LtQ[m, -1]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1897
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p])   Int[x^(m
- 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
 d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
  !IntegerQ[q] && PosQ[n]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.84.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 1400, normalized size of antiderivative = 1.51

method	result	size
risch	Expression too large to display	1400
default	Expression too large to display	3553

input
```
int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-((a*x^2+b*x+c)/x^2)^(1/2)*(e*x+d)^(1/2)+(2*d*a*(1/e*d-1/2*(b+(-4*a*c+b^2)
^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*
(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*(
(x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1
/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(
1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1
/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+2*b*e*(1/e*d-1/2*(b
+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(
1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2)
)/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^
(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF
(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-
4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+3*a*e*
(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)
^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a
*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+
(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1
/2)*((-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)*EllipticE(((x+1/e*d)/(1/e*d-1/
2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/
(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+1/2*(-b+(-4*a*c+b^2)^(1/...
```

3.84.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="fricas")
```

output
```
Timed out
```

3.84.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = \int \frac{\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

input
```
integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x,x)
```

output
```
Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2)/x, x)
```

3.84.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = \int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

input
```
integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x, x)
```

3.84.8 Giac [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = \int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

input
```
integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x, x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx=\int \frac{\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

input
```
int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x,x)
```

output
```
int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x, x)
```

3.85 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$

3.85.1 Optimal result

Integrand size = 29, antiderivative size = 1287

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd}-\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}$$

$$+\frac{\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)\cdots}\right)}{4\sqrt{2}cd\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$

$$+\frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{\cdots}{2a\cdots}\right)}{\sqrt{2}\sqrt{d+ex}(c+bx+ax^2)}$$

$$-\frac{\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\cdots\right)}{2\sqrt{2}c\sqrt{d+ex}(c+bx+ax^2)}$$

$$-\frac{(ad+be)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\mathrm{Ellipt}\cdots}{\sqrt{2}\sqrt{a}d(c+bx+ax^2)}$$

$$+\frac{(bd+ce)^2\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\mathrm{Ellipt}\cdots}{4\sqrt{2}\sqrt{a}cd^2(c+bx+ax^2)}$$

output
```
-1/4*(b*d+c*e)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/c/d-1/2*(a+c/x^2+b/x)^(1/
2)*(e*x+d)^(1/2)/x+1/8*(b*d+c*e)*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(
b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x
+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/c/d/(a*x^2+b*x+c)*2^(1/2)/
(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+3/2*e*x*EllipticF(1/2*(
(b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a
*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*
(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*x+c)*2^(1/2)/(e*x+d)^(1/2)-1/4
*(b*d+c*e)*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2)))
)^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+
b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/(a*x^2+b*
x+c)*2^(1/2)/(e*x+d)^(1/2)-1/2*(a*d+b*e)*x*EllipticPi(2^(1/2)*a^(1/2)*(e*x
+d)^(1/2)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(2*a*d-b*e+e*(-4*a*c+
b^2)^(1/2))/a/d,((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*d/e+(-4*a*c+b^2)^(1
/2)))^(1/2))*(a+c/x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b-(-4*a*c+b^2)^(
1/2))))^(1/2)*(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*a*(e*x+d)/(2*a*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/d/(a*x^2+b*x+c)*2^(1/2)/a^(1/2)+1/8*(...
```

3.85.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.61 (sec) , antiderivative size = 1392, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = \frac{1}{16}x\sqrt{d+ex}\sqrt{a+\frac{c+bx}{x^2}}\left(-\frac{4(2cd+bdx+cex)}{cdx^2}\right.$$

$$+\frac{(d+ex)\left(\frac{4de^2(bd+ce)\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}(c+x(b+ax))}{(d+ex)^2}-\frac{i\sqrt{2}d(bd+ce)\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{\frac{-2ce^2+d\sqrt{(b^2-4ac)e^2}+2ade\cdots}{\left(2ad-be+\sqrt{(b^2\cdots}\right.}}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

3.85. $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$

output
```
(x*Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]*((-4*(2*c*d + b*d*x + c*e*x))/(c*
d*x^2) + ((d + e*x)*((4*d*e^2*(b*d + c*e)*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/
(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))/(d + e*x)^2 -
 (I*Sqrt[2]*d*(b*d + c*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2
*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]
*x + b*e*(d - e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*S
qrt[(2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c
)*e^2]*x + b*e*(-d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d +
e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d +
 b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x
] + (I*Sqrt[2]*(b^2*d^2*e + b*d*(-5*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]) + c
*e*(4*a*d^2 + 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*Sqrt[(-2*c*e^2 + d*Sqr
t[(b^2 - 4*a*c)*e^2] + 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d -
e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 +
 d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e
*(-d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Ellipti
cF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] - ((2*I)*...
```

3.85.3 Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 1486, normalized size of antiderivative = 1.15, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {1897, 1271, 2154, 1282, 2154, 25, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}dx$$

$$\Big\downarrow \text{ 1897}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\int \frac{\sqrt{d+ex}\sqrt{ax^2+bx+c}}{x^3}dx}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{ 1271}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{4}\int \frac{3aex^2+2(ad+be)x+bd+ce}{x^2\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx-\frac{\sqrt{d+ex}\sqrt{ax^2+bx+c}}{2x^2}\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{2154}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{4}\left((bd+ce)\int \frac{1}{x^2\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+\int \frac{2ad+2be+3aex}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx\right)-\frac{\sqrt{d+ex}\sqrt{ax^2+bx+c}}{2x^2}\right)}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{1282}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{4}\left(\int \frac{2ad+2be+3aex}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+(bd+ce)\left(-\frac{\int \frac{-aex^2+bd+ce}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx}{2cd}-\frac{\sqrt{d+ex}\sqrt{ax^2+bx+c}}{cdx}\right)\right)-\frac{\sqrt{d+ex}\sqrt{ax^2-}}{2x^2}\right.}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{2154}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{4}\left(\int \frac{3ae}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+2(ad+be)\int \frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+(bd+ce)\left(-\frac{(bd+ce)\int \frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx-}}}{}\right.\right.\right.}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{4}\left(\int \frac{3ae}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+2(ad+be)\int \frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+(bd+ce)\left(-\frac{(bd+ce)\int \frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx-}}}{2}\right.\right.\right.}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{4}\left(3ae\int \frac{1}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+2(ad+be)\int \frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+(bd+ce)\left(-\frac{(bd+ce)\int \frac{1}{x\sqrt{d+ex}\sqrt{ax^{}}}}{}\right.\right.\right.}{\sqrt{ax^2+bx+c}}$$

$$\Big\downarrow \text{1172}$$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{4}\left(\frac{6\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2ax+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2ax+\sqrt{b^2-4ac}\right)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.$$

$$\Big\downarrow \text{321}$$

3.85. $\displaystyle\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{4}\left(2(ad+be)\int\frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+(bd+ce)\left(-\frac{(bd+ce)\int\frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx-ae\int\frac{x}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}}{2cd}\right.\right.\right.$$

$$\downarrow \text{1269}$$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{4}\left(2(ad+be)\int\frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx+(bd+ce)\left(-\frac{(bd+ce)\int\frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx-ae\left(\frac{\int\frac{\sqrt{d+ex}}{\sqrt{ax^2+bx+c}}dx}{e}-\right.}{2cd}\right.\right.\right.$$

$$\downarrow \text{1172}$$

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{1}{4}\left(\frac{6\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.$$

$$\downarrow \text{321}$$

3.85. $\int\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{1}{4}\left(\frac{6\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.$$

$\downarrow$ 327

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{1}{4}\left((bd+ce)\left(-\frac{(bd+ce)\int\frac{1}{x\sqrt{d+ex}\sqrt{ax^2+bx+c}}dx-ae\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{ae\sqrt{ax^2+bx+c}\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}\right.}{}\right.\right.\right.$$

$\downarrow$ 1279

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{1}{4}\left(\frac{6\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.$$

$\downarrow$ 187

3.85. $\int\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}dx$

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x\left(\frac{1}{4}\left(\frac{6\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.$$

$\downarrow$ 413

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x\left(\frac{1}{4}\left(\frac{6\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.$$

$\downarrow$ 413

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x\left(\frac{1}{4}\left(\frac{6\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.$$

$\downarrow$ 412

3.85. $\displaystyle\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$

$$\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\left(\frac{1}{4}\left(\frac{6\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{ax^2+bx+c}}\right.\right.$$

input
```
Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

output
```
(Sqrt[a + c/x^2 + b/x]*x*(-1/2*(Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2])/x^2 +
 ((6*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2
 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[d + e*x]*S
qrt[c + b*x + a*x^2]) - (2*Sqrt[2]*(a*d + b*e)*Sqrt[2*a*d - (b - Sqrt[b^2
 - 4*a*c])*e]*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*a*x]*Sqrt[b + Sqrt[b^2 - 4*a*c
] + 2*a*x]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*S
qrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2
*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d
+ e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (2*a*d - (b - Sqrt[b^2 -
 4*a*c])*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[a]*d*Sqrt[c + b*x
+ a*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e + (2*a*(d + e*x))/e]*Sqrt[
b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e + (2*a*(d + e*x))/e]) + (b*d + c*e)*(-((
Sqrt[d + e*x]*Sqrt[c + b*x + a*x^2])/(c*d*x)) - (-(a*e*((Sqrt[2]*Sqrt[b^2
 - 4*a*c]*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Ellipt
icE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*e*Sqr
t[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[c + b*x + a*x^2]
) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqr...
```

3.85. $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

3.85. $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1271
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((d + e*x
)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f +
b*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[2*m] && LtQ[m, -1]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

3.85. $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2} \, dx$

rule 1897
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p])   Int[x^(m
- 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
 d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
  !IntegerQ[q] && PosQ[n]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

3.85.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 1597, normalized size of antiderivative = 1.24

method	result	size
risch	Expression too large to display	1597
default	Expression too large to display	4957

input
```
int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(e*x+d)^(1/2)*(b*d*x+c*e*x+2*c*d)/x/c/d*((a*x^2+b*x+c)/x^2)^(1/2)+1/8
/c/d*(2*e^2*a*c*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2
*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e
*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)
/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d
*x+c*e*x+c*d)^(1/2)*((-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)*EllipticE(((x+
1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c
+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+1/2*(-b+(-4
*a*c+b^2)^(1/2))/a*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/
a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b
^2)^(1/2))/a))^(1/2)))+12*a*c*d*e*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x
+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^
2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*
a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a
*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(
-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*
d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+2*a*b*d*e*(1/e*d-1/2*(b+(-4*a*c+b
^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1
/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2
)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/...
```

3.85.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.85.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = \int \frac{\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

input
```
integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2)/x**2, x)
```

3.85.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = \int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

input
```
integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2, x)
```

3.85.8 Giac [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = \int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

input
```
integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2, x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = \int \frac{\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

input
```
int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x^2,x)
```

output
```
int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x^2, x)
```

3.86 $\int (fx)^m (d+ex^n)^q (a+cx^{2n})^p \, dx$

3.86.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \operatorname{Int}\left((fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p, x\right)$$

output
```
Unintegrable((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

3.86.2 Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

input
```
Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
```

output
```
Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

3.86.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \left(a+cx^{2n}\right)^p (d+ex^n)^q \, dx$$

$$\Big\downarrow \text{1888}$$

$$\int (fx)^m \left(a+cx^{2n}\right)^p (d+ex^n)^q dx$$

input
```
Int[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
```

output
```
$Aborted
```

3.86.3.1 Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

3.86.4 Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (fx)^m (d+e\,x^n)^q \left(a+c\,x^{2n}\right)^p dx$$

input
```
int((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

output
```
int((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

3.86.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (fx)^m (d+ex^n)^q (a+cx^{2n})^p \, dx = \int (cx^{2n}+a)^p (ex^n+d)^q (fx)^m \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^q (a+cx^{2n})^p \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(d+e*x**n)**q*(a+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.86.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (fx)^m (d+ex^n)^q (a+cx^{2n})^p \, dx = \int (cx^{2n}+a)^p (ex^n+d)^q (fx)^m \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

3.86.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int \left(cx^{2n}+a\right)^p (ex^n+d)^q (fx)^m \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

3.86.9 Mupad [N/A]

Not integrable

Time = 8.79 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int \left(a+c\,x^{2\,n}\right)^p\,(f\,x)^m\,(d+e\,x^n)^q \, dx$$

input
```
int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^q,x)
```

output
```
int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^q, x)
```

3.87 $\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p \, dx$

3.87.1 Optimal result

Integrand size = 26, antiderivative size = 358

$$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p \, dx = \frac{d^3(fx)^{1+m}(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{3d^2ex^{1+n}(fx)^m(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+n}{2n},-p,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+n} + \frac{3de^2x^{1+2n}(fx)^m(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+2n}{2n},-p,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+2n} + \frac{e^3x^{1+3n}(fx)^m(a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+3n}{2n},-p,\frac{1+m+5n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+3n}$$

output
```
d^3*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n
],-c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+3*d^2*e*x^(1+n)*(f*x)^m*(a+c*x
^(2*n))^p*hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1
+m+n)/((1+c*x^(2*n)/a)^p)+3*d*e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*n))^p*hyperg
eom([-p, 1/2*(1+m+2*n)/n],[1/2*(1+m+4*n)/n],-c*x^(2*n)/a)/(1+m+2*n)/((1+c*
x^(2*n)/a)^p)+e^3*x^(1+3*n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+
m+3*n)/n],[1/2*(1+m+5*n)/n],-c*x^(2*n)/a)/(1+m+3*n)/((1+c*x^(2*n)/a)^p)
```

3.87.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.70

$$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p \, dx$$
$$= x(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(\frac{d^3 \text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)}{1+m} + ex^n \left(\frac{3d^2 \text{Hypergeometric2F1}\left(\frac{1+m+n}{2n},-p,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+n} + ex^n \left(\frac{3d \text{Hypergeometric2F1}\left(\frac{1+m+2n}{2n},-p,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+2n} + \frac{ex^n \text{Hypergeometric2F1}\left(\frac{1+m+3n}{2n},-p,1+ \right.}{1+m+3n}\right.\right.\right.$$

input
```
Integrate[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

output
```
(x*(f*x)^m*(a + c*x^(2*n))^p*((d^3*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
+ (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((3*d^2*Hypergeometric
2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m
+ n) + e*x^n*((3*d*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n
)/(2*n), -((c*x^(2*n))/a)])/(1 + m + 2*n) + (e*x^n*Hypergeometric2F1[(1 +
m + 3*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + 3*n))
)))/(1 + (c*x^(2*n))/a)^p
```

3.87.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1885, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p \, dx$$
$$\Big\downarrow \text{ 1885}$$

$$\int \left(d^3(fx)^m \left(a+cx^{2n}\right)^p+3d^2ex^n(fx)^m \left(a+cx^{2n}\right)^p+3de^2x^{2n}(fx)^m \left(a+cx^{2n}\right)^p+e^3x^{3n}(fx)^m \left(a+cx^{2n}\right)^p\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3(fx)^{m+1} \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2n},-p,\frac{m+1}{2n}+1,-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{3d^2ex^{n+1}(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+n+1}{2n},-p,\frac{m+3n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{3de^2x^{2n+1}(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+2n+1}{2n},-p,\frac{m+4n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+2n+1}+\frac{e^3x^{3n+1}(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+3n+1}{2n},-p,\frac{m+5n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+3n+1}$$

input
```
Int[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

output
```
(d^3*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p,
1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) +
(3*d^2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)
/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x
^(2*n))/a)^p) + (3*d*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeomet
ric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((
1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p) + (e^3*x^(1 + 3*n)*(f*x)^m*(a + c*x^(2
*n))^p*Hypergeometric2F1[(1 + m + 3*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((
c*x^(2*n))/a)])/((1 + m + 3*n)*(1 + (c*x^(2*n))/a)^p)
```

3.87.3.1 Defintions of rubi rules used

rule 1885
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 &&  !RationalQ[n] && (IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.87.4 Maple [F]

$$\int (fx)^m (d+e\,x^n)^3 \left(a+c\,x^{2n}\right)^p dx$$

input
```
int((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

output
```
int((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

3.87.5 Fricas [F]

$$\int (fx)^m (d+ex^n)^3 \left(a+cx^{2n}\right)^p \,dx = \int (ex^n+d)^3 (cx^{2\,n}+a)^p (fx)^m \,dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x^(2*n) +
a)^p*(f*x)^m, x)
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^3 \left(a+cx^{2n}\right)^p \,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(d+e*x**n)**3*(a+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.87.7 Maxima [F]

$$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p \, dx = \int (ex^n+d)^3 (cx^{2n}+a)^p (fx)^m \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

3.87.8 Giac [F(-2)]

Exception generated.

$$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{96,[1,0,6,4,0,3,5,4,1,2]%%%}+%%%{480,[1,0,6,4,0,3,4,4,1,2]
%%%}+%%%{
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p \, dx = \int (a+c\,x^{2\,n})^p\,(f\,x)^m\,(d+e\,x^n)^3 \,dx$$

input
```
int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^3,x)
```

output
```
int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^3, x)
```

3.88 $\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx$

3.88.1 Optimal result

Integrand size = 26, antiderivative size = 262

$$\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx = \frac{d^2(fx)^{1+m} (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{2dex^{1+n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+n}{2n},-p,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+n} + \frac{e^2x^{1+2n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+2n}{2n},-p,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+2n}$$

output
```
d^2*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n
],-c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+2*d*e*x^(1+n)*(f*x)^m*(a+c*x^(
2*n))^p*hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1+m
+n)/((1+c*x^(2*n)/a)^p)+e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom([-
p, 1/2*(1+m+2*n)/n],[1/2*(1+m+4*n)/n],-c*x^(2*n)/a)/(1+m+2*n)/((1+c*x^(2*n
)/a)^p)
```

3.88.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.72

$$\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx = x(fx)^m \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(\frac{d^2 \text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)}{1+m} + ex^n \left(\frac{2d \text{Hypergeometric2F1}\left(\frac{1+m+n}{2n},-p,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+n} + \frac{ex^n \text{Hypergeometric2F1}\left(\frac{1+m+2n}{2n},-p,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+2n}\right)\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

output
```
(x*(f*x)^m*(a + c*x^(2*n))^p*((d^2*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
+ (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((2*d*Hypergeometric2F
1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m +
n) + (e*x^n*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n)
, -((c*x^(2*n))/a)])/(1 + m + 2*n))))/(1 + (c*x^(2*n))/a)^p
```

3.88.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1885, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx$$

$$\Big\downarrow \text{ 1885}$$

$$\int \left(d^2(fx)^m \left(a+cx^{2n}\right)^p + 2dex^n(fx)^m \left(a+cx^{2n}\right)^p + e^2x^{2n}(fx)^m \left(a+cx^{2n}\right)^p\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^2(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2n},-p,\frac{m+1}{2n}+1,-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{2dex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+n+1}{2n},-p,\frac{m+3n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{e^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2n+1}{2n},-p,\frac{m+4n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+2n+1}$$

input
```
Int[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

output
```
(d^2*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p,
1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) +
(2*d*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(
2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(
2*n))/a)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1
[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m
+ 2*n)*(1 + (c*x^(2*n))/a)^p)
```

3.88.3.1 Defintions of rubi rules used

rule 1885
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 && !RationalQ[n] && (IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.88.4 Maple [F]

$$\int (fx)^m\,(d+e\,x^n)^2\left(a+c\,x^{2n}\right)^p dx$$

input
```
int((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

output
```
int((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

3.88.5 Fricas [F]

$$\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx = \int (ex^n+d)^2 (cx^{2n}+a)^p (fx)^m \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(d+e*x**n)**2*(a+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.88.7 Maxima [F]

$$\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx = \int (ex^n+d)^2 (cx^{2n}+a)^p (fx)^m \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

3.88.8 Giac [F(-2)]

Exception generated.

$$\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{64,[1,0,4,3,0,1,4,3,1,1]%%%}+%%%{256,[1,0,4,3,0,1,3,3,1,1]
%%%}+%%%{
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx = \int \left(a+c\,x^{2\,n}\right)^p\,(f\,x)^m\,\left(d+e\,x^n\right)^2 \,dx$$

input
```
int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^2,x)
```

output
```
int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^2, x)
```

3.89 $\int (fx)^m (d+ex^n) (a+cx^{2n})^p \, dx$

3.89.1 Optimal result

Integrand size = 24, antiderivative size = 166

$$\int (fx)^m (d+ex^n) (a+cx^{2n})^p \, dx = \frac{d(fx)^{1+m} (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{ex^{1+n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+n}{2n},-p,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+n}$$

output
```
d*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n],
-c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+e*x^(1+n)*(f*x)^m*(a+c*x^(2*n))^
p*hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1+m+n)/((
1+c*x^(2*n)/a)^p)
```

3.89.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.82

$$\int (fx)^m (d+ex^n) (a+cx^{2n})^p \, dx = \frac{x(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(d(1+m+n) \text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)+e(1+\right.}{(1+m)(1+m+n)}$$

input
```
Integrate[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

output
```
(x*(f*x)^m*(a + c*x^(2*n))^p*(d*(1 + m + n)*Hypergeometric2F1[(1 + m)/(2*n
), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)] + e*(1 + m)*x^n*Hypergeometric
2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)]))/((1 +
m)*(1 + m + n)*(1 + (c*x^(2*n))/a)^p)
```

3.89.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1885, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx$$

$$\Big\downarrow \text{ 1885}$$

$$\int \left(d(fx)^m \left(a+cx^{2n}\right)^p+ex^n(fx)^m \left(a+cx^{2n}\right)^p\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d(fx)^{m+1} \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2n},-p,\frac{m+1}{2n}+1,-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{ex^{n+1}(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+n+1}{2n},-p,\frac{m+3n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+n+1}$$

input
```
Int[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

output
```
(d*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
+ (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (e
*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n),
-p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n))/
a)^p)
```

3.89.3.1 Defintions of rubi rules used

rule 1885
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 &&  !RationalQ[n] && (IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.89.4 Maple [F]

$$\int (fx)^m \left(d+e\,x^n\right) \left(a+c\,x^{2n}\right)^p dx$$

input
```
int((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x)
```

output
```
int((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x)
```

3.89.5 Fricas [F]

$$\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx = \int (ex^n+d)\left(cx^{2n}+a\right)^p (fx)^m \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

3.89.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(d+e*x**n)*(a+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.89.7 Maxima [F]

$$\int (fx)^m (d + ex^n) \left(a + cx^{2n}\right)^p dx = \int (ex^n + d)\left(cx^{2n} + a\right)^p (fx)^m dx$$

input
```
integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

3.89.8 Giac [F]

$$\int (fx)^m (d + ex^n) \left(a + cx^{2n}\right)^p dx = \int (ex^n + d)\left(cx^{2n} + a\right)^p (fx)^m dx$$

input
```
integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d + ex^n) \left(a + cx^{2n}\right)^p dx = \int \left(a + c\,x^{2\,n}\right)^p (f\,x)^m \left(d + e\,x^n\right) dx$$

input
```
int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n),x)
```

output
```
int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n), x)
```

3.90 $\int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n} \, dx$

3.90.1 Optimal result

Integrand size = 26, antiderivative size = 194

$$\int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n} \, dx = \frac{x(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1+m}{2n},-p,1,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(1+m)} - \frac{ex^{1+n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1+m+n}{2n},-p,1,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+m+n)}$$

output
```
x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,1,-p,1+1/2*(1+m)/n,e^2*x^(2
*n)/d^2,-c*x^(2*n)/a)/d/(1+m)/((1+c*x^(2*n)/a)^p)-e*x^(1+n)*(f*x)^m*(a+c*x
^(2*n))^p*AppellF1(1/2*(1+m+n)/n,1,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,-c*x
^(2*n)/a)/d^2/(1+m+n)/((1+c*x^(2*n)/a)^p)
```

3.90.2 Mathematica [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n} \, dx = \int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n} \, dx$$

input
```
Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
```

output
```
Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n), x]
```

3.90.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1886, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{d+ex^n} \, dx$$

$$\Big\downarrow \text{ 1886}$$

$$x^{-m}(fx)^m \int \left(\frac{dx^m \left(cx^{2n}+a\right)^p}{d^2-e^2x^{2n}}-\frac{ex^{m+n} \left(cx^{2n}+a\right)^p}{d^2-e^2x^{2n}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$x^{-m}(fx)^m \left(\frac{x^{m+1}\left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{2n},-p,1,\frac{m+1}{2n}+1,-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(m+1)} - \frac{ex^{m+n+1}\left(a+cx^{2n}\right.}{}\right.$$

input
```
Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
```

output
```
((f*x)^m*((x^(1 + m)*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 1, 1 +
(1 + m)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*(1 + m)*(1 + (c*x^
(2*n))/a)^p) - (e*x^(1 + m + n)*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*
n), -p, 1, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2
*(1 + m + n)*(1 + (c*x^(2*n))/a)^p)))/x^m
```

3.90.3.1 Defintions of rubi rules used

rule 1886
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Simp[(f*x)^m/x^m   Int[ExpandIntegrand[x^m*(a + c*x^
(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/(d^2 - e^2*x^(2*n))))^(-q), x], x
], x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !RationalQ[
n] &&  !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.90.4 Maple [F]

$$\int \frac{(fx)^m (a + c\,x^{2n})^p}{d + e\,x^n} dx$$

input
```
int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x)
```

output
```
int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x)
```

3.90.5 Fricas [F]

$$\int \frac{(fx)^m (a + cx^{2n})^p}{d + ex^n} dx = \int \frac{(cx^{2n} + a)^p (fx)^m}{ex^n + d} dx$$

input
```
integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a + cx^{2n})^p}{d + ex^n} dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n),x)
```

output
```
Timed out
```

3.90.7 Maxima [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(cx^{2n}+a)^p (fx)^m}{ex^n+d}\,dx$$

input
```
integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

3.90.8 Giac [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(cx^{2n}+a)^p (fx)^m}{ex^n+d}\,dx$$

input
```
integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(a+c\,x^{2\,n})^p\,(f\,x)^m}{d+e\,x^n}\,dx$$

input
```
int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n),x)
```

output
```
int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n), x)
```

3.91 $\int \frac{(fx)^m(a+cx^{2n})^p}{(d+ex^n)^2}\,dx$

3.91.1 Optimal result

Integrand size = 26, antiderivative size = 302

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx = \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{2n},-p,2,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+m)} - \frac{2ex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m+n}{2n},-p,2,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(1+m+n)} + \frac{e^2x^{1+2n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m+2n}{2n},-p,2,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+m+2n)}$$

output
```
x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,2,-p,1+1/2*(1+m)/n,e^2*x^(2
*n)/d^2,-c*x^(2*n)/a)/d^2/(1+m)/((1+c*x^(2*n)/a)^p)-2*e*x^(1+n)*(f*x)^m*(a
+c*x^(2*n))^p*AppellF1(1/2*(1+m+n)/n,2,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,
-c*x^(2*n)/a)/d^3/(1+m+n)/((1+c*x^(2*n)/a)^p)+e^2*x^(1+2*n)*(f*x)^m*(a+c*x
^(2*n))^p*AppellF1(1/2*(1+m+2*n)/n,2,-p,1/2*(1+m+4*n)/n,e^2*x^(2*n)/d^2,-c
*x^(2*n)/a)/d^4/(1+m+2*n)/((1+c*x^(2*n)/a)^p)
```

3.91.2 Mathematica [F]

$$\int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx = \int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

output
```
Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

3.91.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1886, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

$$\Big\downarrow \text{1886}$$

$$x^{-m}(fx)^m \int \left(\frac{d^2\left(cx^{2n}+a\right)^p x^m}{\left(d^2-e^2x^{2n}\right)^2} - \frac{2de\left(cx^{2n}+a\right)^p x^{m+n}}{\left(d^2-e^2x^{2n}\right)^2} + \frac{e^2\left(cx^{2n}+a\right)^p x^{m+2n}}{\left(d^2-e^2x^{2n}\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x^{-m}(fx)^m \left(\frac{x^{m+1}\left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{2n},-p,2,\frac{m+1}{2n}+1,-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+1)} + \frac{e^2x^{m+2n+1}\left(a+cx^{2}\right.}{}\right.$$

input
```
Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

output
```
((f*x)^m*((x^(1 + m)*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 2, 1 +
(1 + m)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m)*(1 + (c*
x^(2*n))/a)^p) - (2*e*x^(1 + m + n)*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)
/(2*n), -p, 2, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/
(d^3*(1 + m + n)*(1 + (c*x^(2*n))/a)^p) + (e^2*x^(1 + m + 2*n)*(a + c*x^(2
*n))^p*AppellF1[(1 + m + 2*n)/(2*n), -p, 2, (1 + m + 4*n)/(2*n), -((c*x^(2
*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p)))/x
^m
```

3.91.3.1 Defintions of rubi rules used

rule 1886
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Simp[(f*x)^m/x^m   Int[ExpandIntegrand[x^m*(a + c*x^
(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/(d^2 - e^2*x^(2*n))))^(-q), x], x
], x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !RationalQ[
n] &&  !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.91.4 Maple [F]

$$\int \frac{(fx)^m \left(a+c\,x^{2n}\right)^p}{\left(d+e\,x^n\right)^2}dx$$

input
```
int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

output
```
int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

3.91.5 Fricas [F]

$$\int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{\left(d+ex^n\right)^2}\,dx = \int \frac{(cx^{2\,n}+a)^p(fx)^m}{\left(ex^n+d\right)^2}\,dx$$

input
```
integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x^n + d^2), x)
```

3.91.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^2} \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

output
```
Timed out
```

3.91.7 Maxima [F]

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^2} \, dx = \int \frac{(cx^{2n} + a)^p (fx)^m}{(ex^n + d)^2} \, dx$$

input
```
integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

3.91.8 Giac [F]

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^2} \, dx = \int \frac{(cx^{2n} + a)^p (fx)^m}{(ex^n + d)^2} \, dx$$

input
```
integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(a+c\,x^{2\,n})^p\,(f\,x)^m}{(d+e\,x^n)^2}\,dx$$

input
```
int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^2,x)
```

output
```
int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^2, x)
```

3.92 $\int \frac{(fx)^m(a+cx^{2n})^p}{(d+ex^n)^3}\,dx$

3.92.1 Optimal result

Integrand size = 26, antiderivative size = 412

$$\begin{aligned}\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^3}\,dx &= \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{2n},-p,3,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(1+m)}\\ &\quad-\frac{3ex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m+n}{2n},-p,3,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+m+n)}\\ &\quad+\frac{3e^2x^{1+2n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m+2n}{2n},-p,3,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(1+m+2n)}\\ &\quad-\frac{e^3x^{1+3n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m+3n}{2n},-p,3,\frac{1+m+5n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(1+m+3n)}\end{aligned}$$

output
```
x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,3,-p,1+1/2*(1+m)/n,e^2*x^(2
*n)/d^2,-c*x^(2*n)/a)/d^3/(1+m)/((1+c*x^(2*n)/a)^p)-3*e*x^(1+n)*(f*x)^m*(a
+c*x^(2*n))^p*AppellF1(1/2*(1+m+n)/n,3,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,
-c*x^(2*n)/a)/d^4/(1+m+n)/((1+c*x^(2*n)/a)^p)+3*e^2*x^(1+2*n)*(f*x)^m*(a+c
*x^(2*n))^p*AppellF1(1/2*(1+m+2*n)/n,3,-p,1/2*(1+m+4*n)/n,e^2*x^(2*n)/d^2,
-c*x^(2*n)/a)/d^5/(1+m+2*n)/((1+c*x^(2*n)/a)^p)-e^3*x^(1+3*n)*(f*x)^m*(a+c
*x^(2*n))^p*AppellF1(1/2*(1+m+3*n)/n,3,-p,1/2*(1+m+5*n)/n,e^2*x^(2*n)/d^2,
-c*x^(2*n)/a)/d^6/(1+m+3*n)/((1+c*x^(2*n)/a)^p)
```

3.92.2 Mathematica [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

input
```
Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
```

output
```
Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3, x]
```

3.92.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1886, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3}\,dx$$

$$\Big\downarrow \text{1886}$$

$$x^{-m}(fx)^m \int \left(\frac{d^3 (cx^{2n}+a)^p x^m}{(d^2-e^2x^{2n})^3} - \frac{3d^2e (cx^{2n}+a)^p x^{m+n}}{(d^2-e^2x^{2n})^3} + \frac{3de^2 (cx^{2n}+a)^p x^{m+2n}}{(d^2-e^2x^{2n})^3} - \frac{e^3 (cx^{2n}+a)^p x^{m+3n}}{(d^2-e^2x^{2n})^3}\right) d$$

$$\Big\downarrow \text{2009}$$

$$x^{-m}(fx)^m \left(-\frac{e^3 x^{m+3n+1} (a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+3n+1}{2n}, -p, 3, \frac{m+5n+1}{2n}, -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^6(m+3n+1)} + \frac{3e^2x^{m+2}}{}\right.$$

input
```
Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
```

output
```
((f*x)^m*((x^(1 + m)*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 3, 1 +
(1 + m)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + m)*(1 + (c*
x^(2*n))/a)^p) - (3*e*x^(1 + m + n)*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)
/(2*n), -p, 3, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/
(d^4*(1 + m + n)*(1 + (c*x^(2*n))/a)^p) + (3*e^2*x^(1 + m + 2*n)*(a + c*x^
(2*n))^p*AppellF1[(1 + m + 2*n)/(2*n), -p, 3, (1 + m + 4*n)/(2*n), -((c*x^
(2*n))/a), (e^2*x^(2*n))/d^2])/(d^5*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p) -
 (e^3*x^(1 + m + 3*n)*(a + c*x^(2*n))^p*AppellF1[(1 + m + 3*n)/(2*n), -p,
3, (1 + m + 5*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^6*(1 + m
+ 3*n)*(1 + (c*x^(2*n))/a)^p)))/x^m
```

3.92.3.1 Defintions of rubi rules used

rule 1886
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Simp[(f*x)^m/x^m   Int[ExpandIntegrand[x^m*(a + c*x^
(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/(d^2 - e^2*x^(2*n))))^(-q), x], x
], x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !RationalQ[
n] &&  !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.92.4 Maple [F]

$$\int \frac{(fx)^m (a + c\,x^{2n})^p}{(d + e\,x^n)^3} dx$$

input
```
int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

output
```
int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

3.92.5 Fricas [F]

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^3} dx = \int \frac{(cx^{2n} + a)^p (fx)^m}{(ex^n + d)^3} dx$$

input
```
integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + a)^p*(f*x)^m/(e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*
e*x^n + d^3), x)
```

3.92.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^3} dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

output
```
Timed out
```

3.92.7 Maxima [F]

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^3} dx = \int \frac{(cx^{2n} + a)^p (fx)^m}{(ex^n + d)^3} dx$$

input
```
integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)
```

3.92.8 Giac [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(cx^{2n}+a)^p (fx)^m}{(ex^n+d)^3}\,dx$$

input
```
integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3}\,dx = \int \frac{(a+c\,x^{2\,n})^p\,(f\,x)^m}{(d+e\,x^n)^3}\,dx$$

input
```
int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^3,x)
```

output
```
int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^3, x)
```

3.93 $\int (b+2cx) \left(a+bx+cx^2\right)^{13} dx$

3.93.1 Optimal result

Integrand size = 19, antiderivative size = 16

$$\int (b+2cx) \left(a+bx+cx^2\right)^{13} dx = \frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

output
```
1/14*(c*x^2+b*x+a)^14
```

3.93.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 12.56

$$\begin{aligned}\int (b+2cx) \left(a+bx+cx^2\right)^{13} dx = \frac{1}{14}x(b+cx)\left(14a^{13}+91a^{12}x(b+cx)+364a^{11}x^2(b+cx)^2\right.\\ +1001a^{10}x^3(b+cx)^3+2002a^9x^4(b+cx)^4\\ +3003a^8x^5(b+cx)^5+3432a^7x^6(b+cx)^6\\ +3003a^6x^7(b+cx)^7+2002a^5x^8(b+cx)^8\\ +1001a^4x^9(b+cx)^9+364a^3x^{10}(b+cx)^{10}\\ \left.+91a^2x^{11}(b+cx)^{11}+14ax^{12}(b+cx)^{12}+x^{13}(b+cx)^{13}\right)\end{aligned}$$

input
```
Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

output
```
(x*(b + c*x)*(14*a^13 + 91*a^12*x*(b + c*x) + 364*a^11*x^2*(b + c*x)^2 + 1
001*a^10*x^3*(b + c*x)^3 + 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*
x)^5 + 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 + 2002*a^5*x^8*
(b + c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 + 364*a^3*x^10*(b + c*x)^10 + 91*a^
2*x^11*(b + c*x)^11 + 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

3.93.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}dx$$

$$\Big\downarrow \text{ 1104}$$

$$\frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

input
```
Int[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

output
```
(a + b*x + c*x^2)^14/14
```

3.93.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.93.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(c x^2+bx+a)^{14}}{14}$	15
norman	Expression too large to display	1224
gosper	Expression too large to display	1447
parallelrisch	Expression too large to display	1447
risch	Expression too large to display	1452

input
```
int((2*c*x+b)*(c*x^2+b*x+a)^13,x,method=_RETURNVERBOSE)
```

output
```
1/14*(c*x^2+b*x+a)^14
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1234 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 1234, normalized size of antiderivative = 77.12

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx=\text{Too large to display}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="fricas")
```

output
```
1/14*c^14*x^28 + b*c^13*x^27 + 1/2*(13*b^2*c^12 + 2*a*c^13)*x^26 + 13*(2*b
^3*c^11 + a*b*c^12)*x^25 + 13/2*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^12)*x
^24 + 13*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^23 + 13/2*(33*b^6*c
^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^22 + 143/7*(12*b^7*c^
7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^21 + 143/2*(3*b^8*c^6
 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^20 + 143*(
b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^
19 + 143/2*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7 + 4
5*a^4*b^2*c^8 + 2*a^5*c^9)*x^18 + 13*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*a^2*
b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^17 + 13/2*(b
^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*b^
4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^16 + (b^13*c + 78*a*b^11*c^2 + 143
0*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6 +
 1716*a^6*b*c^7)*x^15 + a^13*b*x + 1/14*(b^14 + 182*a*b^12*c + 6006*a^2*b^
10*c^2 + 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 840
84*a^6*b^2*c^6 + 3432*a^7*c^7)*x^14 + (a*b^13 + 78*a^2*b^11*c + 1430*a^3*b
^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a
^7*b*c^6)*x^13 + 13/2*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a
^5*b^6*c^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^12 + 13*(2
*a^3*b^11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*...
```

3.93.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 1326, normalized size of antiderivative = 82.88

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate((2*c*x+b)*(c*x**2+b*x+a)**13,x)
```

output
```
a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(a*c**13 + 13*b**2*c**1
2/2) + x**25*(13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 + 78*
a*b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 + 286*a*b**3*c**
10 + 143*b**5*c**9) + x**22*(26*a**3*c**11 + 429*a**2*b**2*c**10 + 715*a*b
**4*c**9 + 429*b**6*c**8/2) + x**21*(286*a**3*b*c**10 + 1430*a**2*b**3*c**
9 + 1287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 + 1430*
a**3*b**2*c**9 + 6435*a**2*b**4*c**8/2 + 1716*a*b**6*c**7 + 429*b**8*c**6/
2) + x**19*(715*a**4*b*c**9 + 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 +
1716*a*b**7*c**6 + 143*b**9*c**5) + x**18*(143*a**5*c**9 + 6435*a**4*b**2*
c**8/2 + 8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 + 1287*a*b**8*c**5 + 14
3*b**10*c**4/2) + x**17*(1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 + 12012*a*
*3*b**5*c**6 + 5148*a**2*b**7*c**5 + 715*a*b**9*c**4 + 26*b**11*c**3) + x*
*16*(429*a**6*c**8/2 + 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 + 12012*
a**3*b**6*c**5 + 6435*a**2*b**8*c**4/2 + 286*a*b**10*c**3 + 13*b**12*c**2/
2) + x**15*(1716*a**6*b*c**7 + 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5
 + 8580*a**3*b**7*c**4 + 1430*a**2*b**9*c**3 + 78*a*b**11*c**2 + b**13*c)
+ x**14*(1716*a**7*c**7/7 + 6006*a**6*b**2*c**6 + 18018*a**5*b**4*c**5 + 1
5015*a**4*b**6*c**4 + 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 + 13*a*b**
12*c + b**14/14) + x**13*(1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 + 18018*
a**5*b**5*c**4 + 8580*a**4*b**7*c**3 + 1430*a**3*b**9*c**2 + 78*a**2*b*...
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx = \frac{1}{14}\left(cx^2+bx+a\right)^{14}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="maxima")
```

output
```
1/14*(c*x^2 + b*x + a)^14
```

3.93.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(14) = 28$.

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 13.50

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx = \frac{1}{14}\left(cx^2+bx\right)^{14} + \left(cx^2+bx\right)^{13}a + \frac{13}{2}\left(cx^2+bx\right)^{12}a^2 + 26\left(cx^2+bx\right)^{11}a^3 + \frac{143}{2}\left(cx^2+bx\right)^{10}a^4 + 143\left(cx^2+bx\right)^{9}a^5 + \frac{429}{2}\left(cx^2+bx\right)^{8}a^6 + \frac{1716}{7}\left(cx^2+bx\right)^{7}a^7 + \frac{429}{2}\left(cx^2+bx\right)^{6}a^8 + 143\left(cx^2+bx\right)^{5}a^9 + \frac{143}{2}\left(cx^2+bx\right)^{4}a^{10} + 26\left(cx^2+bx\right)^{3}a^{11} + \frac{13}{2}\left(cx^2+bx\right)^{2}a^{12} + \left(cx^2+bx\right)a^{13}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="giac")
```

output
```
1/14*(c*x^2 + b*x)^14 + (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 + 2
6*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 + 143*(c*x^2 + b*x)^9*
a^5 + 429/2*(c*x^2 + b*x)^8*a^6 + 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^
2 + b*x)^6*a^8 + 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 + 26
*(c*x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 + (c*x^2 + b*x)*a^13
```

3.93.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 1203, normalized size of antiderivative = 75.19

$$
\begin{aligned}
\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx = {} & x^{12}\left(\frac{429\,a^8\,c^6}{2}+5148\,a^7\,b^2\,c^5+15015\,a^6\,b^4\,c^4\right.\\
&\left.+12012\,a^5\,b^6\,c^3+\frac{6435\,a^4\,b^8\,c^2}{2}+286\,a^3\,b^{10}\,c+\frac{13\,a^2\,b^{12}}{2}\right)\\
&+x^{16}\left(\frac{429\,a^6\,c^8}{2}+5148\,a^5\,b^2\,c^7+15015\,a^4\,b^4\,c^6\right.\\
&\left.+12012\,a^3\,b^6\,c^5+\frac{6435\,a^2\,b^8\,c^4}{2}+286\,a\,b^{10}\,c^3+\frac{13\,b^{12}\,c^2}{2}\right)\\
&+x^{13}\left(1716\,a^7\,b\,c^6+12012\,a^6\,b^3\,c^5+18018\,a^5\,b^5\,c^4\right.\\
&\left.+8580\,a^4\,b^7\,c^3+1430\,a^3\,b^9\,c^2+78\,a^2\,b^{11}\,c+a\,b^{13}\right)\\
&+x^{15}\left(1716\,a^6\,b\,c^7+12012\,a^5\,b^3\,c^6+18018\,a^4\,b^5\,c^5\right.\\
&\left.+8580\,a^3\,b^7\,c^4+1430\,a^2\,b^9\,c^3+78\,a\,b^{11}\,c^2+b^{13}\,c\right)\\
&+x^6\left(26\,a^{11}\,c^3+429\,a^{10}\,b^2\,c^2+715\,a^9\,b^4\,c+\frac{429\,a^8\,b^6}{2}\right)\\
&+x^{22}\left(26\,a^3\,c^{11}+429\,a^2\,b^2\,c^{10}+715\,a\,b^4\,c^9+\frac{429\,b^6\,c^8}{2}\right)\\
&+x^{10}\left(143\,a^9\,c^5+\frac{6435\,a^8\,b^2\,c^4}{2}+8580\,a^7\,b^4\,c^3\right.\\
&\left.+6006\,a^6\,b^6\,c^2+1287\,a^5\,b^8\,c+\frac{143\,a^4\,b^{10}}{2}\right)\\
&+x^{18}\left(143\,a^5\,c^9+\frac{6435\,a^4\,b^2\,c^8}{2}+8580\,a^3\,b^4\,c^7\right.\\
&\left.+6006\,a^2\,b^6\,c^6+1287\,a\,b^8\,c^5+\frac{143\,b^{10}\,c^4}{2}\right)\\
&+x^{14}\left(\frac{1716\,a^7\,c^7}{7}+6006\,a^6\,b^2\,c^6+18018\,a^5\,b^4\,c^5\right.\\
&+15015\,a^4\,b^6\,c^4+4290\,a^3\,b^8\,c^3+429\,a^2\,b^{10}\,c^2+13\,a\,b^{12}\,c\\
&\left.+\frac{b^{14}}{14}\right)+x^8\left(\frac{143\,a^{10}\,c^4}{2}+1430\,a^9\,b^2\,c^3+\frac{6435\,a^8\,b^4\,c^2}{2}\right.\\
&\left.+1716\,a^7\,b^6\,c+\frac{429\,a^6\,b^8}{2}\right)+x^{20}\left(\frac{143\,a^4\,c^{10}}{2}\right.\\
&\left.+1430\,a^3\,b^2\,c^9+\frac{6435\,a^2\,b^4\,c^8}{2}+1716\,a\,b^6\,c^7+\frac{429\,b^8\,c^6}{2}\right)\\
&+\frac{c^{14}\,x^{28}}{14}+x^2\left(c\,a^{13}+\frac{13\,a^{12}\,b^2}{2}\right)\\
&+\frac{13\,a^{10}\,x^4\,(a^2\,c^2+12\,a\,b^2\,c+11\,b^4)}{2}\\
&+\frac{13\,c^{10}\,x^{24}\,(a^2\,c^2+12\,a\,b^2\,c+11\,b^4)}{2}\\
&+b\,c^{13}\,x^{27}+\frac{c^{12}\,x^{26}\,(13\,b^2+2\,a\,c)}{2}+a^{13}\,b\,x\\
&143\,a^7\,b\,x^7\,(14\,a^3\,c^3+70\,a^2\,b^2\,c^2+63\,a\,b^4\,c+12\,b^6)
\end{aligned}
$$

input
```
int((b + 2*c*x)*(a + b*x + c*x^2)^13,x)
```

output
```
x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 + 286*a^3*b^10*c + (6435*a^4*b^8*c
^2)/2 + 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 + 5148*a^7*b^2*c^5) + x^16*(
(429*a^6*c^8)/2 + (13*b^12*c^2)/2 + 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2
+ 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 + 5148*a^5*b^2*c^7) + x^13*(a*b^13
 + 78*a^2*b^11*c + 1716*a^7*b*c^6 + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 +
18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5) + x^15*(b^13*c + 78*a*b^11*c^2 + 17
16*a^6*b*c^7 + 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 1
2012*a^5*b^3*c^6) + x^6*((429*a^8*b^6)/2 + 26*a^11*c^3 + 715*a^9*b^4*c + 4
29*a^10*b^2*c^2) + x^22*(26*a^3*c^11 + (429*b^6*c^8)/2 + 715*a*b^4*c^9 + 4
29*a^2*b^2*c^10) + x^10*((143*a^4*b^10)/2 + 143*a^9*c^5 + 1287*a^5*b^8*c +
 6006*a^6*b^6*c^2 + 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2) + x^18*(143*a
^5*c^9 + (143*b^10*c^4)/2 + 1287*a*b^8*c^5 + 6006*a^2*b^6*c^6 + 8580*a^3*b
^4*c^7 + (6435*a^4*b^2*c^8)/2) + x^14*(b^14/14 + (1716*a^7*c^7)/7 + 429*a^
2*b^10*c^2 + 4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^4 + 18018*a^5*b^4*c^5 + 60
06*a^6*b^2*c^6 + 13*a*b^12*c) + x^8*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 +
1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2 + 1430*a^9*b^2*c^3) + x^20*((143*a^4
*c^10)/2 + (429*b^8*c^6)/2 + 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*
a^3*b^2*c^9) + (c^14*x^28)/14 + x^2*(a^13*c + (13*a^12*b^2)/2) + (13*a^10*
x^4*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/2 + (13*c^10*x^24*(11*b^4 + a^2*c^2 +
 12*a*b^2*c))/2 + b*c^13*x^27 + (c^12*x^26*(2*a*c + 13*b^2))/2 + a^13*b...
```

3.93. $\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx$

3.94 $\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx$

3.94.1 Optimal result

Integrand size = 24, antiderivative size = 18

$$\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx=\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

output
```
1/28*(c*x^4+b*x^2+a)^14
```

3.94.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(18) = 36.

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 12.94

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx&=\frac{1}{28}x^2\left(b+cx^2\right)\Big(14a^{13}+91a^{12}x^2\left(b+cx^2\right)\\&\quad+364a^{11}x^4\left(b+cx^2\right)^2+1001a^{10}x^6\left(b+cx^2\right)^3\\&\quad+2002a^9x^8\left(b+cx^2\right)^4+3003a^8x^{10}\left(b+cx^2\right)^5\\&\quad+3432a^7x^{12}\left(b+cx^2\right)^6+3003a^6x^{14}\left(b+cx^2\right)^7\\&\quad+2002a^5x^{16}\left(b+cx^2\right)^8+1001a^4x^{18}\left(b+cx^2\right)^9\\&\quad+364a^3x^{20}\left(b+cx^2\right)^{10}+91a^2x^{22}\left(b+cx^2\right)^{11}\\&\quad+14ax^{24}\left(b+cx^2\right)^{12}+x^{26}\left(b+cx^2\right)^{13}\Big)\end{aligned}$$

input
```
Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

output
```
(x^2*(b + c*x^2)*(14*a^13 + 91*a^12*x^2*(b + c*x^2) + 364*a^11*x^4*(b + c*
x^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 + 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a
^8*x^10*(b + c*x^2)^5 + 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c
*x^2)^7 + 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 + 364*
a^3*x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 + 14*a*x^24*(b + c*x^
2)^12 + x^26*(b + c*x^2)^13))/28
```

3.94.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1576, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \left(2cx^2+b\right)\left(cx^4+bx^2+a\right)^{13}dx^2$$

$$\Big\downarrow \text{1104}$$

$$\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

input
```
Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

output
```
(a + b*x^2 + c*x^4)^14/28
```

3.94.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.94.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(c x^4+b x^2+a)^{14}}{28}$	17
gosper	Expression too large to display	1455
parallelrisch	Expression too large to display	1455
risch	Expression too large to display	1460

input
```
int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x,method=_RETURNVERBOSE)
```

output
```
1/28*(c*x^4+b*x^2+a)^14
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 1240, normalized size of antiderivative = 68.89

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="fricas")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 + 2*a*c^13)*x^52 + 13/
2*(2*b^3*c^11 + a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c
^12)*x^48 + 13/2*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(
33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^44 + 143/14*(
12*b^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^42 + 143/4*(
3*b^8*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^4
0 + 143/2*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^
4*b*c^9)*x^38 + 143/4*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*
b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 + 55*a*b^9*c
^4 + 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x
^34 + 13/4*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5
+ 2310*a^4*b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c + 78
*a*b^11*c^2 + 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12
012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 + 182*a*b^12*c + 6006*
a^2*b^10*c^2 + 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5
 + 84084*a^6*b^2*c^6 + 3432*a^7*c^7)*x^28 + 1/2*(a*b^13 + 78*a^2*b^11*c +
1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^
5 + 1716*a^7*b*c^6)*x^26 + 13/4*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^
2 + 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^
24 + 13/2*(2*a^3*b^11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^...
```

3.94.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1384 vs. $2(14) = 28$.

Time = 0.14 (sec) , antiderivative size = 1384, normalized size of antiderivative = 76.89

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13} dx = \text{Too large to display}$$

input
```
integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**13,x)
```

output
```
a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(a*c**13/2 + 13*
b**2*c**12/4) + x**50*(13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c*
*12/4 + 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 + 143
*a*b**3*c**10 + 143*b**5*c**9/2) + x**44*(13*a**3*c**11 + 429*a**2*b**2*c*
*10/2 + 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(143*a**3*b*c**10 + 7
15*a**2*b**3*c**9 + 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**
4*c**10/4 + 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 + 858*a*b**6*c**7 +
 429*b**8*c**6/4) + x**38*(715*a**4*b*c**9/2 + 2145*a**3*b**3*c**8 + 2574*
a**2*b**5*c**7 + 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(143*a**5*c**9
/2 + 6435*a**4*b**2*c**8/4 + 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 + 1
287*a*b**8*c**5/2 + 143*b**10*c**4/4) + x**34*(1287*a**5*b*c**8/2 + 4290*a
**4*b**3*c**7 + 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 + 715*a*b**9*c**
4/2 + 13*b**11*c**3) + x**32*(429*a**6*c**8/4 + 2574*a**5*b**2*c**7 + 1501
5*a**4*b**4*c**6/2 + 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 + 143*a*b
**10*c**3 + 13*b**12*c**2/4) + x**30*(858*a**6*b*c**7 + 6006*a**5*b**3*c**
6 + 9009*a**4*b**5*c**5 + 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 + 39*a*
b**11*c**2 + b**13*c/2) + x**28*(858*a**7*c**7/7 + 3003*a**6*b**2*c**6 + 9
009*a**5*b**4*c**5 + 15015*a**4*b**6*c**4/2 + 2145*a**3*b**8*c**3 + 429*a*
*2*b**10*c**2/2 + 13*a*b**12*c/2 + b**14/28) + x**26*(858*a**7*b*c**6 + 60
06*a**6*b**3*c**5 + 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 + 715*a**...
```

3.94.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. $2(16) = 32$.

Time = 0.21 (sec) , antiderivative size = 1240, normalized size of antiderivative = 68.89

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="maxima")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 + 2*a*c^13)*x^52 + 13/
2*(2*b^3*c^11 + a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c
^12)*x^48 + 13/2*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(
33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^44 + 143/14*(
12*b^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^42 + 143/4*(
3*b^8*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^4
0 + 143/2*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^
4*b*c^9)*x^38 + 143/4*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*
b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 + 55*a*b^9*c
^4 + 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x
^34 + 13/4*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5
+ 2310*a^4*b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c + 78
*a*b^11*c^2 + 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12
012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 + 182*a*b^12*c + 6006*
a^2*b^10*c^2 + 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5
 + 84084*a^6*b^2*c^6 + 3432*a^7*c^7)*x^28 + 1/2*(a*b^13 + 78*a^2*b^11*c +
1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^
5 + 1716*a^7*b*c^6)*x^26 + 13/4*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^
2 + 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^
24 + 13/2*(2*a^3*b^11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^...
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(16) = 32$.

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 13.67

$$\begin{aligned}\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx &= \frac{1}{28}\left(cx^4+bx^2\right)^{14}+\frac{1}{2}\left(cx^4+bx^2\right)^{13}a \\ &+ \frac{13}{4}\left(cx^4+bx^2\right)^{12}a^2+13\left(cx^4+bx^2\right)^{11}a^3 \\ &+ \frac{143}{4}\left(cx^4+bx^2\right)^{10}a^4+\frac{143}{2}\left(cx^4+bx^2\right)^{9}a^5 \\ &+ \frac{429}{4}\left(cx^4+bx^2\right)^{8}a^6+\frac{858}{7}\left(cx^4+bx^2\right)^{7}a^7 \\ &+ \frac{429}{4}\left(cx^4+bx^2\right)^{6}a^8+\frac{143}{2}\left(cx^4+bx^2\right)^{5}a^9 \\ &+ \frac{143}{4}\left(cx^4+bx^2\right)^{4}a^{10}+13\left(cx^4+bx^2\right)^{3}a^{11} \\ &+ \frac{13}{4}\left(cx^4+bx^2\right)^{2}a^{12}+\frac{1}{2}\left(cx^4+bx^2\right)a^{13}\end{aligned}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="giac")
```

output
```
1/28*(c*x^4 + b*x^2)^14 + 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^
12*a^2 + 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 + 143/2*
(c*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 + 858/7*(c*x^4 + b*x^2
)^7*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 + 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/
4*(c*x^4 + b*x^2)^4*a^10 + 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2
)^2*a^12 + 1/2*(c*x^4 + b*x^2)*a^13
```

3.94.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 1210, normalized size of antiderivative = 67.22

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

input
```
int(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x)
```

output
```
x^24*((13*a^2*b^12)/4 + (429*a^8*c^6)/4 + 143*a^3*b^10*c + (6435*a^4*b^8*c
^2)/4 + 6006*a^5*b^6*c^3 + (15015*a^6*b^4*c^4)/2 + 2574*a^7*b^2*c^5) + x^3
2*((429*a^6*c^8)/4 + (13*b^12*c^2)/4 + 143*a*b^10*c^3 + (6435*a^2*b^8*c^4)
/4 + 6006*a^3*b^6*c^5 + (15015*a^4*b^4*c^6)/2 + 2574*a^5*b^2*c^7) + x^26*(
(a*b^13)/2 + 39*a^2*b^11*c + 858*a^7*b*c^6 + 715*a^3*b^9*c^2 + 4290*a^4*b^
7*c^3 + 9009*a^5*b^5*c^4 + 6006*a^6*b^3*c^5) + x^30*((b^13*c)/2 + 39*a*b^1
1*c^2 + 858*a^6*b*c^7 + 715*a^2*b^9*c^3 + 4290*a^3*b^7*c^4 + 9009*a^4*b^5*
c^5 + 6006*a^5*b^3*c^6) + x^12*((429*a^8*b^6)/4 + 13*a^11*c^3 + (715*a^9*b
^4*c)/2 + (429*a^10*b^2*c^2)/2) + x^44*(13*a^3*c^11 + (429*b^6*c^8)/4 + (7
15*a*b^4*c^9)/2 + (429*a^2*b^2*c^10)/2) + x^20*((143*a^4*b^10)/4 + (143*a^
9*c^5)/2 + (1287*a^5*b^8*c)/2 + 3003*a^6*b^6*c^2 + 4290*a^7*b^4*c^3 + (643
5*a^8*b^2*c^4)/4) + x^36*((143*a^5*c^9)/2 + (143*b^10*c^4)/4 + (1287*a*b^8
*c^5)/2 + 3003*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 + (6435*a^4*b^2*c^8)/4) + x^
28*(b^14/28 + (858*a^7*c^7)/7 + (429*a^2*b^10*c^2)/2 + 2145*a^3*b^8*c^3 +
(15015*a^4*b^6*c^4)/2 + 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 + (13*a*b^12*c
)/2) + x^16*((429*a^6*b^8)/4 + (143*a^10*c^4)/4 + 858*a^7*b^6*c + (6435*a^
8*b^4*c^2)/4 + 715*a^9*b^2*c^3) + x^40*((143*a^4*c^10)/4 + (429*b^8*c^6)/4
 + 858*a*b^6*c^7 + (6435*a^2*b^4*c^8)/4 + 715*a^3*b^2*c^9) + (c^14*x^56)/2
8 + x^4*((a^13*c)/2 + (13*a^12*b^2)/4) + (13*a^10*x^8*(11*b^4 + a^2*c^2 +
12*a*b^2*c))/4 + (13*c^10*x^48*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/4 + (a^...
```

3.95 $\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx$

3.95.1 Optimal result

Integrand size = 26, antiderivative size = 18

$$\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx=\frac{1}{42}\left(a+bx^3+cx^6\right)^{14}$$

output
```
1/42*(c*x^6+b*x^3+a)^14
```

3.95.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(18) = 36.

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 12.94

$$\begin{aligned}\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx&=\frac{1}{42}x^3(b+cx^3)\Big(14a^{13}+91a^{12}x^3(b+cx^3)\\&\quad+364a^{11}x^6\left(b+cx^3\right)^2+1001a^{10}x^9\left(b+cx^3\right)^3\\&\quad+2002a^9x^{12}\left(b+cx^3\right)^4+3003a^8x^{15}\left(b+cx^3\right)^5\\&\quad+3432a^7x^{18}\left(b+cx^3\right)^6+3003a^6x^{21}\left(b+cx^3\right)^7\\&\quad+2002a^5x^{24}\left(b+cx^3\right)^8+1001a^4x^{27}\left(b+cx^3\right)^9\\&\quad+364a^3x^{30}\left(b+cx^3\right)^{10}+91a^2x^{33}\left(b+cx^3\right)^{11}\\&\quad+14ax^{36}\left(b+cx^3\right)^{12}+x^{39}\left(b+cx^3\right)^{13}\Big)\end{aligned}$$

input
```
Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

output
```
(x^3*(b + c*x^3)*(14*a^13 + 91*a^12*x^3*(b + c*x^3) + 364*a^11*x^6*(b + c*
x^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 + 2002*a^9*x^12*(b + c*x^3)^4 + 3003*
a^8*x^15*(b + c*x^3)^5 + 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b +
c*x^3)^7 + 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 + 364
*a^3*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 + 14*a*x^36*(b + c*x
^3)^12 + x^39*(b + c*x^3)^13))/42
```

3.95.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1798, 1104\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^{13} \, dx$$

$$\Big\downarrow \ 1798$$

$$\frac{1}{3} \int \left(2 c x^3+b\right) \left(c x^6+b x^3+a\right)^{13} dx^3$$

$$\Big\downarrow \ 1104$$

$$\frac{1}{42} \left(a+b x^3+c x^6\right)^{14}$$

input
```
Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

output
```
(a + b*x^3 + c*x^6)^14/42
```

3.95.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.95.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(c x^6+b x^3+a)^{14}}{42}$	17
gosper	Expression too large to display	1455
parallelrisch	Expression too large to display	1455
risch	Expression too large to display	1460

input
```
int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x,method=_RETURNVERBOSE)
```

output
```
1/42*(c*x^6+b*x^3+a)^14
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 1240, normalized size of antiderivative = 68.89

$$\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="fricas")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 + 2*a*c^13)*x^78 + 13/
3*(2*b^3*c^11 + a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c
^12)*x^72 + 13/3*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(
33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^66 + 143/21*(
12*b^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^63 + 143/6*(
3*b^8*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^6
0 + 143/3*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^
4*b*c^9)*x^57 + 143/6*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*
b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 + 55*a*b^9*c
^4 + 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x
^51 + 13/6*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5
+ 2310*a^4*b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c + 78
*a*b^11*c^2 + 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12
012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 + 182*a*b^12*c + 6006*
a^2*b^10*c^2 + 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5
 + 84084*a^6*b^2*c^6 + 3432*a^7*c^7)*x^42 + 1/3*(a*b^13 + 78*a^2*b^11*c +
1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^
5 + 1716*a^7*b*c^6)*x^39 + 13/6*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^
2 + 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^
36 + 13/3*(2*a^3*b^11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^...
```

3.95.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(14) = 28$.

Time = 0.15 (sec) , antiderivative size = 1394, normalized size of antiderivative = 77.44

$$\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**13,x)
```

output
```
a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(a*c**13/3 + 13*
b**2*c**12/6) + x**75*(13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2*
c**12/6 + 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 + 2
86*a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(26*a**3*c**11/3 + 143*a**2*b
**2*c**10 + 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(286*a**3*b*c**10
/3 + 1430*a**2*b**3*c**9/3 + 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(1
43*a**4*c**10/6 + 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 + 572*a*b*
*6*c**7 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 + 1430*a**3*b**3*c**
8 + 1716*a**2*b**5*c**7 + 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(143*
a**5*c**9/3 + 2145*a**4*b**2*c**8/2 + 2860*a**3*b**4*c**7 + 2002*a**2*b**6
*c**6 + 429*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(429*a**5*b*c**8 + 286
0*a**4*b**3*c**7 + 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 + 715*a*b**9*
c**4/3 + 26*b**11*c**3/3) + x**48*(143*a**6*c**8/2 + 1716*a**5*b**2*c**7 +
 5005*a**4*b**4*c**6 + 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 + 286*a
*b**10*c**3/3 + 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 + 4004*a**5*b**3
*c**6 + 6006*a**4*b**5*c**5 + 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3
+ 26*a*b**11*c**2 + b**13*c/3) + x**42*(572*a**7*c**7/7 + 2002*a**6*b**2*c
**6 + 6006*a**5*b**4*c**5 + 5005*a**4*b**6*c**4 + 1430*a**3*b**8*c**3 + 14
3*a**2*b**10*c**2 + 13*a*b**12*c/3 + b**14/42) + x**39*(572*a**7*b*c**6 +
4004*a**6*b**3*c**5 + 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 + 1430*...
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. $2(16) = 32$.

Time = 0.20 (sec) , antiderivative size = 1240, normalized size of antiderivative = 68.89

$$\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="maxima")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 + 2*a*c^13)*x^78 + 13/
3*(2*b^3*c^11 + a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c
^12)*x^72 + 13/3*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(
33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^66 + 143/21*(
12*b^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^63 + 143/6*(
3*b^8*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^6
0 + 143/3*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^
4*b*c^9)*x^57 + 143/6*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*
b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 + 55*a*b^9*c
^4 + 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x
^51 + 13/6*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5
+ 2310*a^4*b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c + 78
*a*b^11*c^2 + 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12
012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 + 182*a*b^12*c + 6006*
a^2*b^10*c^2 + 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5
 + 84084*a^6*b^2*c^6 + 3432*a^7*c^7)*x^42 + 1/3*(a*b^13 + 78*a^2*b^11*c +
1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^
5 + 1716*a^7*b*c^6)*x^39 + 13/6*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^
2 + 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^
36 + 13/3*(2*a^3*b^11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^...
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(16) = 32$.

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 13.67

$$\begin{aligned}\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx &= \frac{1}{42}\left(cx^6+bx^3\right)^{14}+\frac{1}{3}\left(cx^6+bx^3\right)^{13}a \\ &\quad+\frac{13}{6}\left(cx^6+bx^3\right)^{12}a^2+\frac{26}{3}\left(cx^6+bx^3\right)^{11}a^3 \\ &\quad+\frac{143}{6}\left(cx^6+bx^3\right)^{10}a^4+\frac{143}{3}\left(cx^6+bx^3\right)^{9}a^5 \\ &\quad+\frac{143}{2}\left(cx^6+bx^3\right)^{8}a^6+\frac{572}{7}\left(cx^6+bx^3\right)^{7}a^7 \\ &\quad+\frac{143}{2}\left(cx^6+bx^3\right)^{6}a^8+\frac{143}{3}\left(cx^6+bx^3\right)^{5}a^9 \\ &\quad+\frac{143}{6}\left(cx^6+bx^3\right)^{4}a^{10}+\frac{26}{3}\left(cx^6+bx^3\right)^{3}a^{11} \\ &\quad+\frac{13}{6}\left(cx^6+bx^3\right)^{2}a^{12}+\frac{1}{3}\left(cx^6+bx^3\right)a^{13}\end{aligned}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="giac")
```

output
```
1/42*(c*x^6 + b*x^3)^14 + 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^
12*a^2 + 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 + 143/
3*(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 + 572/7*(c*x^6 + b*x
^3)^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 + 143/3*(c*x^6 + b*x^3)^5*a^9 + 14
3/6*(c*x^6 + b*x^3)^4*a^10 + 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b
*x^3)^2*a^12 + 1/3*(c*x^6 + b*x^3)*a^13
```

3.95.9 Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 1210, normalized size of antiderivative = 67.22

$$\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx=\text{Too large to display}$$

input
```
int(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x)
```

output
```
x^36*((13*a^2*b^12)/6 + (143*a^8*c^6)/2 + (286*a^3*b^10*c)/3 + (2145*a^4*b
^8*c^2)/2 + 4004*a^5*b^6*c^3 + 5005*a^6*b^4*c^4 + 1716*a^7*b^2*c^5) + x^48
*((143*a^6*c^8)/2 + (13*b^12*c^2)/6 + (286*a*b^10*c^3)/3 + (2145*a^2*b^8*c
^4)/2 + 4004*a^3*b^6*c^5 + 5005*a^4*b^4*c^6 + 1716*a^5*b^2*c^7) + x^39*((a
*b^13)/3 + 26*a^2*b^11*c + 572*a^7*b*c^6 + (1430*a^3*b^9*c^2)/3 + 2860*a^4
*b^7*c^3 + 6006*a^5*b^5*c^4 + 4004*a^6*b^3*c^5) + x^45*((b^13*c)/3 + 26*a*
b^11*c^2 + 572*a^6*b*c^7 + (1430*a^2*b^9*c^3)/3 + 2860*a^3*b^7*c^4 + 6006*
a^4*b^5*c^5 + 4004*a^5*b^3*c^6) + x^18*((143*a^8*b^6)/2 + (26*a^11*c^3)/3
+ (715*a^9*b^4*c)/3 + 143*a^10*b^2*c^2) + x^66*((26*a^3*c^11)/3 + (143*b^6
*c^8)/2 + (715*a*b^4*c^9)/3 + 143*a^2*b^2*c^10) + x^30*((143*a^4*b^10)/6 +
 (143*a^9*c^5)/3 + 429*a^5*b^8*c + 2002*a^6*b^6*c^2 + 2860*a^7*b^4*c^3 + (
2145*a^8*b^2*c^4)/2) + x^54*((143*a^5*c^9)/3 + (143*b^10*c^4)/6 + 429*a*b^
8*c^5 + 2002*a^2*b^6*c^6 + 2860*a^3*b^4*c^7 + (2145*a^4*b^2*c^8)/2) + x^42
*(b^14/42 + (572*a^7*c^7)/7 + 143*a^2*b^10*c^2 + 1430*a^3*b^8*c^3 + 5005*a
^4*b^6*c^4 + 6006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 + (13*a*b^12*c)/3) + x^24
*((143*a^6*b^8)/2 + (143*a^10*c^4)/6 + 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/
2 + (1430*a^9*b^2*c^3)/3) + x^60*((143*a^4*c^10)/6 + (143*b^8*c^6)/2 + 572
*a*b^6*c^7 + (2145*a^2*b^4*c^8)/2 + (1430*a^3*b^2*c^9)/3) + (c^14*x^84)/42
 + x^6*((a^13*c)/3 + (13*a^12*b^2)/6) + (13*a^10*x^12*(11*b^4 + a^2*c^2 +
12*a*b^2*c))/6 + (13*c^10*x^72*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/6 + (a^...
```

3.96 $\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx$

3.96.1 Optimal result

Integrand size = 30, antiderivative size = 23

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx=\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

output

```
1/14*(a+b*x^n+c*x^(2*n))^14/n
```

3.96.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx=\frac{(a+x^n(b+cx^n))^{14}}{14n}$$

input

```
Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
```

output

```
(a + x^n*(b + c*x^n))^14/(14*n)
```

3.96.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1798, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{\int (2cx^n+b)\left(bx^n+cx^{2n}+a\right)^{13}dx^n}{n}$$

$$\Big\downarrow \text{1104}$$

$$\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

input
```
Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
```

output
```
(a + b*x^n + c*x^(2*n))^14/(14*n)
```

3.96.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.96.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2041 vs. 2(21) = 42.

Time = 0.02 (sec) , antiderivative size = 2042, normalized size of antiderivative = 88.78

Expression too large to display

input
```
int(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x)
```

output
```
286*a^10*b/n*(x^n)^7*c^3+1430*a^9*b^3/n*(x^n)^7*c^2+1287*a^8*b^5/n*(x^n)^7
*c+429*a^10/n*(x^n)^6*b^2*c^2+429/2*a^6/n*(x^n)^8*b^8+143/2*a^10/n*(x^n)^8
*c^4+26*a^11*b^3/n*(x^n)^3+c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)^26*b^2+13/2
*a^12/n*(x^n)^2*b^2+13/2*c^12/n*(x^n)^24*a^2+143/2*c^10/n*(x^n)^24*b^4+143
/2*c^10/n*(x^n)^20*a^4+429/2*c^6/n*(x^n)^20*b^8+26*a^11/n*(x^n)^6*c^3+429/
2*a^8/n*(x^n)^6*b^6+13/2*a^12/n*(x^n)^4*c^2+143/2*a^10/n*(x^n)^4*b^4+1/14*
c^14/n*(x^n)^28+1/14/n*(x^n)^14*b^14+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(
x^n)^12*b^12+143*a^9/n*(x^n)^10*c^5+143/2*a^4/n*(x^n)^10*b^10+a^13/n*(x^n)
^2*c+715*b*c^9/n*(x^n)^19*a^4+4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(
x^n)^19*a^2+1716*b^7*c^6/n*(x^n)^19*a+1716/7/n*(x^n)^14*a^7*c^7+143*a^5*b^
9/n*(x^n)^9+b^13*c/n*(x^n)^15+26*c^11*b^3/n*(x^n)^25+143*c^9/n*(x^n)^18*a^
5+143/2*c^4/n*(x^n)^18*b^10+1716/7*b^7*c^7/n*(x^n)^21+26*c^11/n*(x^n)^22*a
^3+429/2*c^8/n*(x^n)^22*b^6+a*b^13/n*(x^n)^13+b*a^13/n*x^n+5148*a^7/n*(x^n
)^12*b^2*c^5+15015*a^6/n*(x^n)^12*b^4*c^4+12012*a^5/n*(x^n)^12*b^6*c^3+643
5/2*a^4/n*(x^n)^12*b^8*c^2+286*a^3/n*(x^n)^12*b^10*c+1716/7*a^7*b^7/n*(x^n
)^7+143*c^9*b^5/n*(x^n)^23+429/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(x^n)^16*b^
12+b*c^13/n*(x^n)^27+26*b^11*c^3/n*(x^n)^17+26*a^3*b^11/n*(x^n)^11+143*b^9
*c^5/n*(x^n)^19+143*a^9*b^5/n*(x^n)^5+6435/2*a^8/n*(x^n)^10*b^2*c^4+8580*a
^7/n*(x^n)^10*b^4*c^3+6006*a^6/n*(x^n)^10*b^6*c^2+1287*a^5/n*(x^n)^10*b^8*
c+6006/n*(x^n)^14*a^6*b^2*c^6+18018/n*(x^n)^14*a^5*b^4*c^5+15015/n*(x^n...
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1297 vs. 2(21) = 42.

Time = 0.29 (sec) , antiderivative size = 1297, normalized size of antiderivative = 56.39

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas
")
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 14*a^13*b*x^n + 7*(13*b^2*c^12
+ 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 + a*b*c^12)*x^(25*n) + 91*(11*b^4*c
^10 + 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 + 22*a*b^3*c^10
 + 6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^
10 + 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^
9 + 14*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*
c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 + 12*a*b^7*c^6 +
 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4
+ 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5
*c^9)*x^(18*n) + 182*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*a^2*b^7*c^5 + 924*a^
3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 + 44*a
*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 + 792*a^
5*b^2*c^7 + 33*a^6*c^8)*x^(16*n) + 14*(b^13*c + 78*a*b^11*c^2 + 1430*a^2*b
^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6 + 1716*a
^6*b*c^7)*x^(15*n) + (b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 + 60060*a^3*
b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 + 34
32*a^7*c^7)*x^(14*n) + 14*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2 + 858
0*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^
(13*n) + 91*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c^3
 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) + 182*(2*a...
```

3.96.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**13,x)
```

output
```
Timed out
```

3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2041 vs. $2(21) = 42$.

Time = 0.27 (sec) , antiderivative size = 2041, normalized size of antiderivative = 88.74

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima
")
```

output
```
1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + a*c^
13*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n + 13*a*b*c^12*x^(25*n)/n + 143/2*b^
4*c^10*x^(24*n)/n + 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n +
143*b^5*c^9*x^(23*n)/n + 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n
)/n + 429/2*b^6*c^8*x^(22*n)/n + 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^
10*x^(22*n)/n + 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 1287*
a*b^5*c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n + 286*a^3*b*c^10*x^(21*
n)/n + 429/2*b^8*c^6*x^(20*n)/n + 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b
^4*c^8*x^(20*n)/n + 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/
n + 143*b^9*c^5*x^(19*n)/n + 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*
x^(19*n)/n + 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/
2*b^10*c^4*x^(18*n)/n + 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18
*n)/n + 8580*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n + 143*
a^5*c^9*x^(18*n)/n + 26*b^11*c^3*x^(17*n)/n + 715*a*b^9*c^4*x^(17*n)/n + 5
148*a^2*b^7*c^5*x^(17*n)/n + 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c
^7*x^(17*n)/n + 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + 286
*a*b^10*c^3*x^(16*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n + 12012*a^3*b^6*c^5
*x^(16*n)/n + 15015*a^4*b^4*c^6*x^(16*n)/n + 5148*a^5*b^2*c^7*x^(16*n)/n +
 429/2*a^6*c^8*x^(16*n)/n + b^13*c*x^(15*n)/n + 78*a*b^11*c^2*x^(15*n)/n +
 1430*a^2*b^9*c^3*x^(15*n)/n + 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*...
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1693 vs. $2(21) = 42$.

Time = 0.38 (sec) , antiderivative size = 1693, normalized size of antiderivative = 73.61

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 14*a*c^1
3*x^(26*n) + 364*b^3*c^11*x^(25*n) + 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10
*x^(24*n) + 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9
*x^(23*n) + 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6
*c^8*x^(22*n) + 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) + 36
4*a^3*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 18018*a*b^5*c^8*x^(21*n) + 2
0020*a^2*b^3*c^9*x^(21*n) + 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*
n) + 24024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) + 20020*a^3*b^2
*c^9*x^(20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 24024*a*b
^7*c^6*x^(19*n) + 72072*a^2*b^5*c^7*x^(19*n) + 60060*a^3*b^3*c^8*x^(19*n)
+ 10010*a^4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 18018*a*b^8*c^5*x^(1
8*n) + 84084*a^2*b^6*c^6*x^(18*n) + 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^
4*b^2*c^8*x^(18*n) + 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) + 10010
*a*b^9*c^4*x^(17*n) + 72072*a^2*b^7*c^5*x^(17*n) + 168168*a^3*b^5*c^6*x^(1
7*n) + 120120*a^4*b^3*c^7*x^(17*n) + 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^
2*x^(16*n) + 4004*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) + 16816
8*a^3*b^6*c^5*x^(16*n) + 210210*a^4*b^4*c^6*x^(16*n) + 72072*a^5*b^2*c^7*x
^(16*n) + 3003*a^6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) + 1092*a*b^11*c^2*x^(
15*n) + 20020*a^2*b^9*c^3*x^(15*n) + 120120*a^3*b^7*c^4*x^(15*n) + 252252*
a^4*b^5*c^5*x^(15*n) + 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^...
```

3.96.9 Mupad [B] (verification not implemented)

Time = 11.02 (sec) , antiderivative size = 1395, normalized size of antiderivative = 60.65

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

input
```
int(x^(n - 1)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x)
```

output
```
x^(n - 1)*((x^(11*n + 1)*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 + 286*a^3*b^10
*c + (6435*a^4*b^8*c^2)/2 + 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 + 5148*a
^7*b^2*c^5))/n + (x^(15*n + 1)*((429*a^6*c^8)/2 + (13*b^12*c^2)/2 + 286*a*
b^10*c^3 + (6435*a^2*b^8*c^4)/2 + 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 +
5148*a^5*b^2*c^7))/n + (x^(12*n + 1)*(a*b^13 + 78*a^2*b^11*c + 1716*a^7*b*
c^6 + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*
b^3*c^5))/n + (x^(14*n + 1)*(b^13*c + 78*a*b^11*c^2 + 1716*a^6*b*c^7 + 143
0*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6))
/n + (x^(5*n + 1)*((429*a^8*b^6)/2 + 26*a^11*c^3 + 715*a^9*b^4*c + 429*a^1
0*b^2*c^2))/n + (x^(21*n + 1)*(26*a^3*c^11 + (429*b^6*c^8)/2 + 715*a*b^4*c
^9 + 429*a^2*b^2*c^10))/n + (x^(9*n + 1)*((143*a^4*b^10)/2 + 143*a^9*c^5 +
 1287*a^5*b^8*c + 6006*a^6*b^6*c^2 + 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)
/2))/n + (x^(17*n + 1)*(143*a^5*c^9 + (143*b^10*c^4)/2 + 1287*a*b^8*c^5 +
6006*a^2*b^6*c^6 + 8580*a^3*b^4*c^7 + (6435*a^4*b^2*c^8)/2))/n + (x^(13*n
+ 1)*(b^14/14 + (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 + 4290*a^3*b^8*c^3 + 1
5015*a^4*b^6*c^4 + 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6 + 13*a*b^12*c))/n
+ (x^(7*n + 1)*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 + 1716*a^7*b^6*c + (643
5*a^8*b^4*c^2)/2 + 1430*a^9*b^2*c^3))/n + (x^(19*n + 1)*((143*a^4*c^10)/2
+ (429*b^8*c^6)/2 + 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*a^3*b^2*c
^9))/n + (c^14*x^(27*n + 1))/(14*n) + (a^12*x^(n + 1)*(a*c + (13*b^2)/2...
```

3.96. $\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx$

3.97 $\int (b+2cx) \left(-a+bx+cx^2\right)^{13} \, dx$

3.97.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int (b+2cx) \left(-a+bx+cx^2\right)^{13} \, dx = \frac{1}{14} \left(a-bx-cx^2\right)^{14}$$

output
```
1/14*(-c*x^2-b*x+a)^14
```

3.97.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(18) = 36.

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 11.17

$$\begin{aligned}\int (b+2cx) \left(-a+bx+cx^2\right)^{13} \, dx = \frac{1}{14}x(b+cx)\left(-14a^{13}+91a^{12}x(b+cx)-364a^{11}x^2(b+cx)^2\right.\\ +1001a^{10}x^3(b+cx)^3-2002a^9x^4(b+cx)^4\\ +3003a^8x^5(b+cx)^5-3432a^7x^6(b+cx)^6\\ +3003a^6x^7(b+cx)^7-2002a^5x^8(b+cx)^8\\ +1001a^4x^9(b+cx)^9-364a^3x^{10}(b+cx)^{10}\\ \left.+91a^2x^{11}(b+cx)^{11}-14ax^{12}(b+cx)^{12}+x^{13}(b+cx)^{13}\right)\end{aligned}$$

input
```
Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

output
```
(x*(b + c*x)*(-14*a^13 + 91*a^12*x*(b + c*x) - 364*a^11*x^2*(b + c*x)^2 +
1001*a^10*x^3*(b + c*x)^3 - 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c
*x)^5 - 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 - 2002*a^5*x^8
*(b + c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 - 364*a^3*x^10*(b + c*x)^10 + 91*a
^2*x^11*(b + c*x)^11 - 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

3.97.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{1104\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx$$

$$\Big\downarrow \ 1104$$

$$\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

input
```
Int[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

output
```
(a - b*x - c*x^2)^14/14
```

3.97.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.97.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(c x^2+b x-a)^{14}}{14}$	17
norman	Expression too large to display	1228
gosper	Expression too large to display	1451
parallelrisch	Expression too large to display	1451
risch	Expression too large to display	1456

input
```
int((2*c*x+b)*(c*x^2+b*x-a)^13,x,method=_RETURNVERBOSE)
```

output
```
1/14*(c*x^2+b*x-a)^14
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 1238, normalized size of antiderivative = 68.78

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx=\text{Too large to display}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="fricas")
```

output
```
1/14*c^14*x^28 + b*c^13*x^27 + 1/2*(13*b^2*c^12 - 2*a*c^13)*x^26 + 13*(2*b
^3*c^11 - a*b*c^12)*x^25 + 13/2*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^12)*x
^24 + 13*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^23 + 13/2*(33*b^6*c
^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^22 + 143/7*(12*b^7*c^
7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^21 + 143/2*(3*b^8*c^6
 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^20 + 143*(
b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^
19 + 143/2*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7 + 4
5*a^4*b^2*c^8 - 2*a^5*c^9)*x^18 + 13*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*a^2*
b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^17 + 13/2*(b
^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^
4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^16 + (b^13*c - 78*a*b^11*c^2 + 143
0*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6 +
 1716*a^6*b*c^7)*x^15 - a^13*b*x + 1/14*(b^14 - 182*a*b^12*c + 6006*a^2*b^
10*c^2 - 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 840
84*a^6*b^2*c^6 - 3432*a^7*c^7)*x^14 - (a*b^13 - 78*a^2*b^11*c + 1430*a^3*b
^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a
^7*b*c^6)*x^13 + 13/2*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a
^5*b^6*c^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^12 - 13*(2
*a^3*b^11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*...
```

3.97.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. $2(12) = 24$.

Time = 0.15 (sec) , antiderivative size = 1326, normalized size of antiderivative = 73.67

$$\int (b + 2cx) \left(-a + bx + cx^2\right)^{13} \, dx = \text{Too large to display}$$

input
```
integrate((2*c*x+b)*(c*x**2+b*x-a)**13,x)
```

output
```
-a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(-a*c**13 + 13*b**2*c*
*12/2) + x**25*(-13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 -
78*a*b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 - 286*a*b**3*
c**10 + 143*b**5*c**9) + x**22*(-26*a**3*c**11 + 429*a**2*b**2*c**10 - 715
*a*b**4*c**9 + 429*b**6*c**8/2) + x**21*(-286*a**3*b*c**10 + 1430*a**2*b**
3*c**9 - 1287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 -
1430*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/2 - 1716*a*b**6*c**7 + 429*b**8*
c**6/2) + x**19*(715*a**4*b*c**9 - 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c*
*7 - 1716*a*b**7*c**6 + 143*b**9*c**5) + x**18*(-143*a**5*c**9 + 6435*a**4
*b**2*c**8/2 - 8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 - 1287*a*b**8*c**
5 + 143*b**10*c**4/2) + x**17*(-1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 - 1
2012*a**3*b**5*c**6 + 5148*a**2*b**7*c**5 - 715*a*b**9*c**4 + 26*b**11*c**
3) + x**16*(429*a**6*c**8/2 - 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 -
 12012*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/2 - 286*a*b**10*c**3 + 13*b**1
2*c**2/2) + x**15*(1716*a**6*b*c**7 - 12012*a**5*b**3*c**6 + 18018*a**4*b*
*5*c**5 - 8580*a**3*b**7*c**4 + 1430*a**2*b**9*c**3 - 78*a*b**11*c**2 + b*
*13*c) + x**14*(-1716*a**7*c**7/7 + 6006*a**6*b**2*c**6 - 18018*a**5*b**4*
c**5 + 15015*a**4*b**6*c**4 - 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 -
13*a*b**12*c + b**14/14) + x**13*(-1716*a**7*b*c**6 + 12012*a**6*b**3*c**5
 - 18018*a**5*b**5*c**4 + 8580*a**4*b**7*c**3 - 1430*a**3*b**9*c**2 + 7...
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(cx^2+bx-a\right)^{14}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="maxima")
```

output
```
1/14*(c*x^2 + b*x - a)^14
```

3.97.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(16) = 32$.

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 12.11

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx = \frac{1}{14}\left(cx^2+bx\right)^{14} - \left(cx^2+bx\right)^{13}a + \frac{13}{2}\left(cx^2+bx\right)^{12}a^2 - 26\left(cx^2+bx\right)^{11}a^3 + \frac{143}{2}\left(cx^2+bx\right)^{10}a^4 - 143\left(cx^2+bx\right)^{9}a^5 + \frac{429}{2}\left(cx^2+bx\right)^{8}a^6 - \frac{1716}{7}\left(cx^2+bx\right)^{7}a^7 + \frac{429}{2}\left(cx^2+bx\right)^{6}a^8 - 143\left(cx^2+bx\right)^{5}a^9 + \frac{143}{2}\left(cx^2+bx\right)^{4}a^{10} - 26\left(cx^2+bx\right)^{3}a^{11} + \frac{13}{2}\left(cx^2+bx\right)^{2}a^{12} - \left(cx^2+bx\right)a^{13}$$

input

```
integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="giac")
```

output

```
1/14*(c*x^2 + b*x)^14 - (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 - 2
6*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 - 143*(c*x^2 + b*x)^9*
a^5 + 429/2*(c*x^2 + b*x)^8*a^6 - 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^
2 + b*x)^6*a^8 - 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 - 26
*(c*x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 - (c*x^2 + b*x)*a^13
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 1208, normalized size of antiderivative = 67.11

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx = x^{12}\left(\frac{429\,a^8\,c^6}{2}-5148\,a^7\,b^2\,c^5+15015\,a^6\,b^4\,c^4-12012\,a^5\,b^6\,c^3+\frac{6435\,a^4\,b^8\,c^2}{2}-286\,a^3\,b^{10}\,c+\frac{13\,a^2\,b^{12}}{2}\right)$$

$$+x^{16}\left(\frac{429\,a^6\,c^8}{2}-5148\,a^5\,b^2\,c^7+15015\,a^4\,b^4\,c^6-12012\,a^3\,b^6\,c^5+\frac{6435\,a^2\,b^8\,c^4}{2}-286\,a\,b^{10}\,c^3+\frac{13\,b^{12}\,c^2}{2}\right)$$

$$-x^{13}\left(1716\,a^7\,b\,c^6-12012\,a^6\,b^3\,c^5+18018\,a^5\,b^5\,c^4-8580\,a^4\,b^7\,c^3+1430\,a^3\,b^9\,c^2-78\,a^2\,b^{11}\,c+a\,b^{13}\right)$$

$$+x^{15}\left(1716\,a^6\,b\,c^7-12012\,a^5\,b^3\,c^6+18018\,a^4\,b^5\,c^5-8580\,a^3\,b^7\,c^4+1430\,a^2\,b^9\,c^3-78\,a\,b^{11}\,c^2+b^{13}\,c\right)$$

$$+x^6\left(-26\,a^{11}\,c^3+429\,a^{10}\,b^2\,c^2-715\,a^9\,b^4\,c+\frac{429\,a^8\,b^6}{2}\right)-x^{22}\left(26\,a^3\,c^{11}-429\,a^2\,b^2\,c^{10}+715\,a\,b^4\,c^9-\frac{429\,b^6\,c^8}{2}\right)$$

$$+x^{10}\left(-143\,a^9\,c^5+\frac{6435\,a^8\,b^2\,c^4}{2}-8580\,a^7\,b^4\,c^3+6006\,a^6\,b^6\,c^2-1287\,a^5\,b^8\,c+\frac{143\,a^4\,b^{10}}{2}\right)$$

$$-x^{18}\left(143\,a^5\,c^9-\frac{6435\,a^4\,b^2\,c^8}{2}+8580\,a^3\,b^4\,c^7-6006\,a^2\,b^6\,c^6+1287\,a\,b^8\,c^5-\frac{143\,b^{10}\,c^4}{2}\right)$$

$$+x^{14}\left(-\frac{1716\,a^7\,c^7}{7}+6006\,a^6\,b^2\,c^6-18018\,a^5\,b^4\,c^5+15015\,a^4\,b^6\,c^4-4290\,a^3\,b^8\,c^3+429\,a^2\,b^{10}\,c^2-13\,a\,b^{12}\,c+\frac{b^{14}}{14}\right)$$

$$+x^8\left(\frac{143\,a^{10}\,c^4}{2}-1430\,a^9\,b^2\,c^3+\frac{6435\,a^8\,b^4\,c^2}{2}-1716\,a^7\,b^6\,c+\frac{429\,a^6\,b^8}{2}\right)+x^{20}\left(\frac{143\,a^4\,c^{10}}{2}-1430\,a^3\,b^2\,c^9+\frac{6435\,a^2\,b^4\,c^8}{2}-1716\,a\,b^6\,c^7+\frac{429\,b^8\,c^6}{2}\right)$$

$$+\frac{c^{14}\,x^{28}}{14}-x^2\left(a^{13}\,c-\frac{13\,a^{12}\,b^2}{2}\right)+\frac{13\,a^{10}\,x^4\,\left(a^2\,c^2-12\,a\,b^2\,c+11\,b^4\right)}{2}+\frac{13\,c^{10}\,x^{24}\,\left(a^2\,c^2-12\,a\,b^2\,c+11\,b^4\right)}{2}$$

$$c^{12}\,x^{26}\,(2\,a\,c-13\,b^2)$$

input
```
int((b + 2*c*x)*(b*x - a + c*x^2)^13,x)
```

output
```
x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 - 286*a^3*b^10*c + (6435*a^4*b^8*c
^2)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a^7*b^2*c^5) + x^16*(
(429*a^6*c^8)/2 + (13*b^12*c^2)/2 - 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2
- 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 - 5148*a^5*b^2*c^7) - x^13*(a*b^13
 - 78*a^2*b^11*c + 1716*a^7*b*c^6 + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 +
18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5) + x^15*(b^13*c - 78*a*b^11*c^2 + 17
16*a^6*b*c^7 + 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 1
2012*a^5*b^3*c^6) + x^6*((429*a^8*b^6)/2 - 26*a^11*c^3 - 715*a^9*b^4*c + 4
29*a^10*b^2*c^2) - x^22*(26*a^3*c^11 - (429*b^6*c^8)/2 + 715*a*b^4*c^9 - 4
29*a^2*b^2*c^10) + x^10*((143*a^4*b^10)/2 - 143*a^9*c^5 - 1287*a^5*b^8*c +
 6006*a^6*b^6*c^2 - 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2) - x^18*(143*a
^5*c^9 - (143*b^10*c^4)/2 + 1287*a*b^8*c^5 - 6006*a^2*b^6*c^6 + 8580*a^3*b
^4*c^7 - (6435*a^4*b^2*c^8)/2) + x^14*(b^14/14 - (1716*a^7*c^7)/7 + 429*a^
2*b^10*c^2 - 4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^4 - 18018*a^5*b^4*c^5 + 60
06*a^6*b^2*c^6 - 13*a*b^12*c) + x^8*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 -
1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2 - 1430*a^9*b^2*c^3) + x^20*((143*a^4
*c^10)/2 + (429*b^8*c^6)/2 - 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 - 1430*
a^3*b^2*c^9) + (c^14*x^28)/14 - x^2*(a^13*c - (13*a^12*b^2)/2) + (13*a^10*
x^4*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/2 + (13*c^10*x^24*(11*b^4 + a^2*c^2 -
 12*a*b^2*c))/2 + b*c^13*x^27 - (c^12*x^26*(2*a*c - 13*b^2))/2 - a^13*b...
```

3.97. $\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx$

3.98 $\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^{13}\,dx$

3.98.1 Optimal result

Integrand size = 26, antiderivative size = 20

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx=\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

output
```
1/28*(-c*x^4-b*x^2+a)^14
```

3.98.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(20) = 40.

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 11.65

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx &= \frac{1}{28}x^2\left(b+cx^2\right)\Big(-14a^{13}+91a^{12}x^2\left(b+cx^2\right)\\
&\quad-364a^{11}x^4\left(b+cx^2\right)^2+1001a^{10}x^6\left(b+cx^2\right)^3\\
&\quad-2002a^9x^8\left(b+cx^2\right)^4+3003a^8x^{10}\left(b+cx^2\right)^5\\
&\quad-3432a^7x^{12}\left(b+cx^2\right)^6+3003a^6x^{14}\left(b+cx^2\right)^7\\
&\quad-2002a^5x^{16}\left(b+cx^2\right)^8+1001a^4x^{18}\left(b+cx^2\right)^9\\
&\quad-364a^3x^{20}\left(b+cx^2\right)^{10}+91a^2x^{22}\left(b+cx^2\right)^{11}\\
&\quad-14ax^{24}\left(b+cx^2\right)^{12}+x^{26}\left(b+cx^2\right)^{13}\Big)\end{aligned}$$

input
```
Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

output
```
(x^2*(b + c*x^2)*(-14*a^13 + 91*a^12*x^2*(b + c*x^2) - 364*a^11*x^4*(b + c
*x^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 - 2002*a^9*x^8*(b + c*x^2)^4 + 3003*
a^8*x^10*(b + c*x^2)^5 - 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b +
c*x^2)^7 - 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 - 364
*a^3*x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 - 14*a*x^24*(b + c*x
^2)^12 + x^26*(b + c*x^2)^13))/28
```

3.98.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1576, 25, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}dx$$

$$\Big\downarrow \ 1576$$

$$\frac{1}{2}\int -\left(\left(2cx^2+b\right)\left(-cx^4-bx^2+a\right)^{13}\right)dx^2$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{2}\int \left(2cx^2+b\right)\left(-cx^4-bx^2+a\right)^{13}dx^2$$

$$\Big\downarrow \ 1104$$

$$\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

input
```
Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

output
```
(a - b*x^2 - c*x^4)^14/28
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.98.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{(c x^4+b x^2-a)^{14}}{28}$	19
parallelrisch	Expression too large to display	1455
gosper	Expression too large to display	1457
risch	Expression too large to display	1460

input
```
int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x,method=_RETURNVERBOSE)
```

output
```
1/28*(c*x^4+b*x^2-a)^14
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 1242, normalized size of antiderivative = 62.10

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="fricas")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 - 2*a*c^13)*x^52 + 13/
2*(2*b^3*c^11 - a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c
^12)*x^48 + 13/2*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(
33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^44 + 143/14*(
12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^42 + 143/4*(
3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^4
0 + 143/2*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^
4*b*c^9)*x^38 + 143/4*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*
b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 - 55*a*b^9*c
^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x
^34 + 13/4*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5
+ 2310*a^4*b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c - 78
*a*b^11*c^2 + 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12
012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 - 182*a*b^12*c + 6006*
a^2*b^10*c^2 - 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5
 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^28 - 1/2*(a*b^13 - 78*a^2*b^11*c +
1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^
5 + 1716*a^7*b*c^6)*x^26 + 13/4*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^
2 - 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^
24 - 13/2*(2*a^3*b^11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^...
```

3.98.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1384 vs. $2(14) = 28$.

Time = 0.14 (sec) , antiderivative size = 1384, normalized size of antiderivative = 69.20

$$\int x(b + 2cx^2)\left(-a + bx^2 + cx^4\right)^{13} dx = \text{Too large to display}$$

input
```
integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**13,x)
```

output
```
-a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(-a*c**13/2 + 1
3*b**2*c**12/4) + x**50*(-13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2
*c**12/4 - 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 -
143*a*b**3*c**10 + 143*b**5*c**9/2) + x**44*(-13*a**3*c**11 + 429*a**2*b**
2*c**10/2 - 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(-143*a**3*b*c**1
0 + 715*a**2*b**3*c**9 - 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(14
3*a**4*c**10/4 - 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 - 858*a*b**6*c
**7 + 429*b**8*c**6/4) + x**38*(715*a**4*b*c**9/2 - 2145*a**3*b**3*c**8 +
2574*a**2*b**5*c**7 - 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(-143*a**
5*c**9/2 + 6435*a**4*b**2*c**8/4 - 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c*
*6 - 1287*a*b**8*c**5/2 + 143*b**10*c**4/4) + x**34*(-1287*a**5*b*c**8/2 +
 4290*a**4*b**3*c**7 - 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 - 715*a*b
**9*c**4/2 + 13*b**11*c**3) + x**32*(429*a**6*c**8/4 - 2574*a**5*b**2*c**7
 + 15015*a**4*b**4*c**6/2 - 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 -
143*a*b**10*c**3 + 13*b**12*c**2/4) + x**30*(858*a**6*b*c**7 - 6006*a**5*b
**3*c**6 + 9009*a**4*b**5*c**5 - 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3
- 39*a*b**11*c**2 + b**13*c/2) + x**28*(-858*a**7*c**7/7 + 3003*a**6*b**2*
c**6 - 9009*a**5*b**4*c**5 + 15015*a**4*b**6*c**4/2 - 2145*a**3*b**8*c**3
+ 429*a**2*b**10*c**2/2 - 13*a*b**12*c/2 + b**14/28) + x**26*(-858*a**7*b*
c**6 + 6006*a**6*b**3*c**5 - 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 ...
```

3.98.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(18) = 36$.

Time = 0.20 (sec) , antiderivative size = 1242, normalized size of antiderivative = 62.10

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="maxima")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 - 2*a*c^13)*x^52 + 13/
2*(2*b^3*c^11 - a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c
^12)*x^48 + 13/2*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(
33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^44 + 143/14*(
12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^42 + 143/4*(
3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^4
0 + 143/2*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^
4*b*c^9)*x^38 + 143/4*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*
b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 - 55*a*b^9*c
^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x
^34 + 13/4*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5
+ 2310*a^4*b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c - 78
*a*b^11*c^2 + 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12
012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 - 182*a*b^12*c + 6006*
a^2*b^10*c^2 - 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5
 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^28 - 1/2*(a*b^13 - 78*a^2*b^11*c +
1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^
5 + 1716*a^7*b*c^6)*x^26 + 13/4*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^
2 - 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^
24 - 13/2*(2*a^3*b^11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^...
```

3.98.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(18) = 36$.

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 12.30

$$\begin{aligned}\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^{13}\,dx &= \frac{1}{28}\left(cx^4+bx^2\right)^{14}-\frac{1}{2}\left(cx^4+bx^2\right)^{13}a\\&+\frac{13}{4}\left(cx^4+bx^2\right)^{12}a^2-13\left(cx^4+bx^2\right)^{11}a^3\\&+\frac{143}{4}\left(cx^4+bx^2\right)^{10}a^4-\frac{143}{2}\left(cx^4+bx^2\right)^{9}a^5\\&+\frac{429}{4}\left(cx^4+bx^2\right)^{8}a^6-\frac{858}{7}\left(cx^4+bx^2\right)^{7}a^7\\&+\frac{429}{4}\left(cx^4+bx^2\right)^{6}a^8-\frac{143}{2}\left(cx^4+bx^2\right)^{5}a^9\\&+\frac{143}{4}\left(cx^4+bx^2\right)^{4}a^{10}-13\left(cx^4+bx^2\right)^{3}a^{11}\\&+\frac{13}{4}\left(cx^4+bx^2\right)^{2}a^{12}-\frac{1}{2}\left(cx^4+bx^2\right)a^{13}\end{aligned}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="giac")
```

output
```
1/28*(c*x^4 + b*x^2)^14 - 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^
12*a^2 - 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 - 143/2*
(c*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 - 858/7*(c*x^4 + b*x^2
)^7*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 - 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/
4*(c*x^4 + b*x^2)^4*a^10 - 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2
)^2*a^12 - 1/2*(c*x^4 + b*x^2)*a^13
```

3.98.9 Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 1214, normalized size of antiderivative = 60.70

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

input
```
int(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^13,x)
```

output
```
x^24*((13*a^2*b^12)/4 + (429*a^8*c^6)/4 - 143*a^3*b^10*c + (6435*a^4*b^8*c
^2)/4 - 6006*a^5*b^6*c^3 + (15015*a^6*b^4*c^4)/2 - 2574*a^7*b^2*c^5) + x^3
2*((429*a^6*c^8)/4 + (13*b^12*c^2)/4 - 143*a*b^10*c^3 + (6435*a^2*b^8*c^4)
/4 - 6006*a^3*b^6*c^5 + (15015*a^4*b^4*c^6)/2 - 2574*a^5*b^2*c^7) - x^26*(
(a*b^13)/2 - 39*a^2*b^11*c + 858*a^7*b*c^6 + 715*a^3*b^9*c^2 - 4290*a^4*b^
7*c^3 + 9009*a^5*b^5*c^4 - 6006*a^6*b^3*c^5) + x^30*((b^13*c)/2 - 39*a*b^1
1*c^2 + 858*a^6*b*c^7 + 715*a^2*b^9*c^3 - 4290*a^3*b^7*c^4 + 9009*a^4*b^5*
c^5 - 6006*a^5*b^3*c^6) + x^12*((429*a^8*b^6)/4 - 13*a^11*c^3 - (715*a^9*b
^4*c)/2 + (429*a^10*b^2*c^2)/2) - x^44*(13*a^3*c^11 - (429*b^6*c^8)/4 + (7
15*a*b^4*c^9)/2 - (429*a^2*b^2*c^10)/2) + x^20*((143*a^4*b^10)/4 - (143*a^
9*c^5)/2 - (1287*a^5*b^8*c)/2 + 3003*a^6*b^6*c^2 - 4290*a^7*b^4*c^3 + (643
5*a^8*b^2*c^4)/4) - x^36*((143*a^5*c^9)/2 - (143*b^10*c^4)/4 + (1287*a*b^8
*c^5)/2 - 3003*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 - (6435*a^4*b^2*c^8)/4) + x^
28*(b^14/28 - (858*a^7*c^7)/7 + (429*a^2*b^10*c^2)/2 - 2145*a^3*b^8*c^3 +
(15015*a^4*b^6*c^4)/2 - 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 - (13*a*b^12*c
)/2) + x^16*((429*a^6*b^8)/4 + (143*a^10*c^4)/4 - 858*a^7*b^6*c + (6435*a^
8*b^4*c^2)/4 - 715*a^9*b^2*c^3) + x^40*((143*a^4*c^10)/4 + (429*b^8*c^6)/4
 - 858*a*b^6*c^7 + (6435*a^2*b^4*c^8)/4 - 715*a^3*b^2*c^9) + (c^14*x^56)/2
8 - x^4*((a^13*c)/2 - (13*a^12*b^2)/4) + (13*a^10*x^8*(11*b^4 + a^2*c^2 -
12*a*b^2*c))/4 + (13*c^10*x^48*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/4 - (a^...
```

3.99 $\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx$

3.99.1 Optimal result

Integrand size = 28, antiderivative size = 20

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx=\frac{1}{42}\left(a-bx^3-cx^6\right)^{14}$$

output
```
1/42*(-c*x^6-b*x^3+a)^14
```

3.99.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(20) = 40.

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 11.65

$$\begin{aligned}\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx=\frac{1}{42}x^3(b+cx^3)\Big(&-14a^{13}+91a^{12}x^3(b+cx^3)\\&-364a^{11}x^6\left(b+cx^3\right)^2+1001a^{10}x^9\left(b+cx^3\right)^3\\&-2002a^9x^{12}\left(b+cx^3\right)^4+3003a^8x^{15}\left(b+cx^3\right)^5\\&-3432a^7x^{18}\left(b+cx^3\right)^6+3003a^6x^{21}\left(b+cx^3\right)^7\\&-2002a^5x^{24}\left(b+cx^3\right)^8+1001a^4x^{27}\left(b+cx^3\right)^9\\&-364a^3x^{30}\left(b+cx^3\right)^{10}+91a^2x^{33}\left(b+cx^3\right)^{11}\\&-14ax^{36}\left(b+cx^3\right)^{12}+x^{39}\left(b+cx^3\right)^{13}\Big)\end{aligned}$$

input
```
Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

output
```
(x^3*(b + c*x^3)*(-14*a^13 + 91*a^12*x^3*(b + c*x^3) - 364*a^11*x^6*(b + c
*x^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 - 2002*a^9*x^12*(b + c*x^3)^4 + 3003
*a^8*x^15*(b + c*x^3)^5 - 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b +
 c*x^3)^7 - 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 - 36
4*a^3*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 - 14*a*x^36*(b + c*
x^3)^12 + x^39*(b + c*x^3)^13))/42
```

3.99.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1798, 25, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(b+2 c x^3\right) \left(-a+b x^3+c x^6\right)^{13} \, dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{3} \int -\left(\left(2 c x^3+b\right) \left(-c x^6-b x^3+a\right)^{13}\right)dx^3$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3} \int \left(2 c x^3+b\right) \left(-c x^6-b x^3+a\right)^{13}dx^3$$

$$\Big\downarrow \text{1104}$$

$$\frac{1}{42} \left(a-b x^3-c x^6\right)^{14}$$

input
```
Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

output
```
(a - b*x^3 - c*x^6)^14/42
```

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.99.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{(c\,x^{6}+b\,x^{3}-a)^{14}}{42}$	19
parallelrisch	Expression too large to display	1455
gosper	Expression too large to display	1457
risch	Expression too large to display	1460

input
```
int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x,method=_RETURNVERBOSE)
```

output
```
1/42*(c*x^6+b*x^3-a)^14
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 1242, normalized size of antiderivative = 62.10

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="fricas")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 - 2*a*c^13)*x^78 + 13/
3*(2*b^3*c^11 - a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c
^12)*x^72 + 13/3*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(
33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^66 + 143/21*(
12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^63 + 143/6*(
3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^6
0 + 143/3*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^
4*b*c^9)*x^57 + 143/6*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*
b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 - 55*a*b^9*c
^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x
^51 + 13/6*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5
+ 2310*a^4*b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c - 78
*a*b^11*c^2 + 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12
012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 - 182*a*b^12*c + 6006*
a^2*b^10*c^2 - 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5
 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^42 - 1/3*(a*b^13 - 78*a^2*b^11*c +
1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^
5 + 1716*a^7*b*c^6)*x^39 + 13/6*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^
2 - 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^
36 - 13/3*(2*a^3*b^11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^...
```

3.99.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(14) = 28$.

Time = 0.14 (sec) , antiderivative size = 1394, normalized size of antiderivative = 69.70

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**13,x)
```

output
```
-a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(-a*c**13/3 + 1
3*b**2*c**12/6) + x**75*(-13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a*
*2*c**12/6 - 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11
- 286*a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(-26*a**3*c**11/3 + 143*a*
*2*b**2*c**10 - 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(-286*a**3*b*
c**10/3 + 1430*a**2*b**3*c**9/3 - 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**
60*(143*a**4*c**10/6 - 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 - 572
*a*b**6*c**7 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 - 1430*a**3*b**
3*c**8 + 1716*a**2*b**5*c**7 - 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*
(-143*a**5*c**9/3 + 2145*a**4*b**2*c**8/2 - 2860*a**3*b**4*c**7 + 2002*a**
2*b**6*c**6 - 429*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(-429*a**5*b*c**
8 + 2860*a**4*b**3*c**7 - 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 - 715*
a*b**9*c**4/3 + 26*b**11*c**3/3) + x**48*(143*a**6*c**8/2 - 1716*a**5*b**2
*c**7 + 5005*a**4*b**4*c**6 - 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2
- 286*a*b**10*c**3/3 + 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 - 4004*a*
*5*b**3*c**6 + 6006*a**4*b**5*c**5 - 2860*a**3*b**7*c**4 + 1430*a**2*b**9*
c**3/3 - 26*a*b**11*c**2 + b**13*c/3) + x**42*(-572*a**7*c**7/7 + 2002*a**
6*b**2*c**6 - 6006*a**5*b**4*c**5 + 5005*a**4*b**6*c**4 - 1430*a**3*b**8*c
**3 + 143*a**2*b**10*c**2 - 13*a*b**12*c/3 + b**14/42) + x**39*(-572*a**7*
b*c**6 + 4004*a**6*b**3*c**5 - 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**...
```

3.99.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(18) = 36$.

Time = 0.21 (sec) , antiderivative size = 1242, normalized size of antiderivative = 62.10

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="maxima")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 - 2*a*c^13)*x^78 + 13/
3*(2*b^3*c^11 - a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c
^12)*x^72 + 13/3*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(
33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^66 + 143/21*(
12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^63 + 143/6*(
3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^6
0 + 143/3*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^
4*b*c^9)*x^57 + 143/6*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*
b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 - 55*a*b^9*c
^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x
^51 + 13/6*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5
+ 2310*a^4*b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c - 78
*a*b^11*c^2 + 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12
012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 - 182*a*b^12*c + 6006*
a^2*b^10*c^2 - 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5
 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^42 - 1/3*(a*b^13 - 78*a^2*b^11*c +
1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^
5 + 1716*a^7*b*c^6)*x^39 + 13/6*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^
2 - 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^
36 - 13/3*(2*a^3*b^11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^...
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(18) = 36$.

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 12.30

$$\begin{aligned}\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx =& \frac{1}{42}\left(cx^6+bx^3\right)^{14}-\frac{1}{3}\left(cx^6+bx^3\right)^{13}a \\ &+\frac{13}{6}\left(cx^6+bx^3\right)^{12}a^2-\frac{26}{3}\left(cx^6+bx^3\right)^{11}a^3 \\ &+\frac{143}{6}\left(cx^6+bx^3\right)^{10}a^4-\frac{143}{3}\left(cx^6+bx^3\right)^{9}a^5 \\ &+\frac{143}{2}\left(cx^6+bx^3\right)^{8}a^6-\frac{572}{7}\left(cx^6+bx^3\right)^{7}a^7 \\ &+\frac{143}{2}\left(cx^6+bx^3\right)^{6}a^8-\frac{143}{3}\left(cx^6+bx^3\right)^{5}a^9 \\ &+\frac{143}{6}\left(cx^6+bx^3\right)^{4}a^{10}-\frac{26}{3}\left(cx^6+bx^3\right)^{3}a^{11} \\ &+\frac{13}{6}\left(cx^6+bx^3\right)^{2}a^{12}-\frac{1}{3}\left(cx^6+bx^3\right)a^{13}\end{aligned}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="giac")
```

output
```
1/42*(c*x^6 + b*x^3)^14 - 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^
12*a^2 - 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 - 143/
3*(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 - 572/7*(c*x^6 + b*x
^3)^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 - 143/3*(c*x^6 + b*x^3)^5*a^9 + 14
3/6*(c*x^6 + b*x^3)^4*a^10 - 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b
*x^3)^2*a^12 - 1/3*(c*x^6 + b*x^3)*a^13
```

3.99.9 Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 1214, normalized size of antiderivative = 60.70

$$\int x^2 \left(b + 2cx^3\right) \left(-a + bx^3 + cx^6\right)^{13} \, dx = \text{Too large to display}$$

input
```
int(x^2*(b + 2*c*x^3)*(b*x^3 - a + c*x^6)^13,x)
```

output
```
x^36*((13*a^2*b^12)/6 + (143*a^8*c^6)/2 - (286*a^3*b^10*c)/3 + (2145*a^4*b
^8*c^2)/2 - 4004*a^5*b^6*c^3 + 5005*a^6*b^4*c^4 - 1716*a^7*b^2*c^5) + x^48
*((143*a^6*c^8)/2 + (13*b^12*c^2)/6 - (286*a*b^10*c^3)/3 + (2145*a^2*b^8*c
^4)/2 - 4004*a^3*b^6*c^5 + 5005*a^4*b^4*c^6 - 1716*a^5*b^2*c^7) - x^39*((a
*b^13)/3 - 26*a^2*b^11*c + 572*a^7*b*c^6 + (1430*a^3*b^9*c^2)/3 - 2860*a^4
*b^7*c^3 + 6006*a^5*b^5*c^4 - 4004*a^6*b^3*c^5) + x^45*((b^13*c)/3 - 26*a*
b^11*c^2 + 572*a^6*b*c^7 + (1430*a^2*b^9*c^3)/3 - 2860*a^3*b^7*c^4 + 6006*
a^4*b^5*c^5 - 4004*a^5*b^3*c^6) + x^18*((143*a^8*b^6)/2 - (26*a^11*c^3)/3
- (715*a^9*b^4*c)/3 + 143*a^10*b^2*c^2) - x^66*((26*a^3*c^11)/3 - (143*b^6
*c^8)/2 + (715*a*b^4*c^9)/3 - 143*a^2*b^2*c^10) + x^30*((143*a^4*b^10)/6 -
 (143*a^9*c^5)/3 - 429*a^5*b^8*c + 2002*a^6*b^6*c^2 - 2860*a^7*b^4*c^3 + (
2145*a^8*b^2*c^4)/2) - x^54*((143*a^5*c^9)/3 - (143*b^10*c^4)/6 + 429*a*b^
8*c^5 - 2002*a^2*b^6*c^6 + 2860*a^3*b^4*c^7 - (2145*a^4*b^2*c^8)/2) + x^42
*(b^14/42 - (572*a^7*c^7)/7 + 143*a^2*b^10*c^2 - 1430*a^3*b^8*c^3 + 5005*a
^4*b^6*c^4 - 6006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 - (13*a*b^12*c)/3) + x^24
*((143*a^6*b^8)/2 + (143*a^10*c^4)/6 - 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/
2 - (1430*a^9*b^2*c^3)/3) + x^60*((143*a^4*c^10)/6 + (143*b^8*c^6)/2 - 572
*a*b^6*c^7 + (2145*a^2*b^4*c^8)/2 - (1430*a^3*b^2*c^9)/3) + (c^14*x^84)/42
 - x^6*((a^13*c)/3 - (13*a^12*b^2)/6) + (13*a^10*x^12*(11*b^4 + a^2*c^2 -
12*a*b^2*c))/6 + (13*c^10*x^72*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/6 - (a^...
```

3.100 $\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx$

3.100.1 Optimal result

Integrand size = 32, antiderivative size = 25

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx=\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

output
```
1/14*(a-b*x^n-c*x^(2*n))^14/n
```

3.100.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx=\frac{(-a+x^n(b+cx^n))^{14}}{14n}$$

input
```
Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

output
```
(-a + x^n*(b + c*x^n))^14/(14*n)
```

3.100.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1798, 25, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{\int -\left((2cx^n+b)\left(-bx^n-cx^{2n}+a\right)^{13}\right)dx^n}{n}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int (2cx^n+b)\left(-bx^n-cx^{2n}+a\right)^{13}dx^n}{n}$$

$$\Big\downarrow \text{1104}$$

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

input
```
Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

output
```
(a - b*x^n - c*x^(2*n))^14/(14*n)
```

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.100.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2045 vs. $2(23) = 46$.

Time = 0.02 (sec) , antiderivative size = 2046, normalized size of antiderivative = 81.84

Expression too large to display

input
```
int(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x)
```

output
```
286*a^10*b/n*(x^n)^7*c^3-1430*a^9*b^3/n*(x^n)^7*c^2+1287*a^8*b^5/n*(x^n)^7
*c+429*a^10/n*(x^n)^6*b^2*c^2+429/2*a^6/n*(x^n)^8*b^8+143/2*a^10/n*(x^n)^8
*c^4-26*a^11*b^3/n*(x^n)^3-c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)^26*b^2+13/2
*a^12/n*(x^n)^2*b^2+13/2*c^12/n*(x^n)^24*a^2+143/2*c^10/n*(x^n)^24*b^4+143
/2*c^10/n*(x^n)^20*a^4+429/2*c^6/n*(x^n)^20*b^8-26*a^11/n*(x^n)^6*c^3+429/
2*a^8/n*(x^n)^6*b^6+13/2*a^12/n*(x^n)^4*c^2+143/2*a^10/n*(x^n)^4*b^4+1/14*
c^14/n*(x^n)^28+1/14/n*(x^n)^14*b^14+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(
x^n)^12*b^12-143*a^9/n*(x^n)^10*c^5+143/2*a^4/n*(x^n)^10*b^10-a^13/n*(x^n)
^2*c+715*b*c^9/n*(x^n)^19*a^4-4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(
x^n)^19*a^2-1716*b^7*c^6/n*(x^n)^19*a-1716/7/n*(x^n)^14*a^7*c^7-143*a^5*b^
9/n*(x^n)^9+b^13*c/n*(x^n)^15+26*c^11*b^3/n*(x^n)^25-143*c^9/n*(x^n)^18*a^
5+143/2*c^4/n*(x^n)^18*b^10+1716/7*b^7*c^7/n*(x^n)^21-26*c^11/n*(x^n)^22*a
^3+429/2*c^8/n*(x^n)^22*b^6-a*b^13/n*(x^n)^13-b*a^13/n*x^n-5148*a^7/n*(x^n
)^12*b^2*c^5+15015*a^6/n*(x^n)^12*b^4*c^4-12012*a^5/n*(x^n)^12*b^6*c^3+643
5/2*a^4/n*(x^n)^12*b^8*c^2-286*a^3/n*(x^n)^12*b^10*c-1716/7*a^7*b^7/n*(x^n
)^7+143*c^9*b^5/n*(x^n)^23+429/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(x^n)^16*b^
12+b*c^13/n*(x^n)^27+26*b^11*c^3/n*(x^n)^17-26*a^3*b^11/n*(x^n)^11+143*b^9
*c^5/n*(x^n)^19-143*a^9*b^5/n*(x^n)^5+6435/2*a^8/n*(x^n)^10*b^2*c^4-8580*a
^7/n*(x^n)^10*b^4*c^3+6006*a^6/n*(x^n)^10*b^6*c^2-1287*a^5/n*(x^n)^10*b^8*
c+6006/n*(x^n)^14*a^6*b^2*c^6-18018/n*(x^n)^14*a^5*b^4*c^5+15015/n*(x^n...
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(23) = 46$.

Time = 0.33 (sec) , antiderivative size = 1299, normalized size of antiderivative = 51.96

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="frica
s")
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) - 14*a^13*b*x^n + 7*(13*b^2*c^12
- 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 - a*b*c^12)*x^(25*n) + 91*(11*b^4*c
^10 - 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 - 22*a*b^3*c^10
 + 6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^
10 - 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^
9 - 14*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*
c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 - 12*a*b^7*c^6 +
 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4
- 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5
*c^9)*x^(18*n) + 182*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*a^2*b^7*c^5 - 924*a^
3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 - 44*a
*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792*a^
5*b^2*c^7 + 33*a^6*c^8)*x^(16*n) + 14*(b^13*c - 78*a*b^11*c^2 + 1430*a^2*b
^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6 + 1716*a
^6*b*c^7)*x^(15*n) + (b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 - 60060*a^3*
b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 - 34
32*a^7*c^7)*x^(14*n) - 14*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2 - 858
0*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^
(13*n) + 91*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3
 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) - 182*(2*a...
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**13,x)
```

output
```
Timed out
```

3.100.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2045 vs. 2(23) = 46.

Time = 0.27 (sec) , antiderivative size = 2045, normalized size of antiderivative = 81.80

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="maxim
a")
```

output
```
1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n - a*c^
13*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n - 13*a*b*c^12*x^(25*n)/n + 143/2*b^
4*c^10*x^(24*n)/n - 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n +
143*b^5*c^9*x^(23*n)/n - 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n
)/n + 429/2*b^6*c^8*x^(22*n)/n - 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^
10*x^(22*n)/n - 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n - 1287*
a*b^5*c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n - 286*a^3*b*c^10*x^(21*
n)/n + 429/2*b^8*c^6*x^(20*n)/n - 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b
^4*c^8*x^(20*n)/n - 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/
n + 143*b^9*c^5*x^(19*n)/n - 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*
x^(19*n)/n - 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/
2*b^10*c^4*x^(18*n)/n - 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18
*n)/n - 8580*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n - 143*
a^5*c^9*x^(18*n)/n + 26*b^11*c^3*x^(17*n)/n - 715*a*b^9*c^4*x^(17*n)/n + 5
148*a^2*b^7*c^5*x^(17*n)/n - 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c
^7*x^(17*n)/n - 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n - 286
*a*b^10*c^3*x^(16*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n - 12012*a^3*b^6*c^5
*x^(16*n)/n + 15015*a^4*b^4*c^6*x^(16*n)/n - 5148*a^5*b^2*c^7*x^(16*n)/n +
 429/2*a^6*c^8*x^(16*n)/n + b^13*c*x^(15*n)/n - 78*a*b^11*c^2*x^(15*n)/n +
 1430*a^2*b^9*c^3*x^(15*n)/n - 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*...
```

3.100.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1693 vs. $2(23) = 46$.

Time = 0.37 (sec) , antiderivative size = 1693, normalized size of antiderivative = 67.72

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="giac"
)
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) - 14*a*c^1
3*x^(26*n) + 364*b^3*c^11*x^(25*n) - 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10
*x^(24*n) - 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9
*x^(23*n) - 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6
*c^8*x^(22*n) - 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) - 36
4*a^3*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) - 18018*a*b^5*c^8*x^(21*n) + 2
0020*a^2*b^3*c^9*x^(21*n) - 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*
n) - 24024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) - 20020*a^3*b^2
*c^9*x^(20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) - 24024*a*b
^7*c^6*x^(19*n) + 72072*a^2*b^5*c^7*x^(19*n) - 60060*a^3*b^3*c^8*x^(19*n)
+ 10010*a^4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) - 18018*a*b^8*c^5*x^(1
8*n) + 84084*a^2*b^6*c^6*x^(18*n) - 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^
4*b^2*c^8*x^(18*n) - 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) - 10010
*a*b^9*c^4*x^(17*n) + 72072*a^2*b^7*c^5*x^(17*n) - 168168*a^3*b^5*c^6*x^(1
7*n) + 120120*a^4*b^3*c^7*x^(17*n) - 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^
2*x^(16*n) - 4004*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) - 16816
8*a^3*b^6*c^5*x^(16*n) + 210210*a^4*b^4*c^6*x^(16*n) - 72072*a^5*b^2*c^7*x
^(16*n) + 3003*a^6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) - 1092*a*b^11*c^2*x^(
15*n) + 20020*a^2*b^9*c^3*x^(15*n) - 120120*a^3*b^7*c^4*x^(15*n) + 252252*
a^4*b^5*c^5*x^(15*n) - 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^...
```

3.100.9 Mupad [B] (verification not implemented)

Time = 11.07 (sec) , antiderivative size = 1401, normalized size of antiderivative = 56.04

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

input
```
int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n - a + c*x^(2*n))^13,x)
```

output
```
x^(n - 1)*((x^(11*n + 1)*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 - 286*a^3*b^10
*c + (6435*a^4*b^8*c^2)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a
^7*b^2*c^5))/n + (x^(15*n + 1)*((429*a^6*c^8)/2 + (13*b^12*c^2)/2 - 286*a*
b^10*c^3 + (6435*a^2*b^8*c^4)/2 - 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 -
5148*a^5*b^2*c^7))/n - (x^(12*n + 1)*(a*b^13 - 78*a^2*b^11*c + 1716*a^7*b*
c^6 + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*
b^3*c^5))/n + (x^(14*n + 1)*(b^13*c - 78*a*b^11*c^2 + 1716*a^6*b*c^7 + 143
0*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6))
/n + (x^(5*n + 1)*((429*a^8*b^6)/2 - 26*a^11*c^3 - 715*a^9*b^4*c + 429*a^1
0*b^2*c^2))/n - (x^(21*n + 1)*(26*a^3*c^11 - (429*b^6*c^8)/2 + 715*a*b^4*c
^9 - 429*a^2*b^2*c^10))/n + (x^(9*n + 1)*((143*a^4*b^10)/2 - 143*a^9*c^5 -
 1287*a^5*b^8*c + 6006*a^6*b^6*c^2 - 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)
/2))/n - (x^(17*n + 1)*(143*a^5*c^9 - (143*b^10*c^4)/2 + 1287*a*b^8*c^5 -
6006*a^2*b^6*c^6 + 8580*a^3*b^4*c^7 - (6435*a^4*b^2*c^8)/2))/n + (x^(13*n
+ 1)*(b^14/14 - (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 - 4290*a^3*b^8*c^3 + 1
5015*a^4*b^6*c^4 - 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6 - 13*a*b^12*c))/n
+ (x^(7*n + 1)*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 - 1716*a^7*b^6*c + (643
5*a^8*b^4*c^2)/2 - 1430*a^9*b^2*c^3))/n + (x^(19*n + 1)*((143*a^4*c^10)/2
+ (429*b^8*c^6)/2 - 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 - 1430*a^3*b^2*c
^9))/n + (c^14*x^(27*n + 1))/(14*n) - (a^12*x^(n + 1)*(a*c - (13*b^2)/2...
```

3.100. $\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx$

3.101 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

3.101.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

output
```
1/14*(c*x^2+b*x)^14
```

3.101.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(15) = 30.

Time = 0.01 (sec) , antiderivative size = 172, normalized size of antiderivative = 11.47

$$\begin{aligned}\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=&\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}\\&+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}\\&+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}\\&+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}\end{aligned}$$

input
```
Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

output
```
(b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (
143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7
*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^2
4)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)
/14
```

3.101.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{1104\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$$
$$\Big\downarrow \text{1104}$$
$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

input
```
Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

output
```
(b*x + c*x^2)^14/14
```

3.101.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.101.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result
gosper	$\frac{(cx+b)^{14} x^{14}}{14}$
default	$\frac{(c\,x^2+bx)^{14}}{14}$
norman	$26b^3c^{11}x^{25} + \frac{13}{2}x^{26}b^2c^{12} + b\,c^{13}x^{27} + \frac{1}{14}c^{14}x^{28} + \frac{1}{14}x^{14}b^{14} + b^{13}c\,x^{15} + \frac{13}{2}x^{16}b^{12}c^2 + 26b^{11}c^3x^1$
risch	$26b^3c^{11}x^{25} + \frac{13}{2}x^{26}b^2c^{12} + b\,c^{13}x^{27} + \frac{1}{14}c^{14}x^{28} + \frac{1}{14}x^{14}b^{14} + b^{13}c\,x^{15} + \frac{13}{2}x^{16}b^{12}c^2 + 26b^{11}c^3x^1$
parallelrisch	$26b^3c^{11}x^{25} + \frac{13}{2}x^{26}b^2c^{12} + b\,c^{13}x^{27} + \frac{1}{14}c^{14}x^{28} + \frac{1}{14}x^{14}b^{14} + b^{13}c\,x^{15} + \frac{13}{2}x^{16}b^{12}c^2 + 26b^{11}c^3x^1$

input
```
int((2*c*x+b)*(c*x^2+b*x)^13,x,method=_RETURNVERBOSE)
```

output
```
1/14*(c*x+b)^14*x^14
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 10.27

$$\begin{aligned}\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx =& \frac{1}{14}\,c^{14}x^{28} + bc^{13}x^{27} + \frac{13}{2}\,b^2c^{12}x^{26} + 26\,b^3c^{11}x^{25} \\ &+ \frac{143}{2}\,b^4c^{10}x^{24} + 143\,b^5c^9x^{23} + \frac{429}{2}\,b^6c^8x^{22} \\ &+ \frac{1716}{7}\,b^7c^7x^{21} + \frac{429}{2}\,b^8c^6x^{20} + 143\,b^9c^5x^{19} + \frac{143}{2}\,b^{10}c^4x^{18} \\ &+ 26\,b^{11}c^3x^{17} + \frac{13}{2}\,b^{12}c^2x^{16} + b^{13}cx^{15} + \frac{1}{14}\,b^{14}x^{14}\end{aligned}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
```

output
```
1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143
/2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*
x^21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^
11*c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

3.101.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 11.67

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

input
```
integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

output
```
b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**
17 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2
+ 1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 1
43*b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c*
*13*x**27 + c**14*x**28/14
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{1}{14}\left(cx^2+bx\right)^{14}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
```

output
```
1/14*(c*x^2 + b*x)^14
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(cx^2+bx\right)^{14}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
```

output
```
1/14*(c*x^2 + b*x)^14
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 10.27

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c^8\,x^{22}}{2}+143\,b^5\,c^9\,x^{23}+\frac{143\,b^4\,c^{10}\,x^{24}}{2}+26\,b^3\,c^{11}\,x^{25}+\frac{13\,b^2\,c^{12}\,x^{26}}{2}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}$$

input
```
int((b*x + c*x^2)^13*(b + 2*c*x),x)
```

output
```
(b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2
*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (
429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b
^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26
)/2
```

3.102 $\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx$

3.102.1 Optimal result

Integrand size = 23, antiderivative size = 16

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

output
```
1/28*x^28*(c*x^2+b)^14
```

3.102.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx=&\ \frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}\\&+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}\\&+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}\\&+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}\end{aligned}$$

input
```
Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

output
```
(b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (
858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b
^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/
2 + (c^14*x^56)/28
```

3.102.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {9, 354, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int x^{26}\left(cx^2+b\right)^{13}\left(2cx^2+b\right)dx^2$$

$$\Big\downarrow \; 83$$

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

input
```
Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

output
```
(x^28*(b + c*x^2)^14)/28
```

3.102.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.102.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{x^{28}\left(c\,x^{2}+b\right)^{14}}{28}$
default	$\frac{\left(b^{2}-\left(2c\,x^{2}+b\right)^{2}\right)^{14}}{7516192768c^{14}}$
risch	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{143}{2}x^{46}b^{5}c^{9}+\frac{143}{4}x^{48}b^{4}c^{10}+13x^{50}b^{3}c^{11}+\frac{13}{4}x^{52}b^{2}c^{12}+\frac{429}{4}x^{44}b^{6}c^{8}+\frac{858}{7}$
parallelrisch	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{143}{2}x^{46}b^{5}c^{9}+\frac{143}{4}x^{48}b^{4}c^{10}+13x^{50}b^{3}c^{11}+\frac{13}{4}x^{52}b^{2}c^{12}+\frac{429}{4}x^{44}b^{6}c^{8}+\frac{858}{7}$

input
```
int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x,method=_RETURNVERBOSE)
```

output
```
1/28*x^28*(c*x^2+b)^14
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx = \frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6x^{40} + \frac{143}{2}\,b^9c^5x^{38} + \frac{143}{4}\,b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}\,b^{12}c^2x^{32} + \frac{1}{2}\,b^{13}cx^{30} + \frac{1}{28}\,b^{14}x^{28}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="fricas")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.102.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx = \frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4} + \frac{bc^{13}x^{54}}{2} + \frac{c^{14}x^{56}}{28}$$

input
```
integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**13,x)
```

output
```
b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x
**34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**4
0/4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/
2 + 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 +
 b*c**13*x**54/2 + c**14*x**56/28
```

3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\begin{aligned}\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx = {} & \frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50} \\ & +\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42} \\ & +\frac{429}{4}\,b^8c^6x^{40}+\frac{143}{2}\,b^9c^5x^{38}+\frac{143}{4}\,b^{10}c^4x^{36} \\ & +13\,b^{11}c^3x^{34}+\frac{13}{4}\,b^{12}c^2x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}\end{aligned}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="maxima")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx = \frac{1}{28}\left(cx^4+bx^2\right)^{14}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="giac")
```

output
```
1/28*(c*x^4 + b*x^2)^14
```

3.102.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx = \frac{b^{14}\,x^{28}}{28}+\frac{b^{13}\,c\,x^{30}}{2}+\frac{13\,b^{12}\,c^2\,x^{32}}{4}+13\,b^{11}\,c^3\,x^{34}+\frac{143\,b^{10}\,c^4\,x^{36}}{4}+\frac{143\,b^9\,c^5\,x^{38}}{2}+\frac{429\,b^8\,c^6\,x^{40}}{4}+\frac{858\,b^7\,c^7\,x^{42}}{7}+\frac{429\,b^6\,c^8\,x^{44}}{4}+\frac{143\,b^5\,c^9\,x^{46}}{2}+\frac{143\,b^4\,c^{10}\,x^{48}}{4}+13\,b^3\,c^{11}\,x^{50}+\frac{13\,b^2\,c^{12}\,x^{52}}{4}+\frac{b\,c^{13}\,x^{54}}{2}+\frac{c^{14}\,x^{56}}{28}$$

input
```
int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x)
```

output
```
(b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*
b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5
*x^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44
)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (1
3*b^2*c^12*x^52)/4
```

3.103 $\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13}\,dx$

3.103.1 Optimal result

Integrand size = 25, antiderivative size = 16

$$\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^{13}\,dx=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

output
```
1/42*x^42*(c*x^3+b)^14
```

3.103.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 186, normalized size of antiderivative = 11.62

$$\begin{aligned}\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^{13}\,dx=&\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}\\&+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}\\&+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}\\&+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}\end{aligned}$$

input
```
Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

output
```
(b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^5
1)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2
 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (1
43*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^1
3*x^81)/3 + (c^14*x^84)/42
```

3.103.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {9, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (b+2cx^3) (bx^3+cx^6)^{13} \, dx$$

$$\Big\downarrow 9$$

$$\int x^{41} (b+cx^3)^{13} (b+2cx^3) \, dx$$

$$\Big\downarrow 948$$

$$\frac{1}{3} \int x^{39} (cx^3+b)^{13} (2cx^3+b) \, dx^3$$

$$\Big\downarrow 83$$

$$\frac{1}{42} x^{42} (b+cx^3)^{14}$$

input
```
Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

output
```
(x^42*(b + c*x^3)^14)/42
```

3.103.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.103.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{x^{42}\left(c\,x^{3}+b\right)^{14}}{42}$
default	$\frac{\left(b^{2}-\left(2c\,x^{3}+b\right)^{2}\right)^{14}}{11274289152c^{14}}$
risch	$\frac{1}{42}c^{14}x^{84}+\frac{13}{6}x^{78}b^{2}c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{143}{3}x^{69}b^{5}c^{9}+\frac{143}{6}x^{72}b^{4}c^{10}+\frac{26}{3}x^{75}b^{3}c^{11}+\frac{572}{7}x^{63}b^{7}c^{7}+\frac{143}{2}$
parallelrisch	$\frac{1}{42}c^{14}x^{84}+\frac{13}{6}x^{78}b^{2}c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{143}{3}x^{69}b^{5}c^{9}+\frac{143}{6}x^{72}b^{4}c^{10}+\frac{26}{3}x^{75}b^{3}c^{11}+\frac{572}{7}x^{63}b^{7}c^{7}+\frac{143}{2}$

input
```
int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x,method=_RETURNVERBOSE)
```

output
```
1/42*x^42*(c*x^3+b)^14
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13}\,dx = \frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6x^{60}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{6}b^{10}c^4x^{54}+\frac{26}{3}b^{11}c^3x^{51}+\frac{13}{6}b^{12}c^2x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="fricas")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

3.103.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 11.56

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13}\,dx = \frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^2x^{48}}{6}+\frac{26b^{11}c^3x^{51}}{3}+\frac{143b^{10}c^4x^{54}}{6}+\frac{143b^9c^5x^{57}}{3}+\frac{143b^8c^6x^{60}}{2}+\frac{572b^7c^7x^{63}}{7}+\frac{143b^6c^8x^{66}}{2}+\frac{143b^5c^9x^{69}}{3}+\frac{143b^4c^{10}x^{72}}{6}+\frac{26b^3c^{11}x^{75}}{3}+\frac{13b^2c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

input
```
integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**13,x)
```

output
```
b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x
**51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x*
*60/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**6
9/3 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78
/6 + b*c**13*x**81/3 + c**14*x**84/42
```

3.103.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13}\,dx = \frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c^6x^{60}+\frac{143}{3}\,b^9c^5x^{57}+\frac{143}{6}\,b^{10}c^4x^{54}+\frac{26}{3}\,b^{11}c^3x^{51}+\frac{13}{6}\,b^{12}c^2x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="maxima")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13}\,dx = \frac{1}{42}\left(cx^6+bx^3\right)^{14}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="giac")
```

output
```
1/42*(c*x^6 + b*x^3)^14
```

3.103.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13}\,dx=\frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2}+\frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3}+\frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

input
```
int(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x)
```

output
```
(b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*
b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9
*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*
x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75
)/3 + (13*b^2*c^12*x^78)/6
```

3.104 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx$

3.104.1 Optimal result

Integrand size = 29, antiderivative size = 21

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

output
```
1/14*x^(14*n)*(b+c*x^n)^14/n
```

3.104.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

input
```
Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

output
```
(x^(14*n)*(b + c*x^n)^14)/(14*n)
```

3.104.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {10, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}dx$$
$$\Big\downarrow \; 10$$
$$\int x^{14n-1}(b+cx^n)^{13}(b+2cx^n)\,dx$$
$$\Big\downarrow \; 948$$
$$\frac{\int x^{13n}(cx^n+b)^{13}(2cx^n+b)\,dx^n}{n}$$
$$\Big\downarrow \; 83$$
$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

input
```
Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

output
```
(x^(14*n)*(b + c*x^n)^14)/(14*n)
```

3.104.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(19) = 38$.

Time = 0.02 (sec) , antiderivative size = 230, normalized size of antiderivative = 10.95

$$\frac{c^{14}x^{28n}}{14n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{13c^{12}x^{26n}b^{2}}{2n}+\frac{26c^{11}b^{3}x^{25n}}{n}+\frac{143c^{10}x^{24n}b^{4}}{2n}+\frac{143c^{9}b^{5}x^{23n}}{n}+\frac{429c^{8}x^{22n}b^{6}}{2n}+\frac{1716b^{7}c^{7}x^{21n}}{7n}+\frac{42}{}$$

input
```
int(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x)
```

output
```
1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*c^12/n*(x^n)^26*b^2+26*c^11*b^
3/n*(x^n)^25+143/2*c^10/n*(x^n)^24*b^4+143*c^9*b^5/n*(x^n)^23+429/2*c^8/n*
(x^n)^22*b^6+1716/7*b^7*c^7/n*(x^n)^21+429/2*c^6/n*(x^n)^20*b^8+143*b^9*c^
5/n*(x^n)^19+143/2*c^4/n*(x^n)^18*b^10+26*b^11*c^3/n*(x^n)^17+13/2*c^2/n*(
x^n)^16*b^12+b^13*c/n*(x^n)^15+1/14/n*(x^n)^14*b^14
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(19) = 38$.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{-1+n}(b+2cx^{n})\left(bx^{n}+cx^{2n}\right)^{13}\,dx$$
$$=\frac{c^{14}x^{28\,n}+14\,bc^{13}x^{27\,n}+91\,b^{2}c^{12}x^{26\,n}+364\,b^{3}c^{11}x^{25\,n}+1001\,b^{4}c^{10}x^{24\,n}+2002\,b^{5}c^{9}x^{23\,n}+3003\,b^{6}c^{8}x^{22\,n}+}{}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="fricas")
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*
c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*
c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^
5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*
x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**13,x)
```

output
```
Timed out
```

3.104.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(19) = 38$.

Time = 0.20 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx=\frac{c^{14}x^{28\,n}}{14\,n}+\frac{bc^{13}x^{27\,n}}{n}+\frac{13\,b^2c^{12}x^{26\,n}}{2\,n}+\frac{26\,b^3c^{11}x^{25\,n}}{n}+\frac{143\,b^4c^{10}x^{24\,n}}{2\,n}+\frac{143\,b^5c^9x^{23\,n}}{n}+\frac{429\,b^6c^8x^{22\,n}}{2\,n}+\frac{1716\,b^7c^7x^{21\,n}}{7\,n}+\frac{429\,b^8c^6x^{20\,n}}{2\,n}+\frac{143\,b^9c^5x^{19\,n}}{n}+\frac{143\,b^{10}c^4x^{18\,n}}{2\,n}+\frac{26\,b^{11}c^3x^{17\,n}}{n}+\frac{13\,b^{12}c^2x^{16\,n}}{2\,n}+\frac{b^{13}cx^{15\,n}}{n}+\frac{b^{14}x^{14\,n}}{14\,n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="maxima")
```

output
```
1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b
^3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n +
429/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20
*n)/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x
^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14
*n)/n
```

3.104.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(19) = 38.

Time = 0.29 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx = \frac{c^{14}x^{28\,n}+14\,bc^{13}x^{27\,n}+91\,b^2c^{12}x^{26\,n}+364\,b^3c^{11}x^{25\,n}+1001\,b^4c^{10}x^{24\,n}+2002\,b^5c^9x^{23\,n}+3003\,b^6c^8x^{22\,n}+\cdots}{\cdots}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*
c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*
c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^
5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*
x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

3.104.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx &= \frac{b^{14}\,x^{14\,n}}{14\,n}+\frac{c^{14}\,x^{28\,n}}{14\,n}+\frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n}\\ &\quad+\frac{26\,b^{11}\,c^3\,x^{17\,n}}{n}+\frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n}+\frac{143\,b^9\,c^5\,x^{19\,n}}{n}\\ &\quad+\frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n}+\frac{1716\,b^7\,c^7\,x^{21\,n}}{7\,n}+\frac{429\,b^6\,c^8\,x^{22\,n}}{2\,n}\\ &\quad+\frac{143\,b^5\,c^9\,x^{23\,n}}{n}+\frac{143\,b^4\,c^{10}\,x^{24\,n}}{2\,n}+\frac{26\,b^3\,c^{11}\,x^{25\,n}}{n}\\ &\quad+\frac{13\,b^2\,c^{12}\,x^{26\,n}}{2\,n}+\frac{b^{13}\,c\,x^{15\,n}}{n}+\frac{b\,c^{13}\,x^{27\,n}}{n}\end{aligned}$$

input
```
int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x)
```

output
```
(b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(
2*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9
*c^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/
(7*n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4
*c^10*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/
(2*n) + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

3.105 $\int \frac{b+2cx}{a+bx+cx^2} dx$

3.105.1 Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{b+2cx}{a+bx+cx^2} dx = \log \left(a+bx+cx^2\right)$$

output
```
ln(c*x^2+b*x+a)
```

3.105.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{b+2cx}{a+bx+cx^2} dx = \log(a+x(b+cx))$$

input
```
Integrate[(b + 2*c*x)/(a + b*x + c*x^2),x]
```

output
```
Log[a + x*(b + c*x)]
```

3.105.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \ 1103$$
$$\log\left(a+bx+cx^2\right)$$

input
```
Int[(b + 2*c*x)/(a + b*x + c*x^2),x]
```

output
```
Log[a + b*x + c*x^2]
```

3.105.3.1 Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.105.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\ln\left(c\,x^2+bx+a\right)$	12
default	$\ln\left(c\,x^2+bx+a\right)$	12
norman	$\ln\left(c\,x^2+bx+a\right)$	12
risch	$\ln\left(c\,x^2+bx+a\right)$	12
parallelrisch	$\ln\left(c\,x^2+bx+a\right)$	12

input
```
int((2*c*x+b)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(c*x^2+b*x+a)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx = \log\left(cx^2+bx+a\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
log(c*x^2 + b*x + a)
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx = \log\left(a+bx+cx^2\right)$$

input
```
integrate((2*c*x+b)/(c*x**2+b*x+a),x)
```

output
```
log(a + b*x + c*x**2)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx = \log\left(cx^2+bx+a\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
log(c*x^2 + b*x + a)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx = \log\left(\left|cx^2+bx+a\right|\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
log(abs(c*x^2 + b*x + a))
```

3.105.9 Mupad [B] (verification not implemented)

Time = 8.63 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx = \ln\left(c\,x^2+b\,x+a\right)$$

input
```
int((b + 2*c*x)/(a + b*x + c*x^2),x)
```

output
```
log(a + b*x + c*x^2)
```

3.106 $\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx$

3.106.1 Optimal result

Integrand size = 24, antiderivative size = 17

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

output
```
1/2*ln(c*x^4+b*x^2+a)
```

3.106.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

input
```
Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
Log[a + b*x^2 + c*x^4]/2
```

3.106.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1576, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

input
```
Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
Log[a + b*x^2 + c*x^4]/2
```

3.106.3.1 Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.106.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln(c\,x^4+b\,x^2+a)}{2}$	16
norman	$\frac{\ln(c\,x^4+b\,x^2+a)}{2}$	16
risch	$\frac{\ln(c\,x^4+b\,x^2+a)}{2}$	16
parallelrisch	$\frac{\ln(c\,x^4+b\,x^2+a)}{2}$	16

input
```
int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*x^4+b*x^2+a)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(cx^4+bx^2+a\right)$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*log(c*x^4 + b*x^2 + a)
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{\log\left(a+bx^2+cx^4\right)}{2}$$

input
```
integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a),x)
```

output
```
log(a + b*x**2 + c*x**4)/2
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(cx^4+bx^2+a\right)$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*log(c*x^4 + b*x^2 + a)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(\left|cx^4+bx^2+a\right|\right)$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*log(abs(c*x^4 + b*x^2 + a))
```

3.106.9 Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{\ln\left(c\,x^4+b\,x^2+a\right)}{2}$$

input
```
int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
log(a + b*x^2 + c*x^4)/2
```

3.107 $\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx$

3.107.1 Optimal result

Integrand size = 26, antiderivative size = 17

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

output
```
1/3*ln(c*x^6+b*x^3+a)
```

3.107.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

input
```
Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
Log[a + b*x^3 + c*x^6]/3
```

3.107.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1798, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (b+2 c x^3)}{a+b x^3+c x^6} \, dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{3} \int \frac{2 c x^3+b}{c x^6+b x^3+a}dx^3$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3} \log \left(a+b x^3+c x^6\right)$$

input
```
Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
Log[a + b*x^3 + c*x^6]/3
```

3.107.3.1 Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.107.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln(c\,x^6+b\,x^3+a)}{3}$	16
norman	$\frac{\ln(c\,x^6+b\,x^3+a)}{3}$	16
risch	$\frac{\ln(c\,x^6+b\,x^3+a)}{3}$	16
parallelrisch	$\frac{\ln(c\,x^6+b\,x^3+a)}{3}$	16

input
```
int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(c*x^6+b*x^3+a)
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(cx^6+bx^3+a\right)$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*log(c*x^6 + b*x^3 + a)
```

3.107.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{\log\left(a+bx^3+cx^6\right)}{3}$$

input
```
integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a),x)
```

output
```
log(a + b*x**3 + c*x**6)/3
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(cx^6+bx^3+a\right)$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*log(c*x^6 + b*x^3 + a)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(\left|cx^6+bx^3+a\right|\right)$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/3*log(abs(c*x^6 + b*x^3 + a))
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{\ln\left(c\,x^6+b\,x^3+a\right)}{3}$$

input
```
int((x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
log(a + b*x^3 + c*x^6)/3
```

3.108 $\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx$

3.108.1 Optimal result

Integrand size = 30, antiderivative size = 19

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx=\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

output
```
ln(a+b*x^n+c*x^(2*n))/n
```

3.108.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx=-\frac{2\log\left(x^{-n}\right)}{n}+\frac{\log\left(c+ax^{-2n}+bx^{-n}\right)}{n}$$

input
```
Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-2*Log[x^(-n)])/n + Log[c + a/x^(2*n) + b/x^n]/n
```

3.108.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1798, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \; 1798$$

$$\frac{\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \; 1103$$

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

input
```
Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

output
```
Log[a + b*x^n + c*x^(2*n)]/n
```

3.108.3.1 Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.108.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
norman	$\frac{\ln\left(a+b\,\mathrm{e}^{n\ln(x)}+c\,\mathrm{e}^{2n\ln(x)}\right)}{n}$	24
risch	$\frac{\ln\left(x^{2n}+\frac{b\,x^{n}}{c}+\frac{a}{c}\right)}{n}$	25

input
```
int(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/n*ln(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx = \frac{\log\left(cx^{2\,n}+bx^n+a\right)}{n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
log(c*x^(2*n) + b*x^n + a)/n
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx=\frac{\log\left(\frac{cx^{2\,n}+bx^n+a}{c}\right)}{n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
log((c*x^(2*n) + b*x^n + a)/c)/n
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx=\frac{\log\left(cx^{2\,n}+bx^n+a\right)}{n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
log(c*x^(2*n) + b*x^n + a)/n
```

3.108.9 Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 121, normalized size of antiderivative = 6.37

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx=-\frac{2\,b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^n}{\sqrt{4\,a\,c-b^2}}\right)-\ln\left(a+b\,x^n+c\,x^{2\,n}\right)\,\sqrt{4\,a\,c-b^2}}{n\,\sqrt{4\,a\,c-b^2}}-\frac{2\,b\,\mathrm{atanh}\left(\frac{b+2\,c\,x^n}{\sqrt{b^2-4\,a\,c}}\right)}{n\,\sqrt{b^2-4\,a\,c}}$$

input
```
int((x^(n - 1)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x)
```

output
```
- (2*b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^n)/(4*a*c - b^2)^(1/2)) - log(a
 + b*x^n + c*x^(2*n))*(4*a*c - b^2)^(1/2))/(n*(4*a*c - b^2)^(1/2)) - (2*b*
atanh((b + 2*c*x^n)/(b^2 - 4*a*c)^(1/2)))/(n*(b^2 - 4*a*c)^(1/2))
```

3.109 $\int \frac{b+2cx}{(a+bx+cx^2)^8} dx$

3.109.1 Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{b+2cx}{(a+bx+cx^2)^8} dx = -\frac{1}{7(a+bx+cx^2)^7}$$

output
```
-1/7/(c*x^2+b*x+a)^7
```

3.109.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx}{(a+bx+cx^2)^8} dx = -\frac{1}{7(a+x(b+cx))^7}$$

input
```
Integrate[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

output
```
-1/7*1/(a + x*(b + c*x))^7
```

3.109.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{(a+bx+cx^2)^8} \, dx$$
$$\Big\downarrow \text{1104}$$
$$-\frac{1}{7(a+bx+cx^2)^7}$$

input
```
Int[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

output
```
-1/7*1/(a + b*x + c*x^2)^7
```

3.109.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.109.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{7(c x^2+bx+a)^7}$	15
derivativedivides	$-\frac{1}{7(c x^2+bx+a)^7}$	15
default	$-\frac{1}{7(c x^2+bx+a)^7}$	15
norman	$-\frac{1}{7(c x^2+bx+a)^7}$	15
risch	$-\frac{1}{7(c x^2+bx+a)^7}$	15
parallelrisch	$-\frac{1}{7(c x^2+bx+a)^7}$	15

input
```
int((2*c*x+b)/(c*x^2+b*x+a)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7/(c*x^2+b*x+a)^7
```

3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 350, normalized size of antiderivative = 21.88

$$\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx =$$

$$-\frac{}{7\,(c^7x^{14}+7\,bc^6x^{13}+7\,(3\,b^2c^5+ac^6)x^{12}+7\,(5\,b^3c^4+6\,abc^5)x^{11}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{10}+7\,(}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="fricas")
```

output
```
-1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 + a*c^6)*x^12 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b
^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a
^2*b^2*c^3 + 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 + 42*a*b^5*c + 210*a^2*b^3*
c^2 + 140*a^3*b*c^3)*x^7 + a^7 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2
+ 5*a^4*c^3)*x^6 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 + 7*(5*
a^3*b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^3 +
7*(3*a^5*b^2 + a^6*c)*x^2)
```

3.109.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(15) = 30$.

Time = 2.55 (sec) , antiderivative size = 359, normalized size of antiderivative = 22.44

$$\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx =$$

$$-\frac{}{7a^7+49a^6bx+49bc^6x^{13}+7c^7x^{14}+x^{12}\cdot(49ac^6+147b^2c^5)+x^{11}\cdot(294abc^5+245b^3c^4)+x^{10}\cdot(147a^2c}$$

input
```
integrate((2*c*x+b)/(c*x**2+b*x+a)**8,x)
```

output
```
-1/(7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(49*a*c*
*6 + 147*b**2*c**5) + x**11*(294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a*
*2*c**5 + 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 + 980*a
*b**3*c**3 + 147*b**5*c**2) + x**8*(245*a**3*c**4 + 1470*a**2*b**2*c**3 +
735*a*b**4*c**2 + 49*b**6*c) + x**7*(980*a**3*b*c**3 + 1470*a**2*b**3*c**2
 + 294*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 + 1470*a**3*b**2*c**2 + 73
5*a**2*b**4*c + 49*a*b**6) + x**5*(735*a**4*b*c**2 + 980*a**3*b**3*c + 147
*a**2*b**5) + x**4*(147*a**5*c**2 + 735*a**4*b**2*c + 245*a**3*b**4) + x**
3*(294*a**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c + 147*a**5*b**2))
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx+a)^7}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="maxima")
```

output
```
-1/7/(c*x^2 + b*x + a)^7
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx+a)^7}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="giac")
```

output
```
-1/7/(c*x^2 + b*x + a)^7
```

3.109.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 358, normalized size of antiderivative = 22.38

$$\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx =$$

$$-\frac{}{7\,\left(x^5\,\left(105\,a^4\,b\,c^2+140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^9\,\left(105\,a^2\,b\,c^4+140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^7\,\left(140\,a^3\,b\,c^3+2\right.\right.}$$

input
```
int((b + 2*c*x)/(a + b*x + c*x^2)^8,x)
```

output
```
-1/(7*(x^5*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2
+ 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^3*
c^2 + 42*a*b^5*c) + x^3*(35*a^4*b^3 + 42*a^5*b*c) + x^11*(35*b^3*c^4 + 42*
a*b*c^5) + x^4*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^10*(21*a^2*c^
5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^6*(7*a*b^6 + 35*a^4*c^3 + 105*a^
2*b^4*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 2
10*a^2*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c + 21*a^5*b^2) + x^12*(7*a*c^6 +
21*b^2*c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))
```

3.110 $\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx$

3.110.1 Optimal result

Integrand size = 24, antiderivative size = 18

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(a+bx^2+cx^4)^7}$$

output
```
-1/14/(c*x^4+b*x^2+a)^7
```

3.110.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(a+bx^2+cx^4)^7}$$

input
```
Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

output
```
-1/14*1/(a + b*x^2 + c*x^4)^7
```

3.110.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1576, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{2cx^2+b}{(cx^4+bx^2+a)^8}dx^2$$

$$\Big\downarrow \text{1104}$$

$$-\frac{1}{14(a+bx^2+cx^4)^7}$$

input
```
Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

output
```
-1/14*1/(a + b*x^2 + c*x^4)^7
```

3.110.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.110.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{14(c\,x^4+b\,x^2+a)^7}$	17
default	$-\frac{1}{14(c\,x^4+b\,x^2+a)^7}$	17
norman	$-\frac{1}{14(c\,x^4+b\,x^2+a)^7}$	17
risch	$-\frac{1}{14(c\,x^4+b\,x^2+a)^7}$	17
parallelrisch	$-\frac{1}{14(c\,x^4+b\,x^2+a)^7}$	17

input
```
int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/14/(c*x^4+b*x^2+a)^7
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(16) = 32.

Time = 0.30 (sec) , antiderivative size = 352, normalized size of antiderivative = 19.56

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,(c^7x^{28}+7\,bc^6x^{26}+7\,(3\,b^2c^5+ac^6)x^{24}+7\,(5\,b^3c^4+6\,abc^5)x^{22}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{20}+7}$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="fricas")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4
 + 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*
b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30
*a^2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140
*a^3*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x
^12 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(
5*a^3*b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c
)*x^6 + 7*(3*a^5*b^2 + a^6*c)*x^4)
```

3.110.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(17) = 34$.

Time = 4.19 (sec) , antiderivative size = 360, normalized size of antiderivative = 20.00

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14a^7+98a^6bx^2+98bc^6x^{26}+14c^7x^{28}+x^{24}\cdot(98ac^6+294b^2c^5)+x^{22}\cdot(588abc^5+490b^3c^4)+x^{20}\cdot(294}$$

input
```
integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a)**8,x)
```

output
```
-1/(14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(98
*a*c**6 + 294*b**2*c**5) + x**22*(588*a*b*c**5 + 490*b**3*c**4) + x**20*(2
94*a**2*c**5 + 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c**4
 + 1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(490*a**3*c**4 + 2940*a**2*b*
*2*c**3 + 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(1960*a**3*b*c**3 + 2940*a
**2*b**3*c**2 + 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 + 2940*a**3
*b**2*c**2 + 1470*a**2*b**4*c + 98*a*b**6) + x**10*(1470*a**4*b*c**2 + 196
0*a**3*b**3*c + 294*a**2*b**5) + x**8*(294*a**5*c**2 + 1470*a**4*b**2*c +
490*a**3*b**4) + x**6*(588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**6*c + 2
94*a**5*b**2))
```

3.110.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 19.56

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,(c^7x^{28}+7\,bc^6x^{26}+7\,(3\,b^2c^5+ac^6)x^{24}+7\,(5\,b^3c^4+6\,abc^5)x^{22}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{20}+7}$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="maxima")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4
 + 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*
b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30
*a^2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140
*a^3*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x
^12 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(
5*a^3*b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c
)*x^6 + 7*(3*a^5*b^2 + a^6*c)*x^4)
```

3.110.8 Giac [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(cx^4+bx^2+a)^7}$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="giac")
```

output
```
-1/14/(c*x^4 + b*x^2 + a)^7
```

3.110.9 Mupad [B] (verification not implemented)

Time = 15.83 (sec) , antiderivative size = 360, normalized size of antiderivative = 20.00

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,\left(x^{10}\,\left(105\,a^4\,b\,c^2+140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{18}\,\left(105\,a^2\,b\,c^4+140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{14}\,\left(140\,a^3\,b\,c^3\right.\right.}$$

input
```
int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x)
```

output
```
-1/(14*(x^10*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^18*(21*b^5*c
^2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^14*(b^7 + 140*a^3*b*c^3 + 210*a^2*
b^3*c^2 + 42*a*b^5*c) + x^6*(35*a^4*b^3 + 42*a^5*b*c) + x^22*(35*b^3*c^4 +
 42*a*b*c^5) + x^8*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^20*(21*a^
2*c^5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^12*(7*a*b^6 + 35*a^4*c^3 + 1
05*a^2*b^4*c + 210*a^3*b^2*c^2) + x^16*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c
^2 + 210*a^2*b^2*c^3) + c^7*x^28 + x^4*(7*a^6*c + 21*a^5*b^2) + x^24*(7*a*
c^6 + 21*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^26))
```

3.111 $\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx$

3.111.1 Optimal result

Integrand size = 26, antiderivative size = 18

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx = -\frac{1}{21(a+bx^3+cx^6)^7}$$

output
```
-1/21/(c*x^6+b*x^3+a)^7
```

3.111.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx = -\frac{1}{21(a+bx^3+cx^6)^7}$$

input
```
Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

output
```
-1/21*1/(a + b*x^3 + c*x^6)^7
```

3.111.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1798, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (b+2cx^3)}{(a+bx^3+cx^6)^8} \, dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{3} \int \frac{2cx^3+b}{(cx^6+bx^3+a)^8} dx^3$$

$$\Big\downarrow \text{1104}$$

$$-\frac{1}{21 (a+bx^3+cx^6)^7}$$

input
```
Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

output
```
-1/21*1/(a + b*x^3 + c*x^6)^7
```

3.111.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.111.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{21(c x^6+b x^3+a)^7}$	17
default	$-\frac{1}{21(c x^6+b x^3+a)^7}$	17
risch	$-\frac{1}{21(c x^6+b x^3+a)^7}$	17
parallelrisch	$-\frac{1}{21(c x^6+b x^3+a)^7}$	17

input
```
int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21/(c*x^6+b*x^3+a)^7
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(16) = 32.

Time = 0.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 19.56

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\,(c^7x^{42}+7\,bc^6x^{39}+7\,(3\,b^2c^5+ac^6)x^{36}+7\,(5\,b^3c^4+6\,abc^5)x^{33}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{30}+7}$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="fricas")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
 + 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*
b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30
*a^2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140
*a^3*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x
^18 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15
*a^4*b^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9
 + a^7 + 7*(3*a^5*b^2 + a^6*c)*x^6)
```

3.111.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(17) = 34$.

Time = 14.85 (sec) , antiderivative size = 360, normalized size of antiderivative = 20.00

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21a^7+147a^6bx^3+147bc^6x^{39}+21c^7x^{42}+x^{36}\cdot(147ac^6+441b^2c^5)+x^{33}\cdot(882abc^5+735b^3c^4)+x^{30}\cdot($$

input
```
integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a)**8,x)
```

output
```
-1/(21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*(
147*a*c**6 + 441*b**2*c**5) + x**33*(882*a*b*c**5 + 735*b**3*c**4) + x**30
*(441*a**2*c**5 + 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*b*c
**4 + 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(735*a**3*c**4 + 4410*a**2
*b**2*c**3 + 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(2940*a**3*b*c**3 + 44
10*a**2*b**3*c**2 + 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 + 4410*
a**3*b**2*c**2 + 2205*a**2*b**4*c + 147*a*b**6) + x**15*(2205*a**4*b*c**2
+ 2940*a**3*b**3*c + 441*a**2*b**5) + x**12*(441*a**5*c**2 + 2205*a**4*b**
2*c + 735*a**3*b**4) + x**9*(882*a**5*b*c + 735*a**4*b**3) + x**6*(147*a**
6*c + 441*a**5*b**2))
```

3.111.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 352, normalized size of antiderivative = 19.56

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\,(c^7x^{42}+7\,bc^6x^{39}+7\,(3\,b^2c^5+ac^6)x^{36}+7\,(5\,b^3c^4+6\,abc^5)x^{33}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{30}+7}$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="maxima")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
 + 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*
b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30
*a^2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140
*a^3*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x
^18 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15
*a^4*b^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9
 + a^7 + 7*(3*a^5*b^2 + a^6*c)*x^6)
```

3.111.8 Giac [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx = -\frac{1}{21\,(cx^6+bx^3+a)^7}$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="giac")
```

output
```
-1/21/(c*x^6 + b*x^3 + a)^7
```

3.111.9 Mupad [B] (verification not implemented)

Time = 18.83 (sec) , antiderivative size = 360, normalized size of antiderivative = 20.00

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\,\left(x^{15}\,\left(105\,a^4\,b\,c^2+140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{27}\,\left(105\,a^2\,b\,c^4+140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{21}\,\left(140\,a^3\,b\,c^3\right.\right.}$$

input
```
int((x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x)
```

output
```
-1/(21*(x^15*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^27*(21*b^5*c
^2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^21*(b^7 + 140*a^3*b*c^3 + 210*a^2*
b^3*c^2 + 42*a*b^5*c) + x^9*(35*a^4*b^3 + 42*a^5*b*c) + x^33*(35*b^3*c^4 +
 42*a*b*c^5) + x^12*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^30*(21*a
^2*c^5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^18*(7*a*b^6 + 35*a^4*c^3 +
105*a^2*b^4*c + 210*a^3*b^2*c^2) + x^24*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*
c^2 + 210*a^2*b^2*c^3) + c^7*x^42 + x^6*(7*a^6*c + 21*a^5*b^2) + x^36*(7*a
*c^6 + 21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^39))
```

3.112 $\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx$

3.112.1 Optimal result

Integrand size = 30, antiderivative size = 23

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx = -\frac{1}{7n\,(a+bx^n+cx^{2n})^7}$$

output
```
-1/7/n/(a+b*x^n+c*x^(2*n))^7
```

3.112.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx = -\frac{1}{7n\,(a+x^n\,(b+cx^n))^7}$$

input
```
Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

output
```
-1/7*1/(n*(a + x^n*(b + c*x^n))^7)
```

3.112.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1798, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}dx$$
$$\Big\downarrow \text{1798}$$
$$\frac{\int \frac{2cx^n+b}{(bx^n+cx^{2n}+a)^8}dx^n}{n}$$
$$\Big\downarrow \text{1104}$$
$$-\frac{1}{7n(a+bx^n+cx^{2n})^7}$$

input
```
Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

output
```
-1/7*1/(n*(a + b*x^n + c*x^(2*n))^7)
```

3.112.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.112.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$-\frac{1}{7n\,(a+b\,x^{n}+c\,x^{2n})^{7}}$$

input
```
int(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x)
```

output
```
-1/7/n/(a+b*x^n+c*(x^n)^2)^7
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(21) = 42.

Time = 0.35 (sec) , antiderivative size = 394, normalized size of antiderivative = 17.13

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx =$$

$$-\frac{}{7\,(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+7\,a^6bnx^n+a^7n+7\,(3\,b^2c^5+ac^6)nx^{12\,n}+7\,(5\,b^3c^4+6\,abc^5)nx^{11\,n}+7\,(5\,b^4c^3}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas"
)
```

output
```
-1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b
^2*c^5 + a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 + 6*a*b*c^5)*n*x^(11*n) + 7*(5*b
^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 + 20*a*b^3*c^
3 + 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5
*a^3*c^4)*n*x^(8*n) + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)
*n*x^(7*n) + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*n*x^(6*
n) + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) + 7*(5*a^3*b^4
+ 15*a^4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 + 6*a^5*b*c)*n*x^(3*n
) + 7*(3*a^5*b^2 + a^6*c)*n*x^(2*n))
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n))**8,x)
```

output
```
Timed out
```

3.112.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(21) = 42.

Time = 0.54 (sec) , antiderivative size = 416, normalized size of antiderivative = 18.09

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx =$$

$$-\frac{}{7\,(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+7\,a^6bnx^n+a^7n+7\,(3\,b^2c^5n+ac^6n)x^{12\,n}+7\,(5\,b^3c^4n+6\,abc^5n)x^{11\,n}+7\,(5\,b}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima"
)
```

output
```
-1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b
^2*c^5*n + a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n + 6*a*b*c^5*n)*x^(11*n) + 7*
(5*b^4*c^3*n + 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n + 2
0*a*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n + 15*a*b^4*c^2*n + 30
*a^2*b^2*c^3*n + 5*a^3*c^4*n)*x^(8*n) + (b^7*n + 42*a*b^5*c*n + 210*a^2*b^
3*c^2*n + 140*a^3*b*c^3*n)*x^(7*n) + 7*(a*b^6*n + 15*a^2*b^4*c*n + 30*a^3*
b^2*c^2*n + 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n + 20*a^3*b^3*c*n + 15*a^
4*b*c^2*n)*x^(5*n) + 7*(5*a^3*b^4*n + 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n
) + 7*(5*a^4*b^3*n + 6*a^5*b*c*n)*x^(3*n) + 7*(3*a^5*b^2*n + a^6*c*n)*x^(2
*n))
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx = -\frac{1}{7\left(cx^{2n}+bx^n+a\right)^7 n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

output
```
-1/7/((c*x^(2*n) + b*x^n + a)^7*n)
```

3.112.9 Mupad [B] (verification not implemented)

Time = 22.40 (sec) , antiderivative size = 496, normalized size of antiderivative = 21.57

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx =$$

$$-\frac{}{7\,a^7\,n+7\,b^7\,n\,x^{7\,n}+7\,c^7\,n\,x^{14\,n}+49\,a^6\,b\,n\,x^n+49\,a\,b^6\,n\,x^{6\,n}+49\,a^6\,c\,n\,x^{2\,n}+49\,a\,c^6\,n\,x^{12\,n}+49\,b^6\,c\,n}$$

input
```
int((x^(n - 1)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x)
```

output
```
-1/(7*a^7*n + 7*b^7*n*x^(7*n) + 7*c^7*n*x^(14*n) + 49*a^6*b*n*x^n + 49*a*b
^6*n*x^(6*n) + 49*a^6*c*n*x^(2*n) + 49*a*c^6*n*x^(12*n) + 49*b^6*c*n*x^(8*
n) + 49*b*c^6*n*x^(13*n) + 147*a^5*b^2*n*x^(2*n) + 245*a^4*b^3*n*x^(3*n) +
 245*a^3*b^4*n*x^(4*n) + 147*a^2*b^5*n*x^(5*n) + 147*a^5*c^2*n*x^(4*n) + 2
45*a^4*c^3*n*x^(6*n) + 245*a^3*c^4*n*x^(8*n) + 147*a^2*c^5*n*x^(10*n) + 14
7*b^5*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(11*n) + 14
7*b^2*c^5*n*x^(12*n) + 735*a^4*b^2*c*n*x^(4*n) + 980*a^3*b^3*c*n*x^(5*n) +
 735*a^4*b*c^2*n*x^(5*n) + 735*a^2*b^4*c*n*x^(6*n) + 980*a^3*b*c^3*n*x^(7*
n) + 735*a*b^4*c^2*n*x^(8*n) + 980*a*b^3*c^3*n*x^(9*n) + 735*a^2*b*c^4*n*x
^(9*n) + 735*a*b^2*c^4*n*x^(10*n) + 1470*a^3*b^2*c^2*n*x^(6*n) + 1470*a^2*
b^3*c^2*n*x^(7*n) + 1470*a^2*b^2*c^3*n*x^(8*n) + 294*a^5*b*c*n*x^(3*n) + 2
94*a*b^5*c*n*x^(7*n) + 294*a*b*c^5*n*x^(11*n))
```

3.113 $\int \frac{b+2cx}{-a+bx+cx^2} \, dx$

3.113.1 Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{b+2cx}{-a+bx+cx^2} \, dx = \log \left(a-bx-cx^2\right)$$

output
```
ln(-c*x^2-b*x+a)
```

3.113.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{b+2cx}{-a+bx+cx^2} \, dx = \log(-a+x(b+cx))$$

input
```
Integrate[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

output
```
Log[-a + x*(b + c*x)]
```

3.113.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{1103}$$
$$\log\left(a-bx-cx^2\right)$$

input
```
Int[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

output
```
Log[a - b*x - c*x^2]
```

3.113.3.1 Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.113.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\ln\left(c\,x^2+bx-a\right)$	14
default	$\ln\left(-c\,x^2-bx+a\right)$	14
norman	$\ln\left(-c\,x^2-bx+a\right)$	14
risch	$\ln\left(-c\,x^2-bx+a\right)$	14
parallelrisch	$\ln\left(c\,x^2+bx-a\right)$	14

input
```
int((2*c*x+b)/(c*x^2+b*x-a),x,method=_RETURNVERBOSE)
```

output
```
ln(c*x^2+b*x-a)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx = \log\left(cx^2+bx-a\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="fricas")
```

output
```
log(c*x^2 + b*x - a)
```

3.113.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx = \log\left(-a+bx+cx^2\right)$$

input
```
integrate((2*c*x+b)/(c*x**2+b*x-a),x)
```

output
```
log(-a + b*x + c*x**2)
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx = \log\left(cx^2+bx-a\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="maxima")
```

output
```
log(c*x^2 + b*x - a)
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx = \log\left(\left|cx^2+bx-a\right|\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="giac")
```

output
```
log(abs(c*x^2 + b*x - a))
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx = \ln\left(c\,x^2+b\,x-a\right)$$

input
```
int((b + 2*c*x)/(b*x - a + c*x^2),x)
```

output
```
log(b*x - a + c*x^2)
```

3.114 $\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx$

3.114.1 Optimal result

Integrand size = 26, antiderivative size = 19

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

output
```
1/2*ln(-c*x^4-b*x^2+a)
```

3.114.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(-a+bx^2+cx^4\right)$$

input
```
Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

output
```
Log[-a + b*x^2 + c*x^4]/2
```

3.114.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1576, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int -\frac{2cx^2+b}{-cx^4-bx^2+a}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{2cx^2+b}{-cx^4-bx^2+a}dx^2$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

input
```
Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

output
```
Log[a - b*x^2 - c*x^4]/2
```

3.114.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.114.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(-c\,x^4-b\,x^2+a)}{2}$	18
norman	$\frac{\ln(-c\,x^4-b\,x^2+a)}{2}$	18
risch	$\frac{\ln(-c\,x^4-b\,x^2+a)}{2}$	18
parallelrisch	$\frac{\ln(c\,x^4+b\,x^2-a)}{2}$	18

input
```
int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(-c*x^4-b*x^2+a)
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(cx^4+bx^2-a\right)$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="fricas")
```

output
```
1/2*log(c*x^4 + b*x^2 - a)
```

3.114.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{\log\left(-a+bx^2+cx^4\right)}{2}$$

input
```
integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a),x)
```

output
```
log(-a + b*x**2 + c*x**4)/2
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{1}{2}\,\log\left(cx^4+bx^2-a\right)$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="maxima")
```

output
```
1/2*log(c*x^4 + b*x^2 - a)
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{1}{2}\,\log\left(\left|cx^4+bx^2-a\right|\right)$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="giac")
```

output
```
1/2*log(abs(c*x^4 + b*x^2 - a))
```

3.114.9 Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{\ln\left(c\,x^4+b\,x^2-a\right)}{2}$$

input
```
int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4),x)
```

output
```
log(b*x^2 - a + c*x^4)/2
```

3.115 $\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx$

3.115.1 Optimal result

Integrand size = 28, antiderivative size = 19

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

output
```
1/3*ln(-c*x^6-b*x^3+a)
```

3.115.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(-a+bx^3+cx^6\right)$$

input
```
Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

output
```
Log[-a + b*x^3 + c*x^6]/3
```

3.115.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1798, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (b+2cx^3)}{-a+bx^3+cx^6} \, dx$$

$$\Big\downarrow \text{ 1798}$$

$$\frac{1}{3} \int -\frac{2cx^3+b}{-cx^6-bx^3+a} dx^3$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{3} \int \frac{2cx^3+b}{-cx^6-bx^3+a} dx^3$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{3} \log \left(a-bx^3-cx^6\right)$$

input
```
Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

output
```
Log[a - b*x^3 - c*x^6]/3
```

3.115.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.115.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(-c\,x^{6}-b\,x^{3}+a)}{3}$	18
norman	$\frac{\ln(-c\,x^{6}-b\,x^{3}+a)}{3}$	18
risch	$\frac{\ln(-c\,x^{6}-b\,x^{3}+a)}{3}$	18
parallelrisch	$\frac{\ln(c\,x^{6}+b\,x^{3}-a)}{3}$	18

input
```
int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(-c*x^6-b*x^3+a)
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(cx^6+bx^3-a\right)$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="fricas")
```

output
```
1/3*log(c*x^6 + b*x^3 - a)
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{\log\left(-a+bx^3+cx^6\right)}{3}$$

input
```
integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a),x)
```

output
```
log(-a + b*x**3 + c*x**6)/3
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(cx^6+bx^3-a\right)$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="maxima")
```

output
```
1/3*log(c*x^6 + b*x^3 - a)
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(\left|cx^6+bx^3-a\right|\right)$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="giac")
```

output
```
1/3*log(abs(c*x^6 + b*x^3 - a))
```

3.115.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{\ln\left(c\,x^6+b\,x^3-a\right)}{3}$$

input
```
int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6),x)
```

output
```
log(b*x^3 - a + c*x^6)/3
```

3.116 $\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} dx$

3.116.1 Optimal result

Integrand size = 32, antiderivative size = 21

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} dx = \frac{\log(a-bx^n-cx^{2n})}{n}$$

output
```
ln(a-b*x^n-c*x^(2*n))/n
```

3.116.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} dx = -\frac{2\log(x^{-n})}{n} + \frac{\log(c-ax^{-2n}+bx^{-n})}{n}$$

input
```
Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
```

output
```
(-2*Log[x^(-n)])/n + Log[c - a/x^(2*n) + b/x^n]/n
```

3.116.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1798, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \; 1798$$

$$\frac{\int -\frac{2cx^n+b}{-bx^n-cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{2cx^n+b}{-bx^n-cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \; 1103$$

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

input
```
Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
```

output
```
Log[a - b*x^n - c*x^(2*n)]/n
```

3.116.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.116.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
norman	$\frac{\ln\left(-c\,{\mathrm e}^{2 n \ln\left(x \right)}-b\,{\mathrm e}^{n \ln\left(x \right)}+a \right)}{n}$	26
risch	$\frac{\ln\left(x^{2 n}+\frac{b\,x^{n}}{c}-\frac{a}{c}\right)}{n}$	26

input
```
int(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/n*ln(-c*exp(n*ln(x))^2-b*exp(n*ln(x))+a)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx=\frac{\log\left(cx^{2\,n}+bx^n-a\right)}{n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
log(c*x^(2*n) + b*x^n - a)/n
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx = \frac{\log\left(\frac{cx^{2\,n}+bx^n-a}{c}\right)}{n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
log((c*x^(2*n) + b*x^n - a)/c)/n
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx = \frac{\log\left(cx^{2\,n}+bx^n-a\right)}{n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
log(c*x^(2*n) + b*x^n - a)/n
```

3.116.9 Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 199, normalized size of antiderivative = 9.48

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx = \ln\left(\frac{2\,c\,x^n}{n}-\left(\frac{1}{n}+\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)\,(b+2\,c\,x^n)\right)\,\left(\frac{1}{n}+\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)+\ln\left(\frac{2\,c\,x^n}{n}-\left(\frac{1}{n}-\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)\,(b+2\,c\,x^n)\right)\,\left(\frac{1}{n}-\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)-\frac{2\,b\,\mathrm{atanh}\left(\frac{b+2\,c\,x^n}{\sqrt{b^2+4\,a\,c}}\right)}{n\,\sqrt{b^2+4\,a\,c}}$$

input
```
int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n - a + c*x^(2*n)),x)
```

output
```
log((2*c*x^n)/n - (1/n + (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n))*(b + 2
*c*x^n))*(1/n + (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n)) + log((2*c*x^n)
/n - (1/n - (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n))*(b + 2*c*x^n))*(1/n
 - (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n)) - (2*b*atanh((b + 2*c*x^n)/(
4*a*c + b^2)^(1/2)))/(n*(4*a*c + b^2)^(1/2))
```

3.117 $\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx$

3.117.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx = \frac{1}{7(a-bx-cx^2)^7}$$

output
```
1/7/(-c*x^2-b*x+a)^7
```

3.117.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx = \frac{1}{7(a-x(b+cx))^7}$$

input
```
Integrate[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
```

output
```
1/(7*(a - x*(b + c*x))^7)
```

3.117.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx$$

$$\Big\downarrow \; 1104$$

$$\frac{1}{7(a-bx-cx^2)^7}$$

input
```
Int[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
```

output
```
1/(7*(a - b*x - c*x^2)^7)
```

3.117.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.117.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{1}{7(-cx^2-bx+a)^7}$	17
derivativedivides	$-\frac{1}{7(cx^2+bx-a)^7}$	17
default	$\frac{1}{7(-cx^2-bx+a)^7}$	17
norman	$\frac{1}{7(-cx^2-bx+a)^7}$	17
risch	$\frac{1}{7(-cx^2-bx+a)^7}$	17
parallelrisch	$-\frac{1}{7(cx^2+bx-a)^7}$	17

input
```
int((2*c*x+b)/(c*x^2+b*x-a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/7/(-c*x^2-b*x+a)^7
```

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 354, normalized size of antiderivative = 19.67

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx =$$

$$-\frac{}{7\,(c^7x^{14}+7\,bc^6x^{13}+7\,(3\,b^2c^5-ac^6)x^{12}+7\,(5\,b^3c^4-6\,abc^5)x^{11}+7\,(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5)x^{10}+7\,(}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="fricas")
```

output
```
-1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 - a*c^6)*x^12 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b
^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a
^2*b^2*c^3 - 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 - 42*a*b^5*c + 210*a^2*b^3*
c^2 - 140*a^3*b*c^3)*x^7 - a^7 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2
- 5*a^4*c^3)*x^6 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 - 7*(5*
a^3*b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^3 -
7*(3*a^5*b^2 - a^6*c)*x^2)
```

3.117.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(14) = 28$.

Time = 2.75 (sec) , antiderivative size = 359, normalized size of antiderivative = 19.94

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx =$$

$$-\frac{}{-7a^7+49a^6bx+49bc^6x^{13}+7c^7x^{14}+x^{12}\left(-49ac^6+147b^2c^5\right)+x^{11}\left(-294abc^5+245b^3c^4\right)+x^{10}\cdot(147}$$

input
```
integrate((2*c*x+b)/(c*x**2+b*x-a)**8,x)
```

output
```
-1/(-7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(-49*a*
c**6 + 147*b**2*c**5) + x**11*(-294*a*b*c**5 + 245*b**3*c**4) + x**10*(147
*a**2*c**5 - 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 - 98
0*a*b**3*c**3 + 147*b**5*c**2) + x**8*(-245*a**3*c**4 + 1470*a**2*b**2*c**
3 - 735*a*b**4*c**2 + 49*b**6*c) + x**7*(-980*a**3*b*c**3 + 1470*a**2*b**3
*c**2 - 294*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 - 1470*a**3*b**2*c**2
 + 735*a**2*b**4*c - 49*a*b**6) + x**5*(735*a**4*b*c**2 - 980*a**3*b**3*c
+ 147*a**2*b**5) + x**4*(-147*a**5*c**2 + 735*a**4*b**2*c - 245*a**3*b**4)
 + x**3*(-294*a**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c - 147*a**5*b**2)
)
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx-a)^7}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="maxima")
```

output
```
-1/7/(c*x^2 + b*x - a)^7
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx-a)^7}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="giac")
```

output
```
-1/7/(c*x^2 + b*x - a)^7
```

3.117.9 Mupad [B] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 358, normalized size of antiderivative = 19.89

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx =$$

$$-\frac{}{7\,\left(x^5\,\left(105\,a^4\,b\,c^2-140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^9\,\left(105\,a^2\,b\,c^4-140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^7\,\left(-140\,a^3\,b\,c^3+\right.\right.}$$

input
```
int((b + 2*c*x)/(b*x - a + c*x^2)^8,x)
```

output
```
-1/(7*(x^5*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2
- 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^3*
c^2 - 42*a*b^5*c) + x^3*(35*a^4*b^3 - 42*a^5*b*c) + x^11*(35*b^3*c^4 - 42*
a*b*c^5) - x^4*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^10*(21*a^2*c^
5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^6*(7*a*b^6 - 35*a^4*c^3 - 105*a^
2*b^4*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + 2
10*a^2*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c - 21*a^5*b^2) - x^12*(7*a*c^6 -
21*b^2*c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))
```

3.118 $\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx$

3.118.1 Optimal result

Integrand size = 26, antiderivative size = 20

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx = \frac{1}{14(a-bx^2-cx^4)^7}$$

output
```
1/14/(-c*x^4-b*x^2+a)^7
```

3.118.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx = -\frac{1}{14(-a+bx^2+cx^4)^7}$$

input
```
Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

output
```
-1/14*1/(-a + b*x^2 + c*x^4)^7
```

3.118.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1576, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{2cx^2+b}{(-cx^4-bx^2+a)^8}dx^2$$

$$\Big\downarrow \text{1104}$$

$$\frac{1}{14(a-bx^2-cx^4)^7}$$

input
```
Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

output
```
1/(14*(a - b*x^2 - c*x^4)^7)
```

3.118.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.118.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{1}{14(-c\,x^{4}-b\,x^{2}+a)^{7}}$	19
default	$\frac{1}{14(-c\,x^{4}-b\,x^{2}+a)^{7}}$	19
norman	$\frac{1}{14(-c\,x^{4}-b\,x^{2}+a)^{7}}$	19
risch	$\frac{1}{14(-c\,x^{4}-b\,x^{2}+a)^{7}}$	19
parallelrisch	$-\frac{1}{14(c\,x^{4}+b\,x^{2}-a)^{7}}$	19

input
```
int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/14/(-c*x^4-b*x^2+a)^7
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(18) = 36.

Time = 0.30 (sec) , antiderivative size = 356, normalized size of antiderivative = 17.80

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,(c^7x^{28}+7\,bc^6x^{26}+7\,(3\,b^2c^5-ac^6)x^{24}+7\,(5\,b^3c^4-6\,abc^5)x^{22}+7\,(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5)x^{20}+7}$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="fricas")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
 - 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*
b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30
*a^2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140
*a^3*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x
^12 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(
5*a^3*b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c
)*x^6 - 7*(3*a^5*b^2 - a^6*c)*x^4)
```

3.118.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(15) = 30$.

Time = 4.36 (sec) , antiderivative size = 360, normalized size of antiderivative = 18.00

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{-14a^7+98a^6bx^2+98bc^6x^{26}+14c^7x^{28}+x^{24}\left(-98ac^6+294b^2c^5\right)+x^{22}\left(-588abc^5+490b^3c^4\right)+x^{20}\cdot(}$$

input
```
integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a)**8,x)
```

output
```
-1/(-14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(-
98*a*c**6 + 294*b**2*c**5) + x**22*(-588*a*b*c**5 + 490*b**3*c**4) + x**20
*(294*a**2*c**5 - 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c
**4 - 1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(-490*a**3*c**4 + 2940*a**
2*b**2*c**3 - 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(-1960*a**3*b*c**3 + 2
940*a**2*b**3*c**2 - 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 - 2940
*a**3*b**2*c**2 + 1470*a**2*b**4*c - 98*a*b**6) + x**10*(1470*a**4*b*c**2
- 1960*a**3*b**3*c + 294*a**2*b**5) + x**8*(-294*a**5*c**2 + 1470*a**4*b**
2*c - 490*a**3*b**4) + x**6*(-588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**
6*c - 294*a**5*b**2))
```

3.118.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(18) = 36$.

Time = 0.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 17.80

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,(c^7x^{28}+7\,bc^6x^{26}+7\,(3\,b^2c^5-ac^6)x^{24}+7\,(5\,b^3c^4-6\,abc^5)x^{22}+7\,(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5)x^{20}+7}$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="maxima")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
 - 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*
b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30
*a^2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140
*a^3*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x
^12 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(
5*a^3*b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c
)*x^6 - 7*(3*a^5*b^2 - a^6*c)*x^4)
```

3.118.8 Giac [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(cx^4+bx^2-a)^7}$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="giac")
```

output
```
-1/14/(c*x^4 + b*x^2 - a)^7
```

3.118.9 Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 360, normalized size of antiderivative = 18.00

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,\left(x^{10}\,\left(105\,a^4\,b\,c^2-140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{18}\,\left(105\,a^2\,b\,c^4-140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{14}\,\left(-140\,a^3\,b\,c\right.\right.}$$

input
```
int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4)^8,x)
```

output
```
-1/(14*(x^10*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^18*(21*b^5*c
^2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^14*(b^7 - 140*a^3*b*c^3 + 210*a^2*
b^3*c^2 - 42*a*b^5*c) + x^6*(35*a^4*b^3 - 42*a^5*b*c) + x^22*(35*b^3*c^4 -
 42*a*b*c^5) - x^8*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^20*(21*a^
2*c^5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^12*(7*a*b^6 - 35*a^4*c^3 - 1
05*a^2*b^4*c + 210*a^3*b^2*c^2) + x^16*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c
^2 + 210*a^2*b^2*c^3) + c^7*x^28 + x^4*(7*a^6*c - 21*a^5*b^2) - x^24*(7*a*
c^6 - 21*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^26))
```

3.119 $\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx$

3.119.1 Optimal result

Integrand size = 28, antiderivative size = 20

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx = \frac{1}{21\,(a-bx^3-cx^6)^7}$$

output
```
1/21/(-c*x^6-b*x^3+a)^7
```

3.119.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx = -\frac{1}{21\,(-a+bx^3+cx^6)^7}$$

input
```
Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

output
```
-1/21*1/(-a + b*x^3 + c*x^6)^7
```

3.119.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1798, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (b+2cx^3)}{(-a+bx^3+cx^6)^8} dx$$

$$\Big\downarrow \text{ 1798}$$

$$\frac{1}{3} \int \frac{2cx^3+b}{(-cx^6-bx^3+a)^8} dx^3$$

$$\Big\downarrow \text{ 1104}$$

$$\frac{1}{21 (a-bx^3-cx^6)^7}$$

input
```
Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

output
```
1/(21*(a - b*x^3 - c*x^6)^7)
```

3.119.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.119.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{1}{21(-c\,x^{6}-b\,x^{3}+a)^{7}}$	19
default	$\frac{1}{21(-c\,x^{6}-b\,x^{3}+a)^{7}}$	19
risch	$\frac{1}{21(-c\,x^{6}-b\,x^{3}+a)^{7}}$	19
parallelrisch	$-\frac{1}{21(c\,x^{6}+b\,x^{3}-a)^{7}}$	19

input
```
int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/21/(-c*x^6-b*x^3+a)^7
```

3.119.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(18) = 36.

Time = 0.30 (sec) , antiderivative size = 356, normalized size of antiderivative = 17.80

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\,(c^7x^{42}+7\,bc^6x^{39}+7\,(3\,b^2c^5-ac^6)x^{36}+7\,(5\,b^3c^4-6\,abc^5)x^{33}+7\,(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5)x^{30}+7}$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="fricas")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
 - 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*
b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30
*a^2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140
*a^3*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x
^18 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15
*a^4*b^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9
 - a^7 - 7*(3*a^5*b^2 - a^6*c)*x^6)
```

3.119.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(15) = 30.

Time = 14.85 (sec) , antiderivative size = 360, normalized size of antiderivative = 18.00

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{-21a^7+147a^6bx^3+147bc^6x^{39}+21c^7x^{42}+x^{36}\left(-147ac^6+441b^2c^5\right)+x^{33}\left(-882abc^5+735b^3c^4\right)+x^{30}}$$

input
```
integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a)**8,x)
```

output
```
-1/(-21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*
(-147*a*c**6 + 441*b**2*c**5) + x**33*(-882*a*b*c**5 + 735*b**3*c**4) + x*
*30*(441*a**2*c**5 - 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*
b*c**4 - 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(-735*a**3*c**4 + 4410*
a**2*b**2*c**3 - 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(-2940*a**3*b*c**3
 + 4410*a**2*b**3*c**2 - 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 -
4410*a**3*b**2*c**2 + 2205*a**2*b**4*c - 147*a*b**6) + x**15*(2205*a**4*b*
c**2 - 2940*a**3*b**3*c + 441*a**2*b**5) + x**12*(-441*a**5*c**2 + 2205*a*
*4*b**2*c - 735*a**3*b**4) + x**9*(-882*a**5*b*c + 735*a**4*b**3) + x**6*(
147*a**6*c - 441*a**5*b**2))
```

3.119.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 356, normalized size of antiderivative = 17.80

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\left(c^7x^{42}+7\,bc^6x^{39}+7\left(3\,b^2c^5-ac^6\right)x^{36}+7\left(5\,b^3c^4-6\,abc^5\right)x^{33}+7\left(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5\right)x^{30}+7\right.}$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="maxima")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
 - 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*
b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30
*a^2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140
*a^3*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x
^18 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15
*a^4*b^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9
 - a^7 - 7*(3*a^5*b^2 - a^6*c)*x^6)
```

3.119.8 Giac [A] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx = -\frac{1}{21\,(cx^6+bx^3-a)^7}$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="giac")
```

output
```
-1/21/(c*x^6 + b*x^3 - a)^7
```

3.119.9 Mupad [B] (verification not implemented)

Time = 17.24 (sec) , antiderivative size = 360, normalized size of antiderivative = 18.00

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\,\left(x^{15}\,\left(105\,a^4\,b\,c^2-140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{27}\,\left(105\,a^2\,b\,c^4-140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{21}\,\left(-140\,a^3\,b\,c\right.\right.}$$

input
```
int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6)^8,x)
```

output
```
-1/(21*(x^15*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^27*(21*b^5*c
^2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^21*(b^7 - 140*a^3*b*c^3 + 210*a^2*
b^3*c^2 - 42*a*b^5*c) + x^9*(35*a^4*b^3 - 42*a^5*b*c) + x^33*(35*b^3*c^4 -
 42*a*b*c^5) - x^12*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^30*(21*a
^2*c^5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^18*(7*a*b^6 - 35*a^4*c^3 -
105*a^2*b^4*c + 210*a^3*b^2*c^2) + x^24*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*
c^2 + 210*a^2*b^2*c^3) + c^7*x^42 + x^6*(7*a^6*c - 21*a^5*b^2) - x^36*(7*a
*c^6 - 21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^39))
```

3.120 $\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx$

3.120.1 Optimal result

Integrand size = 32, antiderivative size = 25

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx=\frac{1}{7n\,(a-bx^n-cx^{2n})^7}$$

output
```
1/7/n/(a-b*x^n-c*x^(2*n))^7
```

3.120.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx=\frac{1}{7n\,(a-x^n\,(b+cx^n))^7}$$

input
```
Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

output
```
1/(7*n*(a - x^n*(b + c*x^n))^7)
```

3.120.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1798, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx$$

$$\Big\downarrow \text{ 1798}$$

$$\frac{\int \frac{2cx^n+b}{(-bx^n-cx^{2n}+a)^8}dx^n}{n}$$

$$\Big\downarrow \text{ 1104}$$

$$\frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

input
```
Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

output
```
1/(7*n*(a - b*x^n - c*x^(2*n))^7)
```

3.120.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.120.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\frac{1}{7n\,(a-b\,x^{n}-c\,x^{2n})^{7}}$$

input
```
int(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x)
```

output
```
1/7/n/(-c*(x^n)^2-b*x^n+a)^7
```

3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(23) = 46.

Time = 0.31 (sec) , antiderivative size = 397, normalized size of antiderivative = 15.88

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx =$$

$$-\frac{}{7\,(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+7\,a^6bnx^n-a^7n+7\,(3\,b^2c^5-ac^6)nx^{12\,n}+7\,(5\,b^3c^4-6\,abc^5)nx^{11\,n}+7\,(5\,b^4c^3}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas
")
```

output
```
-1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b
^2*c^5 - a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 - 6*a*b*c^5)*n*x^(11*n) + 7*(5*b
^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 - 20*a*b^3*c^
3 + 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5
*a^3*c^4)*n*x^(8*n) + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)
*n*x^(7*n) - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*n*x^(6*
n) + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) - 7*(5*a^3*b^4
- 15*a^4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 - 6*a^5*b*c)*n*x^(3*n
) - 7*(3*a^5*b^2 - a^6*c)*n*x^(2*n))
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n))**8,x)
```

output
```
Timed out
```

3.120.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(23) = 46$.

Time = 0.56 (sec) , antiderivative size = 419, normalized size of antiderivative = 16.76

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx =$$

$$-\frac{}{7\,(c^7nx^{14\,n} + 7\,bc^6nx^{13\,n} + 7\,a^6bnx^n - a^7n + 7\,(3\,b^2c^5n - ac^6n)x^{12\,n} + 7\,(5\,b^3c^4n - 6\,abc^5n)x^{11\,n} + 7\,(5\,b}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima
")
```

output
```
-1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b
^2*c^5*n - a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n - 6*a*b*c^5*n)*x^(11*n) + 7*
(5*b^4*c^3*n - 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n - 2
0*a*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n - 15*a*b^4*c^2*n + 30
*a^2*b^2*c^3*n - 5*a^3*c^4*n)*x^(8*n) + (b^7*n - 42*a*b^5*c*n + 210*a^2*b^
3*c^2*n - 140*a^3*b*c^3*n)*x^(7*n) - 7*(a*b^6*n - 15*a^2*b^4*c*n + 30*a^3*
b^2*c^2*n - 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n - 20*a^3*b^3*c*n + 15*a^
4*b*c^2*n)*x^(5*n) - 7*(5*a^3*b^4*n - 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n
) + 7*(5*a^4*b^3*n - 6*a^5*b*c*n)*x^(3*n) - 7*(3*a^5*b^2*n - a^6*c*n)*x^(2
*n))
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx = -\frac{1}{7\left(cx^{2n}+bx^n-a\right)^7 n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

output
```
-1/7/((c*x^(2*n) + b*x^n - a)^7*n)
```

3.120.9 Mupad [B] (verification not implemented)

Time = 22.03 (sec) , antiderivative size = 496, normalized size of antiderivative = 19.84

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx =$$

$$-\frac{}{7\,b^7\,n\,x^{7\,n}-7\,a^7\,n+7\,c^7\,n\,x^{14\,n}+49\,a^6\,b\,n\,x^n-49\,a\,b^6\,n\,x^{6\,n}+49\,a^6\,c\,n\,x^{2\,n}-49\,a\,c^6\,n\,x^{12\,n}+49\,b^6\,c\,n}$$

input
```
int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n - a + c*x^(2*n))^8,x)
```

output
```
-1/(7*b^7*n*x^(7*n) - 7*a^7*n + 7*c^7*n*x^(14*n) + 49*a^6*b*n*x^n - 49*a*b
^6*n*x^(6*n) + 49*a^6*c*n*x^(2*n) - 49*a*c^6*n*x^(12*n) + 49*b^6*c*n*x^(8*
n) + 49*b*c^6*n*x^(13*n) - 147*a^5*b^2*n*x^(2*n) + 245*a^4*b^3*n*x^(3*n) -
 245*a^3*b^4*n*x^(4*n) + 147*a^2*b^5*n*x^(5*n) - 147*a^5*c^2*n*x^(4*n) + 2
45*a^4*c^3*n*x^(6*n) - 245*a^3*c^4*n*x^(8*n) + 147*a^2*c^5*n*x^(10*n) + 14
7*b^5*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(11*n) + 14
7*b^2*c^5*n*x^(12*n) + 735*a^4*b^2*c*n*x^(4*n) - 980*a^3*b^3*c*n*x^(5*n) +
 735*a^4*b*c^2*n*x^(5*n) + 735*a^2*b^4*c*n*x^(6*n) - 980*a^3*b*c^3*n*x^(7*
n) - 735*a*b^4*c^2*n*x^(8*n) - 980*a*b^3*c^3*n*x^(9*n) + 735*a^2*b*c^4*n*x
^(9*n) - 735*a*b^2*c^4*n*x^(10*n) - 1470*a^3*b^2*c^2*n*x^(6*n) + 1470*a^2*
b^3*c^2*n*x^(7*n) + 1470*a^2*b^2*c^3*n*x^(8*n) - 294*a^5*b*c*n*x^(3*n) - 2
94*a*b^5*c*n*x^(7*n) - 294*a*b*c^5*n*x^(11*n))
```

3.121 $\int \frac{b+2cx}{bx+cx^2} \, dx$

3.121.1 Optimal result

Integrand size = 18, antiderivative size = 10

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log \left(bx+cx^2\right)$$

output
```
ln(c*x^2+b*x)
```

3.121.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log(x)+\log(b+cx)$$

input
```
Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
```

output
```
Log[x] + Log[b + c*x]
```

3.121.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 1103}$$

$$\log\left(bx+cx^2\right)$$

input
```
Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

output
```
Log[b*x + c*x^2]
```

3.121.3.1 Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.121.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
default	$\ln\left(x(cx+b)\right)$	9
norman	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
parallelrisch	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
derivativedivides	$\ln\left(c\,x^2+bx\right)$	11
risch	$\ln\left(c\,x^2+bx\right)$	11

input
```
int((2*c*x+b)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x*(c*x+b))
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(cx^2+bx\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
log(c*x^2 + b*x)
```

3.121.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(bx+cx^2\right)$$

input
```
integrate((2*c*x+b)/(c*x**2+b*x),x)
```

output
```
log(b*x + c*x**2)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(cx^2+bx\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
log(c*x^2 + b*x)
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(\left|cx^2+bx\right|\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
log(abs(c*x^2 + b*x))
```

3.121.9 Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \ln\left(x\,(b+c\,x)\right)$$

input
```
int((b + 2*c*x)/(b*x + c*x^2),x)
```

output
```
log(x*(b + c*x))
```

3.122 $\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx$

3.122.1 Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx = \frac{1}{2}\log\left(bx^2+cx^4\right)$$

output
```
1/2*ln(c*x^4+b*x^2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx = \log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

input
```
Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
```

output
```
Log[x] + Log[b + c*x^2]/2
```

3.122.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {9, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{b+2cx^2}{x(b+cx^2)}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{2cx^2+b}{x^2(cx^2+b)}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}\int \left(\frac{c}{cx^2+b}+\frac{1}{x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\log\left(b+cx^2\right)+\log\left(x^2\right)\right)$$

input
```
Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
```

output
```
(Log[x^2] + Log[b + c*x^2])/2
```

3.122.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.122.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$\ln(x) + \frac{\ln(c x^2+b)}{2}$	14
norman	$\ln(x) + \frac{\ln(c x^2+b)}{2}$	14
risch	$\ln(x) + \frac{\ln(c x^2+b)}{2}$	14
parallelrisch	$\ln(x) + \frac{\ln(c x^2+b)}{2}$	14

input
```
int(x*(2*c*x^2+b)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/2*ln(c*x^2+b)
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx = \frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/2*log(c*x^2 + b) + log(x)
```

3.122.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx = \log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

input
```
integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2),x)
```

output
```
log(x) + log(b/c + x**2)/2
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx = \frac{1}{2}\log\left(cx^2+b\right)+\frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx = \frac{1}{2}\log\left(\left|cx^4+bx^2\right|\right)$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*log(abs(c*x^4 + b*x^2))
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx = \frac{\ln\left(c\,x^2+b\right)}{2}+\ln\left(x\right)$$

input
```
int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x)
```

output
```
log(b + c*x^2)/2 + log(x)
```

3.123 $\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx$

3.123.1 Optimal result

Integrand size = 25, antiderivative size = 16

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx = \frac{1}{3}\log\left(bx^3+cx^6\right)$$

output
```
1/3*ln(c*x^6+b*x^3)
```

3.123.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx = \log(x)+\frac{1}{3}\log\left(b+cx^3\right)$$

input
```
Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

output
```
Log[x] + Log[b + c*x^3]/3
```

3.123.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {9, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (b+2 c x^3)}{b x^3+c x^6} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{b+2 c x^3}{x (b+c x^3)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3} \int \frac{2 c x^3+b}{x^3 (c x^3+b)}dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3} \int \left(\frac{c}{c x^3+b}+\frac{1}{x^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(\log \left(b+c x^3\right)+\log \left(x^3\right)\right)$$

input
```
Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

output
```
(Log[x^3] + Log[b + c*x^3])/3
```

3.123.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.123.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14
norman	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14
risch	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14
parallelrisch	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14

input
```
int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/3*ln(c*x^3+b)
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx = \frac{1}{3}\log\left(cx^3+b\right)+\log\left(x\right)$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="fricas")
```

output
```
1/3*log(c*x^3 + b) + log(x)
```

3.123.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx = \log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

input
```
integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3),x)
```

output
```
log(x) + log(b/c + x**3)/3
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx = \frac{1}{3}\log\left(cx^3+b\right)+\frac{1}{3}\log\left(x^3\right)$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="maxima")
```

output
```
1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx = \frac{1}{3}\log\left(\left|cx^6+bx^3\right|\right)$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="giac")
```

output
```
1/3*log(abs(c*x^6 + b*x^3))
```

3.123.9 Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx = \frac{\ln\left(c\,x^3+b\right)}{3}+\ln\left(x\right)$$

input
```
int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x)
```

output
```
log(b + c*x^3)/3 + log(x)
```

3.124 $\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx$

3.124.1 Optimal result

Integrand size = 29, antiderivative size = 15

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \log(x) + \frac{\log(b+cx^n)}{n}$$

output
```
ln(x)+ln(b+c*x^n)/n
```

3.124.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \frac{\log(x^n) + \log(n(b+cx^n))}{n}$$

input
```
Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
```

output
```
(Log[x^n] + Log[n*(b + c*x^n)])/n
```

3.124.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {10, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}(b+2cx^n)}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \ 10$$

$$\int \frac{b+2cx^n}{x(b+cx^n)}dx$$

$$\Big\downarrow \ 948$$

$$\frac{\int \frac{x^{-n}(2cx^n+b)}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \ 86$$

$$\frac{\int \left(x^{-n}+\frac{c}{cx^n+b}\right)dx^n}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{\log(b+cx^n)+\log(x^n)}{n}$$

input
```
Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
```

output
```
(Log[x^n] + Log[b + c*x^n])/n
```

3.124.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.124.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\ln\left(x\right)+\frac{\ln\left(c\,{\mathrm e}^{n\ln\left(x\right)}+b\right)}{n}$	18
risch	$\ln\left(x\right)+\frac{\ln\left(x^{n}+\frac{b}{c}\right)}{n}$	18

input
```
int(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx=\frac{n\log\left(x\right)+\log\left(cx^n+b\right)}{n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
(n*log(x) + log(c*x^n + b))/n
```

3.124.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 7.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \begin{cases} \log(x) & \text{for } c = 0 \wedge n = 0 \\ -\frac{\log(x^{-n})}{n} & \text{for } c = 0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n = 0 \\ \frac{\log(x^n)}{n} + \frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), (-log(x**(-n))/n, Eq(c, 0)), ((b
+ 2*c)*log(x)/(b + c), Eq(n, 0)), (log(x**n)/n + log(b/c + x**n)/n, True))
```

3.124.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = b\left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right) + \frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \frac{\log(|cx^n+b|)}{n} + \log(|x|)$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
log(abs(c*x^n + b))/n + log(abs(x))
```

3.124.9 Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \frac{2\left(\ln(b+c\,x^n) - \operatorname{atanh}\left(\frac{2\,c\,x^n}{b}+1\right)\right)}{n}$$

input
```
int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x)
```

output
```
(2*(log(b + c*x^n) - atanh((2*c*x^n)/b + 1)))/n
```

3.125 $\int \frac{b+2cx}{(bx+cx^2)^8}\,dx$

3.125.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,(bx+cx^2)^7}$$

output
```
-1/7/(c*x^2+b*x)^7
```

3.125.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7x^7(b+cx)^7}$$

input
```
Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

output
```
-1/7*1/(x^7*(b + c*x)^7)
```

3.125.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{\left(bx+cx^2\right)^8} \, dx$$

$$\Big\downarrow \; 1104$$

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

input
```
Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

output
```
-1/7*1/(b*x + c*x^2)^7
```

3.125.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.125.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result
gosper	$-\frac{1}{7x^7(cx+b)^7}$
norman	$-\frac{1}{7x^7(cx+b)^7}$
risch	$-\frac{1}{7x^7(cx+b)^7}$
parallelrisch	$-\frac{1}{7x^7(cx+b)^7}$
derivativedivides	$-\frac{1}{7(c\,x^2+bx)^7}$
default	$-\frac{1}{7b^7x^7}-\frac{132c^6}{b^{13}x}+\frac{66c^5}{b^{12}x^2}-\frac{30c^4}{b^{11}x^3}+\frac{12c^3}{b^{10}x^4}-\frac{4c^2}{b^9x^5}+\frac{c}{b^8x^6}+\frac{132c^7}{b^{13}(cx+b)}+\frac{66c^7}{b^{12}(cx+b)^2}+\frac{30c^7}{b^{11}(cx+b)^3}+$

input
```
int((2*c*x+b)/(c*x^2+b*x)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7/x^7/(c*x+b)^7
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(13) = 26$.

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.40

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx$$
$$= -\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

output
```
-1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4
*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

3.125.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(14) = 28$.

Time = 0.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.80

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx =$$
$$-\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

input
```
integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

output
```
-1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**1
0 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x
**14)
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx)^7}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
```

output
```
-1/7/(c*x^2 + b*x)^7
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx)^7}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
```

output
```
-1/7/(c*x^2 + b*x)^7
```

3.125.9 Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,x^7\,(b+c\,x)^7}$$

input
```
int((b + 2*c*x)/(b*x + c*x^2)^8,x)
```

output
```
-1/(7*x^7*(b + c*x)^7)
```

3.126 $\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} dx$

3.126.1 Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} dx = -\frac{1}{14x^{14}(b+cx^2)^7}$$

output
```
-1/14/x^14/(c*x^2+b)^7
```

3.126.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} dx = -\frac{1}{14x^{14}(b+cx^2)^7}$$

input
```
Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

output
```
-1/14*1/(x^14*(b + c*x^2)^7)
```

3.126.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {9, 354, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8}dx$$
$$\Big\downarrow \ 354$$
$$\frac{1}{2}\int \frac{2cx^2+b}{x^{16}(cx^2+b)^8}dx^2$$
$$\Big\downarrow \ 83$$
$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

input
```
Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

output
```
-1/14*1/(x^14*(b + c*x^2)^7)
```

3.126.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.126.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
norman	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
risch	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
parallelrisch	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
default	$-\frac{1}{14b^7x^{14}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}-\frac{15c^4}{b^{11}x^6}+\frac{6c^3}{b^{10}x^8}-\frac{2c^2}{b^9x^{10}}+\frac{c}{2b^8x^{12}}-\frac{c^8\left(-\frac{12b^3}{c\left(c\,x^2+b\right)^4}-\frac{b^5}{c\left(c\,x^2+b\right)^6}-\frac{66b}{c\left(c\,x^2+b\right)^2}-\cdots\right)}{2\cdots}$

input
```
int(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/14/x^14/(c*x^2+b)^7
```

3.126.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}\,dx =$$
$$-\frac{1}{14\,(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14})}$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="fricas")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^
4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

3.126.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.66 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}\,dx = -\frac{1}{14b^7x^{14}+98b^6cx^{16}+294b^5c^2x^{18}+490b^4c^3x^{20}+490b^3c^4x^{22}+294b^2c^5x^{24}+98bc^6x^{26}+14c^7x^{28}}$$

input
```
integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2)**8,x)
```

output
```
-1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*
x**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c
**7*x**28)
```

3.126.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14})}$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="maxima")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^
4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(cx^4+bx^2)^7}$$

input
```
integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="giac")
```

output
```
-1/14/(c*x^4 + b*x^2)^7
```

3.126.9 Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}\,dx = -\frac{1}{14\,x^{14}\,(c\,x^2+b)^7}$$

input
```
int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x)
```

output
```
-1/(14*x^14*(b + c*x^2)^7)
```

3.127 $\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx$

3.127.1 Optimal result

Integrand size = 25, antiderivative size = 16

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx = -\frac{1}{21x^{21}(b+cx^3)^7}$$

output
```
-1/21/x^21/(c*x^3+b)^7
```

3.127.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx = -\frac{1}{21x^{21}(b+cx^3)^7}$$

input
```
Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

output
```
-1/21*1/(x^21*(b + c*x^3)^7)
```

3.127.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {9, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (b+2cx^3)}{(bx^3+cx^6)^8} \, dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{b+2cx^3}{x^{22} (b+cx^3)^8} dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3} \int \frac{2cx^3+b}{x^{24} (cx^3+b)^8} dx^3$$

$$\Big\downarrow \ 83$$

$$-\frac{1}{21x^{21} (b+cx^3)^7}$$

input
```
Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

output
```
-1/21*1/(x^21*(b + c*x^3)^7)
```

3.127.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.127.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
risch	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
parallelrisch	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
default	$-\frac{1}{21b^{7}x^{21}}-\frac{44c^{6}}{b^{13}x^{3}}+\frac{22c^{5}}{b^{12}x^{6}}-\frac{10c^{4}}{b^{11}x^{9}}+\frac{4c^{3}}{b^{10}x^{12}}-\frac{4c^{2}}{3b^{9}x^{15}}+\frac{c}{3b^{8}x^{18}}-\frac{c^{8}\left(-\frac{12b^{3}}{c\left(c\,x^{3}+b\right)^{4}}-\frac{b^{5}}{c\left(c\,x^{3}+b\right)^{6}}-\frac{66b}{c\left(c\,x^{3}+b\right)^{2}}-\right.}{}$

input
```
int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21/x^21/(c*x^3+b)^7
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx =$$
$$-\frac{1}{21\,(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21})}$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="fricas")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^
4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

3.127.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx = -\frac{1}{21b^7x^{21}+147b^6cx^{24}+441b^5c^2x^{27}+735b^4c^3x^{30}+735b^3c^4x^{33}+441b^2c^5x^{36}+147bc^6x^{39}+21c^7x^{42}}$$

input
```
integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3)**8,x)
```

output
```
-1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3
*x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21
*c**7*x**42)
```

3.127.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx = -\frac{1}{21\,(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21})}$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="maxima")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^
4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx = -\frac{1}{21\,(cx^6+bx^3)^7}$$

input
```
integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="giac")
```

output
```
-1/21/(c*x^6 + b*x^3)^7
```

3.127.9 Mupad [B] (verification not implemented)

Time = 11.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx = -\frac{1}{21\,x^{21}\,(c\,x^3+b)^7}$$

input
```
int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x)
```

output
```
-1/(21*x^21*(b + c*x^3)^7)
```

3.128 $\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx$

3.128.1 Optimal result

Integrand size = 29, antiderivative size = 21

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

output
```
-1/7/n/(x^(7*n))/(b+c*x^n)^7
```

3.128.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

input
```
Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

output
```
-1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

3.128.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {10, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx$$

$$\Big\downarrow \; 10$$

$$\int \frac{x^{-7n-1}(b+2cx^n)}{(b+cx^n)^8}dx$$

$$\Big\downarrow \; 948$$

$$\frac{\int \frac{x^{-8n}(2cx^n+b)}{(cx^n+b)^8}dx^n}{n}$$

$$\Big\downarrow \; 83$$

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

input
```
Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

output
```
-1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

3.128.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.128.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(21) = 42$.

Time = 69.56 (sec) , antiderivative size = 203, normalized size of antiderivative = 9.67

method	result
risch	$-\frac{132c^6x^{-n}}{b^{13}n}+\frac{66c^5x^{-2n}}{b^{12}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{c\,x^{-6n}}{b^8n}-\frac{x^{-7n}}{7b^7n}+\frac{c^7(924x^{6n}c^6+6006b\,c^5x^{5n}+16380b^2$

input
```
int(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x,method=_RETURNVERBOSE)
```

output
```
-132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*
c^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*
c^7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^
3*(x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^
7
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(21) = 42$.

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx =$$

$$-\frac{1}{7\,(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+21\,b^2c^5nx^{12\,n}+35\,b^3c^4nx^{11\,n}+35\,b^4c^3nx^{10\,n}+21\,b^5c^2nx^{9\,n}+7\,b^6cnx^{8\,n}+b^7n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="fricas")
```

output
```
-1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3
*c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n
*x^(8*n) + b^7*n*x^(7*n))
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n))**8,x)
```

output
```
Timed out
```

3.128.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 612, normalized size of antiderivative = 29.14

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = -\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n} + 2342340\,bc^{12}x^{12\,n} + 6426420\,b^2c^{11}x^{11\,n} + 9579570\,b^3c^{10}x^{10\,n} + 8270262\,b^4c^9x^{9\,n}}{b^{14}c^7nx^{14\,n} + 7\,b^{15}c^6nx^{13\,n} + 21\,b^{16}c^5n}\right. + \frac{1}{105}\,c\left(\frac{360360\,c^{12}x^{12\,n} + 2342340\,bc^{11}x^{11\,n} + 6426420\,b^2c^{10}x^{10\,n} + 9579570\,b^3c^9x^{9\,n} + 8270262\,b^4c^8x^{8\,n} +}{b^{13}c^7nx^{13\,n} + 7\,b^{14}c^6nx^{12\,n} + 21\,b^{15}c^5nx^{11\,n} +}\right.$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="maxima")
```

output
```
-1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^
11*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 401801
4*b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*
b^8*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^
2*x^(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x
^(13*n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*
x^(10*n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) +
360360*c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c
*((360360*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10
*n) + 9579570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*
x^(7*n) + 934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^
(4*n) + 1001*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b
^12)/(b^13*c^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n)
 + 35*b^16*c^4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n)
+ 7*b^19*c*n*x^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c
^6*log((c*x^n + b)/c)/(b^14*n))
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = -\frac{1}{7\,(cx^{2\,n}+bx^n)^7 n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

output
```
-1/7/((c*x^(2*n) + b*x^n)^7*n)
```

3.128.9 Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 107, normalized size of antiderivative = 5.10

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx =$$
$$-\frac{1}{7\,b^7\,n\,x^{7\,n}+7\,c^7\,n\,x^{14\,n}+49\,b^6\,c\,n\,x^{8\,n}+49\,b\,c^6\,n\,x^{13\,n}+147\,b^5\,c^2\,n\,x^{9\,n}+245\,b^4\,c^3\,n\,x^{10\,n}+245\,b^3\,c^4\,n\,x}$$

input
```
int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x)
```

output
```
-1/(7*b^7*n*x^(7*n) + 7*c^7*n*x^(14*n) + 49*b^6*c*n*x^(8*n) + 49*b*c^6*n*x
^(13*n) + 147*b^5*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x
^(11*n) + 147*b^2*c^5*n*x^(12*n))
```

3.128. $\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx$

3.129 $\int (b+2cx) \left(a+bx+cx^2\right)^p \, dx$

3.129.1 Optimal result

Integrand size = 19, antiderivative size = 20

$$\int (b+2cx) \left(a+bx+cx^2\right)^p \, dx = \frac{\left(a+bx+cx^2\right)^{1+p}}{1+p}$$

output
```
(c*x^2+b*x+a)^(p+1)/(p+1)
```

3.129.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (b+2cx) \left(a+bx+cx^2\right)^p \, dx = \frac{(a+x(b+cx))^{1+p}}{1+p}$$

input
```
Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
```

output
```
(a + x*(b + c*x))^(1 + p)/(1 + p)
```

3.129.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \;1104$$

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

input
```
Int[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
```

output
```
(a + b*x + c*x^2)^(1 + p)/(1 + p)
```

3.129.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.129.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{\left(c\,x^{2}+bx+a\right)^{1+p}}{1+p}$	21
derivativedivides	$\frac{\left(c\,x^{2}+bx+a\right)^{1+p}}{1+p}$	21
default	$\frac{\left(c\,x^{2}+bx+a\right)^{1+p}}{1+p}$	21
risch	$\frac{\left(c\,x^{2}+bx+a\right)\left(c\,x^{2}+bx+a\right)^{p}}{1+p}$	29
parallelrisch	$\frac{x^{2}\left(c\,x^{2}+bx+a\right)^{p}ac+ab\left(c\,x^{2}+bx+a\right)^{p}x+a^{2}\left(c\,x^{2}+bx+a\right)^{p}}{a(1+p)}$	61
norman	$\frac{a\,\mathrm{e}^{p\ln\left(c\,x^{2}+bx+a\right)}}{1+p}+\frac{bx\,\mathrm{e}^{p\ln\left(c\,x^{2}+bx+a\right)}}{1+p}+\frac{c\,x^{2}\mathrm{e}^{p\ln\left(c\,x^{2}+bx+a\right)}}{1+p}$	69

input
```
int((2*c*x+b)*(c*x^2+b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x+a)^(1+p)/(1+p)
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx=\frac{(cx^2+bx+a)(cx^2+bx+a)^p}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
(c*x^2 + b*x + a)*(c*x^2 + b*x + a)^p/(p + 1)
```

3.129.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(15) = 30$.

Time = 29.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 5.20

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx = \begin{cases} \frac{a(a+bx+cx^2)^p}{p+1} + \frac{bx(a+bx+cx^2)^p}{p+1} + \frac{cx^2(a+bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log\left(\frac{b}{2c} + x - \frac{\sqrt{-4ac+b^2}}{2c}\right) + \log\left(\frac{b}{2c} + x + \frac{\sqrt{-4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((2*c*x+b)*(c*x**2+b*x+a)**p,x)
```

output
```
Piecewise((a*(a + b*x + c*x**2)**p/(p + 1) + b*x*(a + b*x + c*x**2)**p/(p
+ 1) + c*x**2*(a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) + x
- sqrt(-4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)
), True))
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx = \frac{(cx^2+bx+a)^{p+1}}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
(c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx = \frac{(cx^2+bx+a)^{p+1}}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
(c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

3.129.9 Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx=\left(\frac{a}{p+1}+\frac{b\,x}{p+1}+\frac{c\,x^2}{p+1}\right)\,\left(c\,x^2+b\,x+a\right)^p$$

input
```
int((b + 2*c*x)*(a + b*x + c*x^2)^p,x)
```

output
```
(a/(p + 1) + (b*x)/(p + 1) + (c*x^2)/(p + 1))*(a + b*x + c*x^2)^p
```

3.130 $\int x(b+2cx^2)(a+bx^2+cx^4)^p\,dx$

3.130.1 Optimal result

Integrand size = 24, antiderivative size = 25

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p\,dx=\frac{(a+bx^2+cx^4)^{1+p}}{2(1+p)}$$

output
```
1/2*(c*x^4+b*x^2+a)^(p+1)/(p+1)
```

3.130.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p\,dx=\frac{(a+bx^2+cx^4)^{1+p}}{2(1+p)}$$

input
```
Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

3.130.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1576, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \left(2cx^2+b\right)\left(cx^4+bx^2+a\right)^p\,dx^2$$

$$\Big\downarrow \text{1104}$$

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

input
```
Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

3.130.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.130.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(c\,x^4+b\,x^2+a)^{1+p}}{2+2p}$	24
risch	$\frac{(c\,x^4+b\,x^2+a)\,(c\,x^4+b\,x^2+a)^p}{2+2p}$	34
parallelrisch	$\frac{x^4(c\,x^4+b\,x^2+a)^p c^2+x^2(c\,x^4+b\,x^2+a)^p bc+(c\,x^4+b\,x^2+a)^p ac}{2c(1+p)}$	70
norman	$\frac{a\,e^{p\ln\left(c\,x^4+b\,x^2+a\right)}}{2+2p}+\frac{b\,x^2 e^{p\ln\left(c\,x^4+b\,x^2+a\right)}}{2+2p}+\frac{c\,x^4 e^{p\ln\left(c\,x^4+b\,x^2+a\right)}}{2+2p}$	80

input
```
int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^4+b*x^2+a)^(1+p)/(1+p)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2+a)(cx^4+bx^2+a)^p}{2\,(p+1)}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

3.130.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(19) = 38.

Time = 104.63 (sec) , antiderivative size = 201, normalized size of antiderivative = 8.04

$$\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx$$
$$=\begin{cases}\frac{a(a+bx^2+cx^4)^p}{2p+2}+\frac{bx^2(a+bx^2+cx^4)^p}{2p+2}+\frac{cx^4(a+bx^2+cx^4)^p}{2p+2}\\ \frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2}\end{cases}$$

input
```
integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**p,x)
```

output
```
Piecewise((a*(a + b*x**2 + c*x**4)**p/(2*p + 2) + b*x**2*(a + b*x**2 + c*x
**4)**p/(2*p + 2) + c*x**4*(a + b*x**2 + c*x**4)**p/(2*p + 2), Ne(p, -1)),
 (log(x - sqrt(2)*sqrt(-b/c - sqrt(-4*a*c + b**2)/c)/2)/2 + log(x + sqrt(2
)*sqrt(-b/c - sqrt(-4*a*c + b**2)/c)/2)/2 + log(x - sqrt(2)*sqrt(-b/c + sq
rt(-4*a*c + b**2)/c)/2)/2 + log(x + sqrt(2)*sqrt(-b/c + sqrt(-4*a*c + b**2
)/c)/2)/2, True))
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2+a)(cx^4+bx^2+a)^p}{2(p+1)}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2+a)^{p+1}}{2(p+1)}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
1/2*(c*x^4 + b*x^2 + a)^(p + 1)/(p + 1)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p\,dx=\left(c\,x^4+b\,x^2+a\right)^p\left(\frac{a}{2\,p+2}+\frac{b\,x^2}{2\,p+2}+\frac{c\,x^4}{2\,p+2}\right)$$

input
```
int(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x)
```

output
```
(a + b*x^2 + c*x^4)^p*(a/(2*p + 2) + (b*x^2)/(2*p + 2) + (c*x^4)/(2*p + 2)
)
```

3.131 $\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^p\,dx$

3.131.1 Optimal result

Integrand size = 26, antiderivative size = 25

$$\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\frac{\left(a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

output
```
1/3*(c*x^6+b*x^3+a)^(p+1)/(p+1)
```

3.131.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\frac{\left(a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

input
```
Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

3.131.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1798, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (b+2cx^3) (a+bx^3+cx^6)^p \, dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{3}\int (2cx^3+b) (cx^6+bx^3+a)^p \, dx^3$$

$$\Big\downarrow \text{1104}$$

$$\frac{(a+bx^3+cx^6)^{p+1}}{3(p+1)}$$

input
```
Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

3.131.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.131.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(c x^6+b x^3+a)^{1+p}}{3+3p}$	24
risch	$\frac{(c x^6+b x^3+a)(c x^6+b x^3+a)^p}{3+3p}$	34
parallelrisch	$\frac{x^6(c x^6+b x^3+a)^p c^2+x^3(c x^6+b x^3+a)^p bc+(c x^6+b x^3+a)^p ac}{3c(1+p)}$	70
norman	$\frac{a\,e^{p\ln(c x^6+b x^3+a)}}{3+3p}+\frac{b x^3 e^{p\ln(c x^6+b x^3+a)}}{3+3p}+\frac{c x^6 e^{p\ln(c x^6+b x^3+a)}}{3+3p}$	80

input
```
int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x^6+b*x^3+a)^(1+p)/(1+p)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3+a)(cx^6+bx^3+a)^p}{3\,(p+1)}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x^2 (b+2cx^3) (a+bx^3+cx^6)^p \, dx = \frac{(cx^6+bx^3+a)(cx^6+bx^3+a)^p}{3(p+1)}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x^2 (b+2cx^3) (a+bx^3+cx^6)^p \, dx = \frac{(cx^6+bx^3+a)^{p+1}}{3(p+1)}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
1/3*(c*x^6 + b*x^3 + a)^(p + 1)/(p + 1)
```

3.131.9 Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int x^2 (b+2cx^3) (a+bx^3+cx^6)^p \, dx = (c\,x^6+b\,x^3+a)^p \left(\frac{a}{3\,p+3}+\frac{b\,x^3}{3\,p+3}+\frac{c\,x^6}{3\,p+3}\right)$$

input
```
int(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x)
```

output
```
(a + b*x^3 + c*x^6)^p*(a/(3*p + 3) + (b*x^3)/(3*p + 3) + (c*x^6)/(3*p + 3)
)
```

3.132 $\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$

3.132.1 Optimal result

Integrand size = 30, antiderivative size = 27

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{\left(a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

output
```
(a+b*x^n+c*x^(2*n))^(p+1)/n/(p+1)
```

3.132.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{\left(a+x^n(b+cx^n)\right)^{1+p}}{n(1+p)}$$

input
```
Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

3.132.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1798, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{\int (2cx^n+b)\left(bx^n+cx^{2n}+a\right)^p\,dx^n}{n}$$

$$\Big\downarrow \text{1104}$$

$$\frac{\left(a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

input
```
Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

3.132.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.132.4 Maple [A] (verified)

Time = 36.71 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

method	result	size
risch	$\frac{(a+b\,x^n+c\,x^{2n})\left(a+b\,x^n+c\,x^{2n}\right)^p}{n(1+p)}$	40

input
```
int(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x,method=_RETURNVERBOSE)
```

output
```
(a+b*x^n+c*(x^n)^2)/n/(1+p)*(a+b*x^n+c*(x^n)^2)^p
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2\,n}+bx^n+a)(cx^{2\,n}+bx^n+a)^p}{np+n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas"
)
```

output
```
(c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*p + n)
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2n}+bx^n+a)(cx^{2n}+bx^n+a)^p}{n(p+1)}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima"
)
```

output
```
(c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*(p + 1))
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2n}+bx^n+a)^{p+1}}{n(p+1)}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
(c*x^(2*n) + b*x^n + a)^(p + 1)/(n*(p + 1))
```

3.132.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\left(a+b\,x^n+c\,x^{2\,n}\right)^p\left(\frac{a}{n\,(p+1)}+\frac{b\,x^n}{n\,(p+1)}+\frac{c\,x^{2\,n}}{n\,(p+1)}\right)$$

input
```
int(x^(n - 1)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
(a + b*x^n + c*x^(2*n))^p*(a/(n*(p + 1)) + (b*x^n)/(n*(p + 1)) + (c*x^(2*n
))/(n*(p + 1)))
```

3.133 $\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx$

3.133.1 Optimal result

Integrand size = 21, antiderivative size = 22

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\frac{\left(-a+bx+cx^2\right)^{1+p}}{1+p}$$

output
```
(c*x^2+b*x-a)^(p+1)/(p+1)
```

3.133.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\frac{(-a+x(b+cx))^{1+p}}{1+p}$$

input
```
Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
```

output
```
(-a + x*(b + c*x))^(1 + p)/(1 + p)
```

3.133.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx) \left(-a+bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \; 1104$$

$$\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

input
```
Int[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
```

output
```
(-a + b*x + c*x^2)^(1 + p)/(1 + p)
```

3.133.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.133.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{(c\,x^2+bx-a)^{1+p}}{1+p}$	23
derivativedivides	$\frac{(c\,x^2+bx-a)^{1+p}}{1+p}$	23
default	$\frac{(c\,x^2+bx-a)^{1+p}}{1+p}$	23
risch	$-\frac{(-c\,x^2-bx+a)\,(c\,x^2+bx-a)^p}{1+p}$	34
parallelrisch	$\frac{x^2(c\,x^2+bx-a)^p bc+x(c\,x^2+bx-a)^p b^2-ab(c\,x^2+bx-a)^p}{b(1+p)}$	68
norman	$\frac{bx\,\mathrm{e}^{p\ln\left(c\,x^2+bx-a\right)}}{1+p}+\frac{c\,x^2\mathrm{e}^{p\ln\left(c\,x^2+bx-a\right)}}{1+p}-\frac{a\,\mathrm{e}^{p\ln\left(c\,x^2+bx-a\right)}}{1+p}$	76

input
```
int((2*c*x+b)*(c*x^2+b*x-a)^p,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x-a)^(1+p)/(1+p)
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\frac{(cx^2+bx-a)(cx^2+bx-a)^p}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="fricas")
```

output
```
(c*x^2 + b*x - a)*(c*x^2 + b*x - a)^p/(p + 1)
```

3.133.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(15) = 30$.

Time = 29.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.73

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx = \begin{cases} -\frac{a(-a+bx+cx^2)^p}{p+1}+\frac{bx(-a+bx+cx^2)^p}{p+1}+\frac{cx^2(-a+bx+cx^2)^p}{p+1} & \text{for } p\neq -1 \\ \log\left(\frac{b}{2c}+x-\frac{\sqrt{4ac+b^2}}{2c}\right)+\log\left(\frac{b}{2c}+x+\frac{\sqrt{4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((2*c*x+b)*(c*x**2+b*x-a)**p,x)
```

output
```
Piecewise((-a*(-a + b*x + c*x**2)**p/(p + 1) + b*x*(-a + b*x + c*x**2)**p/
(p + 1) + c*x**2*(-a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c)
+ x - sqrt(4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(4*a*c + b**2)/(2*
c)), True))
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx = \frac{(cx^2+bx-a)^{p+1}}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="maxima")
```

output
```
(c*x^2 + b*x - a)^(p + 1)/(p + 1)
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx = \frac{(cx^2+bx-a)^{p+1}}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="giac")
```

output
```
(c*x^2 + b*x - a)^(p + 1)/(p + 1)
```

3.133.9 Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\left(\frac{b\,x}{p+1}-\frac{a}{p+1}+\frac{c\,x^2}{p+1}\right)\,\left(c\,x^2+b\,x-a\right)^p$$

input
```
int((b + 2*c*x)*(b*x - a + c*x^2)^p,x)
```

output
```
((b*x)/(p + 1) - a/(p + 1) + (c*x^2)/(p + 1))*(b*x - a + c*x^2)^p
```

3.134 $\int x(b+2cx^2)(-a+bx^2+cx^4)^p\,dx$

3.134.1 Optimal result

Integrand size = 26, antiderivative size = 27

$$\int x(b+2cx^2)(-a+bx^2+cx^4)^p\,dx=\frac{(-a+bx^2+cx^4)^{1+p}}{2(1+p)}$$

output
```
1/2*(c*x^4+b*x^2-a)^(p+1)/(p+1)
```

3.134.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x(b+2cx^2)(-a+bx^2+cx^4)^p\,dx=\frac{(-a+bx^2+cx^4)^{1+p}}{2(1+p)}$$

input
```
Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
```

output
```
(-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

3.134.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1576, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^p\,dx$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int (2cx^2+b)\left(cx^4+bx^2-a\right)^p dx^2$$

$$\Big\downarrow \text{ 1104}$$

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

input
```
Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
```

output
```
(-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

3.134.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.134.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(c x^4+b x^2-a)^{1+p}}{2+2p}$	26
risch	$-\frac{(-c x^4-b x^2+a)(c x^4+b x^2-a)^p}{2(1+p)}$	38
parallelrisch	$\frac{x^4(c x^4+b x^2-a)^p c^2+x^2(c x^4+b x^2-a)^p bc-(c x^4+b x^2-a)^p ac}{2c(1+p)}$	77
norman	$-\frac{a\,\mathrm{e}^{p\ln(c x^4+b x^2-a)}}{2(1+p)}+\frac{b x^2\mathrm{e}^{p\ln(c x^4+b x^2-a)}}{2+2p}+\frac{c x^4\mathrm{e}^{p\ln(c x^4+b x^2-a)}}{2+2p}$	86

input
```
int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^4+b*x^2-a)^(1+p)/(1+p)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2-a)(cx^4+bx^2-a)^p}{2\,(p+1)}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="fricas")
```

output
```
1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

3.134.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(19) = 38.

Time = 107.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 7.44

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^p\,dx$$
$$=\begin{cases}-\frac{a(-a+bx^2+cx^4)^p}{2p+2}+\frac{bx^2(-a+bx^2+cx^4)^p}{2p+2}+\frac{cx^4(-a+bx^2+cx^4)^p}{2p+2} & \text{for }\\ \frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2} & \text{oth}\end{cases}$$

input
```
integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**p,x)
```

output
```
Piecewise((-a*(-a + b*x**2 + c*x**4)**p/(2*p + 2) + b*x**2*(-a + b*x**2 +
c*x**4)**p/(2*p + 2) + c*x**4*(-a + b*x**2 + c*x**4)**p/(2*p + 2), Ne(p, -
1)), (log(x - sqrt(2)*sqrt(-b/c - sqrt(4*a*c + b**2)/c)/2)/2 + log(x + sqr
t(2)*sqrt(-b/c - sqrt(4*a*c + b**2)/c)/2)/2 + log(x - sqrt(2)*sqrt(-b/c +
sqrt(4*a*c + b**2)/c)/2)/2 + log(x + sqrt(2)*sqrt(-b/c + sqrt(4*a*c + b**2
)/c)/2)/2, True))
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2-a)(cx^4+bx^2-a)^p}{2\,(p+1)}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="maxima")
```

output
```
1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2-a)^{p+1}}{2\,(p+1)}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="giac")
```

output
```
1/2*(c*x^4 + b*x^2 - a)^(p + 1)/(p + 1)
```

3.134.9 Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^p\,dx=\left(c\,x^4+b\,x^2-a\right)^p\left(\frac{b\,x^2}{2\,p+2}-\frac{a}{2\,p+2}+\frac{c\,x^4}{2\,p+2}\right)$$

input
```
int(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^p,x)
```

output
```
(b*x^2 - a + c*x^4)^p*((b*x^2)/(2*p + 2) - a/(2*p + 2) + (c*x^4)/(2*p + 2)
)
```

3.135 $\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx$

3.135.1 Optimal result

Integrand size = 28, antiderivative size = 27

$$\int x^2\left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^p\,dx=\frac{\left(-a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

output
```
1/3*(c*x^6+b*x^3-a)^(p+1)/(p+1)
```

3.135.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^2\left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^p\,dx=\frac{\left(-a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

input
```
Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]
```

output
```
(-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

3.135.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1798, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(b+2 c x^3\right) \left(-a+b x^3+c x^6\right)^p \, dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{3} \int \left(2 c x^3+b\right) \left(c x^6+b x^3-a\right)^p dx^3$$

$$\Big\downarrow \text{1104}$$

$$\frac{\left(-a+b x^3+c x^6\right)^{p+1}}{3 (p+1)}$$

input
```
Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]
```

output
```
(-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

3.135.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.135.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(c x^6+b x^3-a)^{1+p}}{3+3p}$	26
risch	$-\frac{(-c x^6-b x^3+a)(c x^6+b x^3-a)^p}{3(1+p)}$	38
parallelrisch	$\frac{x^6(c x^6+b x^3-a)^p c^2+x^3(c x^6+b x^3-a)^p bc-(c x^6+b x^3-a)^p ac}{3c(1+p)}$	77
norman	$-\frac{a\, e^{p\ln(c x^6+b x^3-a)}}{3(1+p)}+\frac{b x^3 e^{p\ln(c x^6+b x^3-a)}}{3+3p}+\frac{c x^6 e^{p\ln(c x^6+b x^3-a)}}{3+3p}$	86

input
```
int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x^6+b*x^3-a)^(1+p)/(1+p)
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3-a)(cx^6+bx^3-a)^p}{3(p+1)}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="fricas")
```

output
```
1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

3.135.6 Sympy [F(-1)]

Timed out.

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**p,x)
```

output
```
Timed out
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3-a)(cx^6+bx^3-a)^p}{3\,(p+1)}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="maxima")
```

output
```
1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3-a)^{p+1}}{3\,(p+1)}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="giac")
```

output
```
1/3*(c*x^6 + b*x^3 - a)^(p + 1)/(p + 1)
```

3.135.9 Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx=\left(c\,x^6+b\,x^3-a\right)^p\left(\frac{b\,x^3}{3\,p+3}-\frac{a}{3\,p+3}+\frac{c\,x^6}{3\,p+3}\right)$$

input
```
int(x^2*(b + 2*c*x^3)*(b*x^3 - a + c*x^6)^p,x)
```

output
```
(b*x^3 - a + c*x^6)^p*((b*x^3)/(3*p + 3) - a/(3*p + 3) + (c*x^6)/(3*p + 3)
)
```

3.136 $\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx$

3.136.1 Optimal result

Integrand size = 32, antiderivative size = 29

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{\left(-a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

output
```
(-a+b*x^n+c*x^(2*n))^(p+1)/n/(p+1)
```

3.136.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{\left(-a+x^n(b+cx^n)\right)^{1+p}}{n(1+p)}$$

input
```
Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(-a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

3.136.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1798, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{\int (2cx^n+b)\left(bx^n+cx^{2n}-a\right)^p\,dx^n}{n}$$

$$\Big\downarrow \text{1104}$$

$$\frac{\left(-a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

input
```
Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(-a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

3.136.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.136.4 Maple [A] (verified)

Time = 35.81 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

method	result	size
risch	$-\frac{\left(a-b\,x^{n}-c\,x^{2 n}\right) \left(-a+b\,x^{n}+c\,x^{2 n}\right)^{p}}{n \left(1+p\right)}$	45

input
```
int(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x,method=_RETURNVERBOSE)
```

output
```
-(-c*(x^n)^2-b*x^n+a)/n/(1+p)*(c*(x^n)^2+b*x^n-a)^p
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2n}+bx^n-a)(cx^{2n}+bx^n-a)^p}{np+n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas
")
```

output
```
(c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*p + n)
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2n}+bx^n-a)(cx^{2n}+bx^n-a)^p}{n(p+1)}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima
")
```

output
```
(c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*(p + 1))
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2n}+bx^n-a)^{p+1}}{n(p+1)}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
(c*x^(2*n) + b*x^n - a)^(p + 1)/(n*(p + 1))
```

3.136.9 Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\left(\frac{b\,x^n}{n\,(p+1)}-\frac{a}{n\,(p+1)}+\frac{c\,x^{2\,n}}{n\,(p+1)}\right)\,\left(b\,x^n-a+c\,x^{2\,n}\right)^p$$

input
```
int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n - a + c*x^(2*n))^p,x)
```

output
```
((b*x^n)/(n*(p + 1)) - a/(n*(p + 1)) + (c*x^(2*n))/(n*(p + 1)))*(b*x^n - a
 + c*x^(2*n))^p
```

3.137 $\int (b+2cx) (bx+cx^2)^p \, dx$

3.137.1 Optimal result

Integrand size = 18, antiderivative size = 19

$$\int (b+2cx) (bx+cx^2)^p \, dx = \frac{(bx+cx^2)^{1+p}}{1+p}$$

output
```
(c*x^2+b*x)^(p+1)/(p+1)
```

3.137.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (b+2cx) (bx+cx^2)^p \, dx = \frac{(x(b+cx))^{1+p}}{1+p}$$

input
```
Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

output
```
(x*(b + c*x))^(1 + p)/(1 + p)
```

3.137.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx) \left(bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1104}$$

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

input
```
Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

output
```
(b*x + c*x^2)^(1 + p)/(1 + p)
```

3.137.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.137.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\left(c\,x^{2}+bx\right)^{1+p}}{1+p}$	20
default	$\frac{\left(c\,x^{2}+bx\right)^{1+p}}{1+p}$	20
risch	$\frac{x(cx+b)(x(cx+b))^{p}}{1+p}$	22
gosper	$\frac{x(cx+b)\left(c\,x^{2}+bx\right)^{p}}{1+p}$	24
parallelrisch	$\frac{x^{2}(x(cx+b))^{p}bc+x(x(cx+b))^{p}b^{2}}{b(1+p)}$	40
norman	$\frac{bx\,\mathrm{e}^{p\ln\left(c\,x^{2}+bx\right)}}{1+p}+\frac{c\,x^{2}\mathrm{e}^{p\ln\left(c\,x^{2}+bx\right)}}{1+p}$	46

input
```
int((2*c*x+b)*(c*x^2+b*x)^p,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x)^(1+p)/(1+p)
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(cx^2+bx)(cx^2+bx)^p}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
(c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

3.137.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx = \begin{cases} \frac{bx(bx+cx^2)^p}{p+1} + \frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c}+x\right) & \text{otherwise} \end{cases}$$

input
```
integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

output
```
Piecewise(((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1
), Ne(p, -1)), (log(x) + log(b/c + x), True))
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx = \frac{(cx^2+bx)^{p+1}}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
(c*x^2 + b*x)^(p + 1)/(p + 1)
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx = \frac{(cx^2+bx)^{p+1}}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
(c*x^2 + b*x)^(p + 1)/(p + 1)
```

3.137.9 Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{x\,{\left(c\,x^2+b\,x\right)}^p\,\left(b+c\,x\right)}{p+1}$$

input
```
int((b*x + c*x^2)^p*(b + 2*c*x),x)
```

output
```
(x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)
```

3.138 $\int x(b+2cx^2)\left(bx^2+cx^4\right)^p\,dx$

3.138.1 Optimal result

Integrand size = 23, antiderivative size = 24

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^p\,dx=\frac{\left(bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

output
```
1/2*(c*x^4+b*x^2)^(p+1)/(p+1)
```

3.138.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.04

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^p\,dx$$
$$=\frac{x^2(x^2(b+cx^2))^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^2}{b}\right)+2c(1+p)x^2\,\text{Hype}\right.}{2(1+p)(2+p)}$$

input
```
Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

output
```
(x^2*(x^2*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -(
(c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2
)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

3.138.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1940, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^p\,dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{2}\int \left(2cx^2+b\right)\left(cx^4+bx^2\right)^p\,dx^2$$

$$\Big\downarrow \text{1104}$$

$$\frac{\left(bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

input
```
Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

output
```
(b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

3.138.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.138.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

method	result	size
gosper	$\frac{x^{2}\left(c\,x^{2}+b\right)\left(c\,x^{4}+b\,x^{2}\right)^{p}}{2+2p}$	31
risch	$\frac{x^{2}\left(c\,x^{2}+b\right)\left(x^{2}\left(c\,x^{2}+b\right)\right)^{p}}{2+2p}$	31
parallelrisch	$\frac{x^{4}\left(x^{2}\left(c\,x^{2}+b\right)\right)^{p}bc+x^{2}\left(x^{2}\left(c\,x^{2}+b\right)\right)^{p}b^{2}}{2b\left(1+p\right)}$	51
norman	$\frac{b\,x^{2}\mathrm{e}^{p\ln\left(c\,x^{4}+b\,x^{2}\right)}}{2+2p}+\frac{c\,x^{4}\mathrm{e}^{p\ln\left(c\,x^{4}+b\,x^{2}\right)}}{2+2p}$	54

input
```
int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*(c*x^2+b)/(1+p)*(c*x^4+b*x^2)^p
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int x\left(b+2cx^{2}\right)\left(bx^{2}+cx^{4}\right)^{p}\,dx=\frac{(cx^{4}+bx^{2})(cx^{4}+bx^{2})^{p}}{2\,(p+1)}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="fricas")
```

output
```
1/2*(c*x^4 + b*x^2)*(c*x^4 + b*x^2)^p/(p + 1)
```

3.138.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(17) = 34.

Time = 8.59 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.12

$$\int x\left(b+2cx^{2}\right)\left(bx^{2}+cx^{4}\right)^{p}\,dx=\begin{cases}\frac{bx^{2}(bx^{2}+cx^{4})^{p}}{2p+2}+\frac{cx^{4}(bx^{2}+cx^{4})^{p}}{2p+2} & \text{for } p\neq -1\\ \log{(x)}+\frac{\log{\left(x-\sqrt{-\frac{b}{c}}\right)}}{2}+\frac{\log{\left(x+\sqrt{-\frac{b}{c}}\right)}}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**p,x)
```

output
```
Piecewise((b*x**2*(b*x**2 + c*x**4)**p/(2*p + 2) + c*x**4*(b*x**2 + c*x**4
)**p/(2*p + 2), Ne(p, -1)), (log(x) + log(x - sqrt(-b/c))/2 + log(x + sqrt
(-b/c))/2, True))
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2\,p\log(x))}}{2\,(p+1)}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="maxima")
```

output
```
1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2)^{p+1}}{2\,(p+1)}$$

input
```
integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="giac")
```

output
```
1/2*(c*x^4 + b*x^2)^(p + 1)/(p + 1)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^p\,dx=\frac{x^2\,\left(c\,x^2+b\right)\,{\left(c\,x^4+b\,x^2\right)}^p}{2\,\left(p+1\right)}$$

input
```
int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x)
```

output
```
(x^2*(b + c*x^2)*(b*x^2 + c*x^4)^p)/(2*(p + 1))
```

3.139 $\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx$

3.139.1 Optimal result

Integrand size = 25, antiderivative size = 24

$$\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^p\,dx=\frac{\left(bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

output
```
1/3*(c*x^6+b*x^3)^(p+1)/(p+1)
```

3.139.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.04

$$\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^p\,dx$$
$$=\frac{x^3(x^3(b+cx^3))^p\left(1+\frac{cx^3}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^3}{b}\right)+2c(1+p)x^3\,\text{Hype}\right.}{3(1+p)(2+p)}$$

input
```
Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

output
```
(x^3*(x^3*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -(
(c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^3
)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

3.139.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1940, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (b+2cx^3) \left(bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{1}{3} \int \left(2cx^3+b\right) \left(cx^6+bx^3\right)^p dx^3$$

$$\Big\downarrow \text{1104}$$

$$\frac{\left(bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

input
```
Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

output
```
(b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

3.139.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.139.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

method	result	size
gosper	$\frac{(c\,x^3+b)x^3(c\,x^6+b\,x^3)^p}{3+3p}$	31
risch	$\frac{x^3(c\,x^3+b)(x^3(c\,x^3+b))^p}{3+3p}$	31
parallelrisch	$\frac{x^6(x^3(c\,x^3+b))^p bc+x^3(x^3(c\,x^3+b))^p b^2}{3b(1+p)}$	51
norman	$\frac{b\,x^3\mathrm{e}^{p\ln\left(c\,x^6+b\,x^3\right)}}{3+3p}+\frac{c\,x^6\mathrm{e}^{p\ln\left(c\,x^6+b\,x^3\right)}}{3+3p}$	54

input
```
int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x^3+b)*x^3/(1+p)*(c*x^6+b*x^3)^p
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3)(cx^6+bx^3)^p}{3\,(p+1)}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="fricas")
```

output
```
1/3*(c*x^6 + b*x^3)*(c*x^6 + b*x^3)^p/(p + 1)
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**p,x)
```

output
```
Timed out
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3)e^{(p\log(cx^3+b)+3\,p\log(x))}}{3\,(p+1)}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="maxima")
```

output
```
1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3)^{p+1}}{3\,(p+1)}$$

input
```
integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="giac")
```

output
```
1/3*(c*x^6 + b*x^3)^(p + 1)/(p + 1)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx=\frac{x^3\,(c\,x^3+b)\,\left(c\,x^6+b\,x^3\right)^p}{3\,\left(p+1\right)}$$

input
```
int(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x)
```

output
```
(x^3*(b + c*x^3)*(b*x^3 + c*x^6)^p)/(3*(p + 1))
```

3.140 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$

3.140.1 Optimal result

Integrand size = 29, antiderivative size = 26

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx=\frac{\left(bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

output
```
(b*x^n+c*x^(2*n))^(p+1)/n/(p+1)
```

3.140.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.27

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$$
$$=\frac{x^{-np}(x^n(b+cx^n))^p\left(1+\frac{cx^n}{b}\right)^{-p}\left(b(2+p)x^{n(1+p)}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^n}{b}\right)+2c(1+p)\right.}{n(1+p)(2+p)}$$

input
```
Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

output
```
((x^n*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p,
 2 + p, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2
+ p, 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^(n*p)*(1 + (c*x^n)/b)^p)
```

3.140.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1940, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1940}$$

$$\frac{\int (2cx^n+b)\left(bx^n+cx^{2n}\right)^p\,dx^n}{n}$$

$$\Big\downarrow \text{1104}$$

$$\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

input
```
Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

output
```
(b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

3.140.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.140.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 29.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.08

method	result	size
risch	$\frac{x^{n}\left(b+c\,x^{n}\right)\left(x^{n}\right)^{p}\left(b+c\,x^{n}\right)^{p}{\mathrm e}^{-\frac{i \operatorname{csgn}\left(i x^{n} \left(b+c\,x^{n}\right)\right) \pi p \left(-\operatorname{csgn}\left(i x^{n} \left(b+c\,x^{n}\right)\right)+\operatorname{csgn}\left(i x^{n}\right)\right) \left(-\operatorname{csgn}\left(i x^{n} \left(b+c\,x^{n}\right)\right)+\operatorname{csgn}\left(i \left(b+c\,x^{n}\right)\right)\right)}{2}}}{n \left(1+p\right)}$	106

input
```
int(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x,method=_RETURNVERBOSE)
```

output
```
x^n*(b+c*x^n)/n/(1+p)*(x^n)^p*(b+c*x^n)^p*exp(-1/2*I*csgn(I*x^n*(b+c*x^n))
*Pi*p*(-csgn(I*x^n*(b+c*x^n))+csgn(I*x^n))*(-csgn(I*x^n*(b+c*x^n))+csgn(I*
(b+c*x^n))))
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2\,n}+bx^n)(cx^{2\,n}+bx^n)^p}{np+n}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
(c*x^(2*n) + b*x^n)*(c*x^(2*n) + b*x^n)^p/(n*p + n)
```

3.140.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(19) = 38.

Time = 11.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.85

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$$
$$=\begin{cases}\frac{(b+2c)\log(x)}{b+c} & \text{for } n=0\wedge p=-1\\ (b+c)^p\,(b+2c)\log(x) & \text{for } n=0\\ \frac{\log(x^n)}{n}+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{for } p=-1\\ \frac{bxx^{n-1}(bx^n+cx^{2n})^p}{np+n}+\frac{cxx^nx^{n-1}(bx^n+cx^{2n})^p}{np+n} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**p,x)
```

output
```
Piecewise(((b + 2*c)*log(x)/(b + c), Eq(n, 0) & Eq(p, -1)), ((b + c)**p*(b
 + 2*c)*log(x), Eq(n, 0)), (log(x**n)/n + log(b/c + x**n)/n, Eq(p, -1)), (
b*x*x**(n - 1)*(b*x**n + c*x**(2*n))**p/(n*p + n) + c*x*x**n*x**(n - 1)*(b
*x**n + c*x**(2*n))**p/(n*p + n), True))
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2n}+bx^n)e^{(p\log(cx^n+b)+p\log(x^n))}}{n(p+1)}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
(c*x^(2*n) + b*x^n)*e^(p*log(c*x^n + b) + p*log(x^n))/(n*(p + 1))
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2n}+bx^n)^{p+1}}{n(p+1)}$$

input
```
integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
(c*x^(2*n) + b*x^n)^(p + 1)/(n*(p + 1))
```

3.140.9 Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx=\frac{x^n\,(b+c\,x^n)\,\left(b\,x^n+c\,x^{2\,n}\right)^p}{n\,(p+1)}$$

input
```
int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x)
```

output
```
(x^n*(b + c*x^n)*(b*x^n + c*x^(2*n))^p)/(n*(p + 1))
```

3.141 $\int \frac{(fx)^m(d+ex^n)}{a+bx^n+cx^{2n}}\,dx$

3.141.1 Optimal result

Integrand size = 29, antiderivative size = 196

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)f(1+m)}$$

output
```
(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1
/2)))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))+(
f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/
2)))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b+(-4*a*c+b^2)^(1/2))
```

3.141.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \frac{x(fx)^m\left(\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+\left(-bd+\sqrt{b^2-4ac}\right.\right.}{2a\sqrt{b^2-4ac}(1+m)}$$

input
```
Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(x*(f*x)^m*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, (1 +
m)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (-(b*d) + Sqrt[
b^2 - 4*a*c]*d + 2*a*e)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))
```

3.141.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}} \, dx$$

$$\Big\downarrow \text{1884}$$

$$\int \left(\frac{(fx)^m \left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{-\sqrt{b^2-4ac}+b+2cx^n}+\frac{(fx)^m \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}+b+2cx^n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(fx)^{m+1} \left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right) \text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{f(m+1) \left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1} \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right) \text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1) \left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1,
(1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[
b^2 - 4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1
 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

3.141.3.1 Defintions of rubi rules used

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.141.4 Maple [F]

$$\int \frac{(fx)^m (d+e\,x^n)}{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

3.141.5 Fricas [F]

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)(fx)^m}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.141.6 Sympy [F]

$$\int \frac{(fx)^m (d + ex^n)}{a + bx^n + cx^{2n}}\,dx = \int \frac{(fx)^m (d + ex^n)}{a + bx^n + cx^{2n}}\,dx$$

input
```
integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral((f*x)**m*(d + e*x**n)/(a + b*x**n + c*x**(2*n)), x)
```

3.141.7 Maxima [F]

$$\int \frac{(fx)^m (d + ex^n)}{a + bx^n + cx^{2n}}\,dx = \int \frac{(ex^n + d)(fx)^m}{cx^{2n} + bx^n + a}\,dx$$

input
```
integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.141.8 Giac [F]

$$\int \frac{(fx)^m (d + ex^n)}{a + bx^n + cx^{2n}}\,dx = \int \frac{(ex^n + d)(fx)^m}{cx^{2n} + bx^n + a}\,dx$$

input
```
integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx=\int \frac{(f\,x)^m\,(d+e\,x^n)}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)), x)
```

3.142 $\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^2}\,dx$

3.142.1 Optimal result

Integrand size = 29, antiderivative size = 374

$$\int \frac{(fx)^m\,(d+ex^n)}{(a+bx^n+cx^{2n})^2}\,dx = \frac{(fx)^{1+m}\,(b^2d-2acd-abe+c(bd-2ae)x^n)}{a\,(b^2-4ac)\,fn\,(a+bx^n+cx^{2n})}$$
$$-\frac{c\Big((bd-2ae)(1+m-n)-\frac{4acd(1+m-2n)-b^2d(1+m-n)+2aben}{\sqrt{b^2-4ac}}\Big)\,(fx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+}{n}\right.}{a\,(b^2-4ac)\,\big(b-\sqrt{b^2-4ac}\big)\,f(1+m)n}$$
$$-\frac{c\Big((bd-2ae)(1+m-n)+\frac{4acd(1+m-2n)-b^2d(1+m-n)+2aben}{\sqrt{b^2-4ac}}\Big)\,(fx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+}{n}\right.}{a\,(b^2-4ac)\,\big(b+\sqrt{b^2-4ac}\big)\,f(1+m)n}$$

output
```
(f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/f/n/(a
+b*x^n+c*x^(2*n))-c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)))*((-2*a*e+b*d)*(1+m-n)+(-4*a*c*d*(1+m-2*n)+b^2*d*
(1+m-n)-2*a*b*e*n)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/f/(1+m)/n/(b-(-4*a*c
+b^2)^(1/2))-c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+
(-4*a*c+b^2)^(1/2)))*((-2*a*e+b*d)*(1+m-n)+(4*a*c*d*(1+m-2*n)-b^2*d*(1+m-n
)+2*a*b*e*n)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/f/(1+m)/n/(b+(-4*a*c+b^2)^
(1/2))
```

3.142.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5363 vs. $2(374) = 748$.

Time = 7.23 (sec) , antiderivative size = 5363, normalized size of antiderivative = 14.34

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
Result too large to show
```

3.142.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1882, 25, 1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2}\,dx$$

$$\Big\downarrow \; 1882$$

$$\frac{(fx)^{m+1} \left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{afn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-\frac{\int -\frac{(fx)^m\left(-c(bd-2ae)(m-n+1)x^n+abe(m+1)+2acd(m-2n+1)-b^2d(m-n+1)\right)}{bx^n+cx^{2n}+a}dx}{an\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(fx)^m\left(-c(bd-2ae)(m-n+1)x^n+abe(m+1)+2acd(m-2n+1)-b^2d(m-n+1)\right)}{bx^n+cx^{2n}+a}dx}{an\left(b^2-4ac\right)}+\frac{(fx)^{m+1} \left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{afn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 1884$$

$$\frac{\int \left(\frac{\left(-c(bd-2ae)(m-n+1)-\frac{c\left(db^2+dmb^2-dnb^2-2aenb-4acd-4acdm+8acdn\right)}{\sqrt{b^2-4ac}}\right)(fx)^m}{2cx^n+b-\sqrt{b^2-4ac}} + \frac{\left(\frac{c\left(db^2+dmb^2-dnb^2-2aenb-4acd-4acdm+8acdn\right)}{\sqrt{b^2-4ac}}-c(bd\cdots}{2cx^n+b+\sqrt{b^2-4ac}}\right.}{an\left(b^2-4ac\right)}$$

$$\frac{(fx)^{m+1}\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{afn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{c(fx)^{m+1}\left((m-n+1)(bd-2ae)-\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)} - \frac{c(fx)^{m+1}\left(\cdots\right.}{\cdots}}{an\left(b^2-4ac\right)}$$

$$\frac{(fx)^{m+1}\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{afn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

input
```
Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(a*(b^2 -
4*a*c)*f*n*(a + b*x^n + c*x^(2*n))) + (-((c*((b*d - 2*a*e)*(1 + m - n) - (
4*a*c*d*(1 + m - 2*n) - b^2*d*(1 + m - n) + 2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*
(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*f*(1 + m))) - (c*((b*d -
2*a*e)*(1 + m - n) + (4*a*c*d*(1 + m - 2*n) - b^2*d*(1 + m - n) + 2*a*b*e*
n)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*f*(
1 + m)))/(a*(b^2 - 4*a*c)*n)
```

3.142.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1882
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m
+ 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && ILtQ[p + 1, 0]
```

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.142.4 Maple [F]

$$\int \frac{(fx)^m (d+e\,x^n)}{(a+b\,x^n+c\,x^{2n})^2}dx$$

input
```
int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

3.142.5 Fricas [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2\,n}+bx^n+a)^2}\,dx$$

input
```
integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral((e*x^n + d)*(f*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2
+ 2*(b*c*x^n + a*c)*x^(2*n)), x)
```

3.142.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2} \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

3.142.7 Maxima [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2} \, dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2n}+bx^n+a)^2} \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
((b^2*d*f^m - (2*c*d*f^m + b*e*f^m)*a)*x*x^m + (b*c*d*f^m - 2*a*c*e*f^m)*x
*e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*
n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) - integrate(((b^2*d*f^m*(m - n +
 1) - (2*c*d*f^m*(m - 2*n + 1) + b*e*f^m*(m + 1))*a)*x^m + (b*c*d*f^m*(m -
 n + 1) - 2*a*c*e*f^m*(m - n + 1))*e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4
*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n
), x)
```

3.142.8 Giac [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2} \, dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2n}+bx^n+a)^2} \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d + ex^n)}{(a + bx^n + cx^{2n})^2} \, dx = \int \frac{(f\,x)^m\,(d + e\,x^n)}{(a + b\,x^n + c\,x^{2\,n})^2} \, dx$$

input
```
int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2, x)
```

3.143 $\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx$

3.143.1 Optimal result

Integrand size = 29, antiderivative size = 816

$$\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx = \frac{(fx)^{1+m}(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a(b^2-4ac)fn(a+bx^n+cx^{2n})^2} + \frac{(fx)^{1+m}((b^2-2ac)(abe(1+m)+2acd(1+m-4n)-b^2d(1+m-2n))+abc(bd-2ae)(1+m-3n}{2a^2(b^2-4ac)^2fn^2(a+b}$$

$$-\frac{c\Big((ab^2e(1+m)+2abcd(2+2m-7n)-4a^2ce(1+m-3n)-b^3d(1+m-2n))(1+m-n)+\frac{ab^3e(1-}{}}{}$$

$$-\frac{c\Big((ab^2e(1+m)+2abcd(2+2m-7n)-4a^2ce(1+m-3n)-b^3d(1+m-2n))(1+m-n)-\frac{ab^3e(1-}{}}{}$$

output
```
1/2*(f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/f/
n/(a+b*x^n+c*x^(2*n))^2+1/2*(f*x)^(1+m)*((-2*a*c+b^2)*(a*b*e*(1+m)+2*a*c*d
*(1+m-4*n)-b^2*d*(1+m-2*n))+a*b*c*(-2*a*e+b*d)*(1+m-3*n)+c*(a*b^2*e*(1+m)+
2*a*b*c*d*(2+2*m-7*n)-4*a^2*c*e*(1+m-3*n)-b^3*d*(1+m-2*n))*x^n)/a^2/(-4*a*
c+b^2)^2/f/n^2/(a+b*x^n+c*x^(2*n))-1/2*c*(f*x)^(1+m)*hypergeom([1, (1+m)/n
],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((a*b^2*e*(1+m)+2*a*b*c*d*(
2+2*m-7*n)-4*a^2*c*e*(1+m-3*n)-b^3*d*(1+m-2*n))*(1+m-n)+(a*b^3*e*(1+m)*(1+
m-n)-4*a^2*b*c*e*(1+m^2+m*(2-n)-n-3*n^2)-b^4*d*(1+m^2+m*(2-3*n)-3*n+2*n^2)
+6*a*b^2*c*d*(1+m^2+m*(2-4*n)-4*n+3*n^2)-8*a^2*c^2*d*(1+m^2+m*(2-6*n)-6*n+
8*n^2))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/f/(1+m)/n^2/(b-(-4*a*c+b^2)
^(1/2))-1/2*c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))*((a*b^2*e*(1+m)+2*a*b*c*d*(2+2*m-7*n)-4*a^2*c*e*(1+m-3
*n)-b^3*d*(1+m-2*n))*(1+m-n)+(-a*b^3*e*(1+m)*(1+m-n)+4*a^2*b*c*e*(1+m^2+m*
(2-n)-n-3*n^2)+b^4*d*(1+m^2+m*(2-3*n)-3*n+2*n^2)-6*a*b^2*c*d*(1+m^2+m*(2-4
*n)-4*n+3*n^2)+8*a^2*c^2*d*(1+m^2+m*(2-6*n)-6*n+8*n^2))/(-4*a*c+b^2)^(1/2)
)/a^2/(-4*a*c+b^2)^2/f/(1+m)/n^2/(b+(-4*a*c+b^2)^(1/2))
```

3.143.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 20515 vs. $2(816) = 1632$.

Time = 8.44 (sec) , antiderivative size = 20515, normalized size of antiderivative = 25.14

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
Result too large to show
```

3.143.3 Rubi [A] (verified)

Time = 3.48 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1882, 25, 1882, 25, 1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3} dx$$

$$\Big\downarrow \text{1882}$$

$$\frac{(fx)^{m+1} \left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2afn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}-\frac{\int -\frac{(fx)^m(-c(bd-2ae)(m-3n+1)x^n+abe(m+1)+2acd(m-4n+1)-b^2d(m-2n+1))}{(bx^n+cx^{2n}+a)^2}dx}{2an\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(fx)^m(-c(bd-2ae)(m-3n+1)x^n+abe(m+1)+2acd(m-4n+1)-b^2d(m-2n+1))}{(bx^n+cx^{2n}+a)^2}dx}{2an\left(b^2-4ac\right)}+\frac{(fx)^{m+1} \left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2afn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$$\Big\downarrow \text{1882}$$

$$\frac{(fx)^{m+1}\left(cx^n\left(-4a^2ce(m-3n+1)+ab^2e(m+1)+2abcd(2m-7n+2)+b^3(-d)(m-2n+1)\right)+\left(b^2-2ac\right)\left(abe(m+1)+2acd(m-4n+1)+b^2(-d)(m-2n\right.\right.}{afn(b^2-4ac)(a+bx^n+cx^{2n})}$$

$$\frac{(fx)^{m+1} \left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2afn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{(fx)^m\left(-c\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)(m-n+1)x^n+\left(-d(m-2n+1)b^2+ae(m+1)b+2acd(m-4n+1)\right)\left(2ac(m-2n+1)-\right.\right.}{bx^n+cx^{2n}+a}$$
$$an(b^2-4ac)$$

$$\frac{(fx)^{m+1} \left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{2afn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$$\Big\downarrow \text{1884}$$

$$\frac{\left(c(bd-2ae)x^n+b^2d-2acd-abe\right)(fx)^{m+1}}{2a\left(b^2-4ac\right)fn\left(bx^n+cx^{2n}+a\right)^2}+$$

$$\frac{(c(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1))x^n+\left(b^2-2ac\right)\left(-d(m-2n+1)b^2+ae(m+1)b+2acd(m-4n+1)\right)+abc(bd-}{a(b^2-4ac)fn(bx^n+cx^{2n}+a)}$$

$$\Big\downarrow \text{2009}$$

3.143. $\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^3} dx$

$$\frac{\left(c(bd-2ae)x^n+b^2d-2acd-abe\right)(fx)^{m+1}}{2a\left(b^2-4ac\right)fn\left(bx^n+cx^{2n}+a\right)^2}+$$

$$\frac{\frac{\left(c\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)x^n+\left(b^2-2ac\right)\left(-d(m-2n+1)b^2+ae(m+1)b+2acd(m-4n+1)\right)+abc(bd-}{a(b^2-4ac)fn(bx^n+cx^{2n}+a)}}{}$$

input
```
Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(2*a*(b^2
- 4*a*c)*f*n*(a + b*x^n + c*x^(2*n))^2) + (((f*x)^(1 + m)*((b^2 - 2*a*c)*(
a*b*e*(1 + m) + 2*a*c*d*(1 + m - 4*n) - b^2*d*(1 + m - 2*n)) + a*b*c*(b*d
- 2*a*e)*(1 + m - 3*n) + c*(a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) -
4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*x^n))/(a*(b^2 - 4*a*c)*f*n*
(a + b*x^n + c*x^(2*n))) + (-((c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m -
7*n) - 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) + (a*b^3
*e*(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b
^4*d*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 -
 4*n) - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/
Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m +
n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*f*(1 +
 m))) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) - 4*a^2*c*e*(1 +
m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) - (a*b^3*e*(1 + m)*(1 + m - n)
 - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d*(1 + m^2 + m*(2 -
 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n) - 4*n + 3*n^2) -
 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b^2 - 4*a*c])*(f*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m)))/(a*(b^2 - 4*a*c)
*n))/(2*a*(b^2 - 4*a*c)*n)
```

3.143. $\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx$

3.143.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1882
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m
+ 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && ILtQ[p + 1, 0]
```

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.143.4 Maple [F]

$$\int \frac{(fx)^m (d + e\,x^n)}{(a + b\,x^n + c\,x^{2n})^3} dx$$

input
```
int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

output
```
int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

3.143.5 Fricas [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3} \, dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2n}+bx^n+a)^3} \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral((e*x^n + d)*(f*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n)
+ 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2
*a*b*c*x^n + a^2*c)*x^(2*n)), x)
```

3.143.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3} \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Timed out
```

3.143.7 Maxima [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3} \, dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2n}+bx^n+a)^3} \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
-1/2*((a*b^4*d*f^m*(m - 3*n + 1) + 2*(b*c*e*f^m*(2*m - 5*n + 2) + 2*c^2*d*
f^m*(m - 6*n + 1))*a^3 - (b^2*c*d*f^m*(5*m - 21*n + 5) + b^3*e*f^m*(m - n
+ 1))*a^2)*x*x^m + (b^3*c^2*d*f^m*(m - 2*n + 1) + 4*a^2*c^3*e*f^m*(m - 3*n
 + 1) - (2*b*c^3*d*f^m*(2*m - 7*n + 2) + b^2*c^2*e*f^m*(m + 1))*a)*x*e^(m*
log(x) + 3*n*log(x)) + (2*b^4*c*d*f^m*(m - 2*n + 1) + 2*(b*c^2*e*f^m*(4*m
- 9*n + 4) + 2*c^3*d*f^m*(m - 4*n + 1))*a^2 - (b^2*c^2*d*f^m*(9*m - 29*n +
 9) + 2*b^3*c*e*f^m*(m + 1))*a)*x*e^(m*log(x) + 2*n*log(x)) + (b^5*d*f^m*(
m - 2*n + 1) + 4*a^3*c^2*e*f^m*(m - 5*n + 1) + (b^2*c*e*f^m*(3*m - 4*n + 3
) + 2*b*c^2*d*f^m*n)*a^2 - (4*b^3*c*d*f^m*(m - 3*n + 1) + b^4*e*f^m*(m + 1
))*a)*x*e^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c
^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) +
2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^
6*n^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4
*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) + integrate(1/2*(((m^2 - m*(3*n - 2) +
 2*n^2 - 3*n + 1)*b^4*d*f^m + 2*(2*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)
*c^2*d*f^m + (2*m^2 - m*(5*n - 4) - 5*n + 2)*b*c*e*f^m)*a^2 - ((5*m^2 - m*
(21*n - 10) + 16*n^2 - 21*n + 5)*b^2*c*d*f^m + (m^2 - m*(n - 2) - n + 1)*b
^3*e*f^m)*a)*x^m + ((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d*f^m + 4*
(m^2 - 2*m*(2*n - 1) + 3*n^2 - 4*n + 1)*a^2*c^2*e*f^m - (2*(2*m^2 - m*(9*n
 - 4) + 7*n^2 - 9*n + 2)*b*c^2*d*f^m + (m^2 - m*(n - 2) - n + 1)*b^2*c*...
```

3.143.8 Giac [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2\,n}+bx^n+a)^3}\,dx$$

input
```
integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d + ex^n)}{(a + bx^n + cx^{2n})^3}\, dx = \int \frac{(f\,x)^m\,(d + e\,x^n)}{(a + b\,x^n + c\,x^{2\,n})^3}\, dx$$

input
```
int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x)
```

output
```
int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3, x)
```

3.144 $\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx$

3.144.1 Optimal result

Integrand size = 59, antiderivative size = 47

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx=-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

output
```
-3*ln(c^(2/3)-c^(1/3)*d^(1/3)*x^(1/3)+d^(2/3)*x^(2/3))/c^(1/3)/d^(2/3)
```

3.144.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx=-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

input
```
Integrate[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/
3)*x + c^(1/3)*d*x^(4/3)),x]
```

output
```
(-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(
2/3))
```

3.144.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1979, 1798, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}+c\sqrt[3]{d}x^{2/3}}\,dx$$

$$\Big\downarrow \text{1979}$$

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{x^{2/3}\left(-c^{2/3}d^{2/3}\sqrt[3]{x}+\sqrt[3]{c}dx^{2/3}+c\sqrt[3]{d}\right)}dx$$

$$\Big\downarrow \text{1798}$$

$$3\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{\sqrt[3]{d}c-d^{2/3}\sqrt[3]{x}c^{2/3}+dx^{2/3}\sqrt[3]{c}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

input
```
Int[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x +
 c^(1/3)*d*x^(4/3)),x]
```

output
```
(-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(
2/3))
```

3.144.3.1 Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 1979
```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) +
(B_.)*(x_)^(r_.)), x_Symbol] :> Int[x^(p*q)*(A + B*x^(n - q))*(a + b*x^(n -
 q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[r, n
 - q] && EqQ[j, 2*n - q] && IntegerQ[p] && PosQ[n - q]
```

3.144.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{3\ln\left(c^{\frac{2}{3}} d^{\frac{2}{3}} x^{\frac{1}{3}}-c^{\frac{1}{3}} d\, x^{\frac{2}{3}}-c\, d^{\frac{1}{3}}\right)}{d^{\frac{2}{3}} c^{\frac{1}{3}}}$	36
default	$-\frac{3\ln\left(c^{\frac{2}{3}} d^{\frac{2}{3}} x^{\frac{1}{3}}-c^{\frac{1}{3}} d\, x^{\frac{2}{3}}-c\, d^{\frac{1}{3}}\right)}{d^{\frac{2}{3}} c^{\frac{1}{3}}}$	36

input
```
int((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+c^(1/
3)*d*x^(4/3)),x,method=_RETURNVERBOSE)
```

output
```
-3/d^(2/3)/c^(1/3)*ln(c^(2/3)*d^(2/3)*x^(1/3)-c^(1/3)*d*x^(2/3)-c*d^(1/3))
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx = -\frac{3\log\left(dx^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+c^{\frac{2}{3}}d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

input
```
integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x
+c^(1/3)*d*x^(4/3)),x, algorithm="fricas")
```

output
```
-3*log(d*x^(2/3) - c^(1/3)*d^(2/3)*x^(1/3) + c^(2/3)*d^(1/3))/(c^(1/3)*d^(
2/3))
```

3.144.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(44) = 88.

Time = 2.61 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx = -\frac{3\log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}}+\sqrt[3]{x}-\frac{\sqrt{3}\sqrt{-c^{\frac{4}{3}}d^{\frac{4}{3}}}}{2\sqrt[3]{c}d}\right)}{\sqrt[3]{c}d^{\frac{2}{3}}}-\frac{3\log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}}+\sqrt[3]{x}+\frac{\sqrt{3}\sqrt{-c^{\frac{4}{3}}d^{\frac{4}{3}}}}{2\sqrt[3]{c}d}\right)}{\sqrt[3]{c}d^{\frac{2}{3}}}$$

input
```
integrate((c**(1/3)-2*d**(1/3)*x**(1/3))/(c*d**(1/3)*x**(2/3)-c**(2/3)*d**
(2/3)*x+c**(1/3)*d*x**(4/3)),x)
```

output
```
-3*log(-c**(1/3)/(2*d**(1/3)) + x**(1/3) - sqrt(3)*sqrt(-c**(4/3)*d**(4/3)
)/(2*c**(1/3)*d))/(c**(1/3)*d**(2/3)) - 3*log(-c**(1/3)/(2*d**(1/3)) + x**
(1/3) + sqrt(3)*sqrt(-c**(4/3)*d**(4/3))/(2*c**(1/3)*d))/(c**(1/3)*d**(2/3
))
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx = -\frac{3\log\left(c^{\frac{1}{3}}dx^{\frac{2}{3}}-c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+cd^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

input
```
integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x
+c^(1/3)*d*x^(4/3)),x, algorithm="maxima")
```

output
```
-3*log(c^(1/3)*d*x^(2/3) - c^(2/3)*d^(2/3)*x^(1/3) + c*d^(1/3))/(c^(1/3)*d
^(2/3))
```

3.144.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{c} - 2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3} - c^{2/3}d^{2/3}x + \sqrt[3]{c}dx^{4/3}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x
+c^(1/3)*d*x^(4/3)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{%%{[%%%{1,[1]%%%},0]:[1,0,0,%%%{-1,[1]%%%}]%%},[1]
%%%},0]:[
```

3.144.9 Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{c} - 2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3} - c^{2/3}d^{2/3}x + \sqrt[3]{c}dx^{4/3}} \, dx = -\frac{3 \ln\left(x^{2/3} + \frac{c^{2/3}}{d^{2/3}} - \frac{c^{1/3}\,x^{1/3}}{d^{1/3}}\right)}{c^{1/3}\,d^{2/3}}$$

input
```
int((c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x +
 c^(1/3)*d*x^(4/3)),x)
```

output
```
-(3*log(x^(2/3) + c^(2/3)/d^(2/3) - (c^(1/3)*x^(1/3))/d^(1/3)))/(c^(1/3)*d
^(2/3))
```

3.145 $\int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

3.145.1 Optimal result

Integrand size = 31, antiderivative size = 245

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \frac{2c(fx)^{1+m}(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1+m}{n},1,-q,\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)} - \frac{2c(fx)^{1+m}(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1+m}{n},1,-q,\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)f(1+m)}$$

output
```
2*c*(f*x)^(1+m)*(d+e*x^n)^q*AppellF1((1+m)/n,1,-q,(1+m+n)/n,-2*c*x^n/(b-(-
4*a*c+b^2)^(1/2)),-e*x^n/d)/f/(1+m)/((1+e*x^n/d)^q)/(b-(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2)-2*c*(f*x)^(1+m)*(d+e*x^n)^q*AppellF1((1+m)/n,1,-q,(1+m
+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/f/(1+m)/((1+e*x^n/d)^q)/(-
4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

3.145.2 Mathematica [F]

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}} \, dx = \int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}} \, dx$$

input
```
Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

3.145.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1880, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}} \, dx$$

$$\Big\downarrow \ 1880$$

$$\frac{2c\int \frac{(fx)^m(ex^n+d)^q}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}} - \frac{2c\int \frac{(fx)^m(ex^n+d)^q}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \ 1013$$

$$\frac{2c(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q} \int \frac{(fx)^m\left(\frac{ex^n}{d}+1\right)^q}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}} - \frac{2c(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q} \int \frac{(fx)^m\left(\frac{ex^n}{d}+1\right)^q}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \ 1012$$

$$\frac{2c(fx)^{m+1} (d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q} \operatorname{AppellF1}\left(\frac{m+1}{n},1,-q,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} - \frac{2c(fx)^{m+1} (d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q} \operatorname{AppellF1}\left(\frac{m+1}{n},1,-q,\frac{m+n+1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*c*(f*x)^(1 + m)*(d + e*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n,
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b
- Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^n)/d)^q) - (2*c*(f*x)^(1 + m)*(d
+ e*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f
*(1 + m)*(1 + (e*x^n)/d)^q)
```

3.145.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1880
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[
2*(c/r)   Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Simp[2*(c
/r)   Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b
, c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !Rati
onalQ[n]
```

3.145.4 Maple [F]

$$\int \frac{(fx)^m (d + e\,x^n)^q}{a + b\,x^n + c\,x^{2n}} dx$$

input
```
int((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

3.145.5 Fricas [F]

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q(fx)^m}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.145.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((f*x)**m*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.145.7 Maxima [F]

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q(fx)^m}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.145.8 Giac [F]

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q (fx)^m}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(f\,x)^m\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

3.146 $\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

3.146.1 Optimal result

Integrand size = 29, antiderivative size = 210

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = -\frac{2cx^3(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{n},1,-q,\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)} - \frac{2cx^3(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{n},1,-q,\frac{3+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}$$

output
```
-2/3*c*x^3*(d+e*x^n)^q*AppellF1(3/n,1,-q,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^
(1/2)),-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2/3*c*x^
3*(d+e*x^n)^q*AppellF1(3/n,1,-q,(3+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e
*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.146.2 Mathematica [F]

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

input
```
Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

3.146.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1880, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \; 1880$$

$$\frac{2c\int \frac{x^2(ex^n+d)^q}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x^2(ex^n+d)^q}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 1013$$

$$\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\int \frac{x^2\left(\frac{ex^n}{d}+1\right)^q}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\int \frac{x^2\left(\frac{ex^n}{d}+1\right)^q}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 1012$$

$$\frac{2cx^3(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{n},1,-q,\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^3(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{n},1,-q,\frac{n+3}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c
])*(1 + (e*x^n)/d)^q) - (2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n
)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

3.146.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1880
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[
2*(c/r)   Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Simp[2*(c
/r)   Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b
, c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !Rati
onalQ[n]
```

3.146.4 Maple [F]

$$\int \frac{x^2(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

3.146.5 Fricas [F]

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q x^2}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

3.146.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**2*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.146.7 Maxima [F]

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q x^2}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

3.146.8 Giac [F]

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q x^2}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^2\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int((x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

3.147 $\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

3.147.1 Optimal result

Integrand size = 27, antiderivative size = 206

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = -\frac{cx^2(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\text{AppellF1}\left(\frac{2}{n},1,-q,\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{cx^2(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\text{AppellF1}\left(\frac{2}{n},1,-q,\frac{2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

output
```
-c*x^2*(d+e*x^n)^q*AppellF1(2/n,1,-q,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)),-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2*(d+e*x
^n)^q*AppellF1(2/n,1,-q,(2+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/
((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.147.2 Mathematica [F]

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

input
```
Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

3.147.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1880, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1880}$$

$$\frac{2c\int \frac{x(ex^n+d)^q}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x(ex^n+d)^q}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\int \frac{x\left(\frac{ex^n}{d}+1\right)^q}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\int \frac{x\left(\frac{ex^n}{d}+1\right)^q}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{cx^2(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{2}{n},1,-q,\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx^2(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{2}{n},1,-q,\frac{n+2}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(
1 + (e*x^n)/d)^q) - (c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b +
Sqrt[b^2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

3.147.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1880
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[
2*(c/r)   Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Simp[2*(c
/r)   Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b
, c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !Rati
onalQ[n]
```

3.147.4 Maple [F]

$$\int \frac{x(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

3.147.5 Fricas [F]

$$\int \frac{x(d + ex^n)^q}{a + bx^n + cx^{2n}}\,dx = \int \frac{(ex^n + d)^q x}{cx^{2n} + bx^n + a}\,dx$$

input
```
integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

3.147.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x(d + ex^n)^q}{a + bx^n + cx^{2n}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.147.7 Maxima [F]

$$\int \frac{x(d + ex^n)^q}{a + bx^n + cx^{2n}}\,dx = \int \frac{(ex^n + d)^q x}{cx^{2n} + bx^n + a}\,dx$$

input
```
integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

3.147.8 Giac [F]

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q x}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{x\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int((x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

3.148 $\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$

3.148.1 Optimal result

Integrand size = 26, antiderivative size = 194

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}} dx = -\frac{2cx(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

output
```
-2*c*x*(d+e*x^n)^q*AppellF1(1/n,1,-q,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2))
,-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*(d+e*x^n
)^q*AppellF1(1/n,1,-q,1+1/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/((1+
e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.148.2 Mathematica [F]

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}} dx = \int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$$

input
```
Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
```

output
```
Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x]
```

3.148.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1758, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{1758}$$

$$\frac{2c\int \frac{(ex^n+d)^q}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(ex^n+d)^q}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{937}$$

$$\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\int \frac{\left(\frac{ex^n}{d}+1\right)^q}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\int \frac{\left(\frac{ex^n}{d}+1\right)^q}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{936}$$

$$\frac{2cx(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c
])*(1 + (e*x^n)/d)^q) - (2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n
^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

3.148.3.1 Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1758
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/r)   Int[(d + e*x
^n)^q/(b - r + 2*c*x^n), x], x] - Simp[2*(c/r)   Int[(d + e*x^n)^q/(b + r +
 2*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

3.148.4 Maple [F]

$$\int \frac{(d + e\,x^n)^q}{a + b\,x^n + c\,x^{2n}} dx$$

input
```
int((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

3.148.5 Fricas [F]

$$\int \frac{(d + ex^n)^q}{a + bx^n + cx^{2n}}\, dx = \int \frac{(ex^n + d)^q}{cx^{2n} + bx^n + a}\, dx$$

input
```
integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

3.148.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.148.7 Maxima [F]

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

3.148.8 Giac [F]

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx=\int \frac{(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int((d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x)
```

3.149 $\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx$

3.149.1 Optimal result

Integrand size = 29, antiderivative size = 263

$$\begin{aligned}\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx &= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^n)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)n(1+q)} \\ &\quad+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^n)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{a\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)n(1+q)} \\ &\quad-\frac{(d+ex^n)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,1+\frac{ex^n}{d}\right)}{adn(1+q)}\end{aligned}$$

output
```
-(d+e*x^n)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^n/d)/a/d/n/(1+q)+c*(d+e*x^
n)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(d+e*x^n)/(2*c*d-e*(b-(-4*a*c+b^2)^(
1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/a/n/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))
)+c*(d+e*x^n)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(d+e*x^n)/(2*c*d-e*(b+(-4
*a*c+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/a/n/(1+q)/(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2)))
```

3.149.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx = \frac{(d+ex^n)^{1+q}\left(\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^n)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q\right.}{2cd-\left(b+\sqrt{b^2-4ac}\right)}\right.}{an(1+q)}$$

input
```
Integrate[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
```

output
```
((d + e*x^n)^(1 + q)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1
+ q, 2 + q, (2*c*(d + e*x^n))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeomet
ric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + q, 2
+ q, 1 + (e*x^n)/d]/d))/(a*n*(1 + q))
```

3.149.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{\int \frac{x^{-n}(ex^n+d)^q}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\int \left(\frac{(ex^n+d)^q x^{-n}}{a}+\frac{(-cx^n-b)(ex^n+d)^q}{a(bx^n+cx^{2n}+a)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^n)^{q+1}\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^n+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{q+1}\text{Hypergeometric2F1}\left(1,q+1\right.}{a(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}}{n}$$

input
```
Int[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
```

output
```
((c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d
 + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*
(1 + q)) - ((d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*
x^n)/d])/(a*d*(1 + q)))/n
```

3.149.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.149. $\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx$

3.149.4 Maple [F]

$$\int \frac{(d+e\,x^n)^q}{x\,(a+b\,x^n+c\,x^{2n})}dx$$

input
```
int((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x)
```

3.149.5 Fricas [F]

$$\int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2\,n}+bx^n+a)x}\,dx$$

input
```
integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e*x^n + d)^q/(c*x*x^(2*n) + b*x*x^n + a*x), x)
```

3.149.6 Sympy [F]

$$\int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx = \int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx$$

input
```
integrate((d+e*x**n)**q/x/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral((d + e*x**n)**q/(x*(a + b*x**n + c*x**(2*n))), x)
```

3.149.7 Maxima [F]

$$\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x}\,dx$$

input
```
integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

3.149.8 Giac [F]

$$\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x}\,dx$$

input
```
integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx = \int \frac{(d+e\,x^n)^q}{x\,(a+b\,x^n+c\,x^{2\,n})}\,dx$$

input
```
int((d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int((d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))), x)
```

3.150 $\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx$

3.150.1 Optimal result

Integrand size = 29, antiderivative size = 212

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \frac{2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(-\frac{1}{n},1,-q,-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x} + \frac{2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(-\frac{1}{n},1,-q,-\frac{1-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x}$$

output
```
2*c*(d+e*x^n)^q*AppellF1(-1/n,1,-q,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)
),-e*x^n/d)/x/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+2*c*(d+e*x^
n)^q*AppellF1(-1/n,1,-q,(-1+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)
/x/((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.150.2 Mathematica [F]

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx$$

input
```
Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

output
```
Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x]
```

3.150.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1880, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^q}{x^2 (a+bx^n+cx^{2n})} \, dx$$

$$\Big\downarrow \text{1880}$$

$$\frac{2c\int \frac{(ex^n+d)^q}{x^2\left(2cx^n+b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(ex^n+d)^q}{x^2\left(2cx^n+b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{2c(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q}\int \frac{\left(\frac{ex^n}{d}+1\right)^q}{x^2\left(2cx^n+b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{2c(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q}\int \frac{\left(\frac{ex^n}{d}+1\right)^q}{x^2\left(2cx^n+b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{2c(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(-\frac{1}{n},1,-q,-\frac{1-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(-\frac{1}{n},1,-q,-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$

input
```
Int[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(-2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n), (-2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a
*c])*x*(1 + (e*x^n)/d)^q) + (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -(
(1 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 -
 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*x*(1 + (e*x^n)/d)^q)
```

3.150.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1880
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[
2*(c/r)   Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Simp[2*(c
/r)   Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b
, c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !Rati
onalQ[n]
```

3.150.4 Maple [F]

$$\int \frac{(d+e\,x^{n})^{q}}{x^{2}\left(a+b\,x^{n}+c\,x^{2n}\right)}dx$$

input
```
int((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x)
```

3.150.5 Fricas [F]

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx=\int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x^2}\,dx$$

input
```
integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e*x^n + d)^q/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

3.150.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d+e*x**n)**q/x**2/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.150.7 Maxima [F]

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx=\int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x^2}\,dx$$

input
```
integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.150.8 Giac [F]

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x^2}\,dx$$

input
```
integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \int \frac{(d+e\,x^n)^q}{x^2\,(a+b\,x^n+c\,x^{2\,n})}\,dx$$

input
```
int((d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int((d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x)
```

3.151 $\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx$

3.151.1 Optimal result

Integrand size = 29, antiderivative size = 210

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(-\frac{2}{n},1,-q,-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2} + \frac{c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(-\frac{2}{n},1,-q,-\frac{2-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x^2}$$

output
```
c*(d+e*x^n)^q*AppellF1(-2/n,1,-q,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),
-e*x^n/d)/x^2/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+c*(d+e*x^n)
^q*AppellF1(-2/n,1,-q,(-2+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/x
^2/((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.151.2 Mathematica [F]

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx = \int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx$$

input
```
Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

output
```
Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x]
```

3.151.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1880, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}dx$$

$$\Big\downarrow \text{1880}$$

$$\frac{2c\int \frac{(ex^n+d)^q}{x^3\left(2cx^n+b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(ex^n+d)^q}{x^3\left(2cx^n+b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{1013}$$

$$\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\int \frac{\left(\frac{ex^n}{d}+1\right)^q}{x^3\left(2cx^n+b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\int \frac{\left(\frac{ex^n}{d}+1\right)^q}{x^3\left(2cx^n+b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(-\frac{2}{n},1,-q,-\frac{2-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}-\frac{c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(-\frac{2}{n},1,-q,-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$

input
```
Int[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

output
```
-((c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])
*x^2*(1 + (e*x^n)/d)^q)) + (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n
)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c
]*(b + Sqrt[b^2 - 4*a*c])*x^2*(1 + (e*x^n)/d)^q)
```

3.151.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1880
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[
2*(c/r)   Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Simp[2*(c
/r)   Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b
, c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !Rati
onalQ[n]
```

3.151.4 Maple [F]

$$\int \frac{(d+e\,x^n)^q}{x^3\,(a+b\,x^n+c\,x^{2n})}dx$$

input
```
int((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x)
```

3.151.5 Fricas [F]

$$\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x^3}\,dx$$

input
```
integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((e*x^n + d)^q/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x**n)**q/x**3/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

3.151.7 Maxima [F]

$$\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x^3}\,dx$$

input
```
integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.151.8 Giac [F]

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x^3}\,dx$$

input
```
integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx = \int \frac{(d+e\,x^n)^q}{x^3\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int((d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int((d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x)
```

3.152 $\int (fx)^m (d+ex^n)^2 (a+bx^n+cx^{2n})^p \, dx$

3.152.1 Optimal result

Integrand size = 31, antiderivative size = 498

$$\int (fx)^m (d+ex^n)^2 (a+bx^n+cx^{2n})^p \, dx$$
$$= \frac{d^2(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2}{b-\sqrt{}}\right.}{f(1+m)}$$
$$+\frac{2dex^{1+n}(fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m+n}{n},-p,-p,\frac{1+m+}{n}\right.}{1+m+n}$$
$$+\frac{e^2x^{1+2n}(fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m+2n}{n},-p,-p,\frac{1+m}{n}\right.}{1+m+2n}$$

output
```
d^2*(f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+
2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+
2*d*e*x^(1+n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+n)/n,-p,-p,(1+m+
2*n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1
+m+n)/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(
1/2)))^p)+e^2*x^(1+2*n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+2*n)/n
,-p,-p,(1+m+3*n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2
)^(1/2)))/(1+m+2*n)/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+
(-4*a*c+b^2)^(1/2)))^p)
```

3.152.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.79

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$= \frac{x(fx)^m \left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+x^n(b+cx^n))^p \left(d^2(1+m^2+3n+2n^2+m(2+3n))\right.}{}$$

input
```
Integrate[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d^2*(1 + m^2 + 3*n + 2*n^2 + m*(2 + 3*
n))*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^n*(2*d*(1 + m +
 2*n)*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m + n)*x^n*A
ppellF1[(1 + m + 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 + m)*(1 + m + n)*(1
 + m + 2*n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*
((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.152.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$$

$$\Big\downarrow \; 1884$$

$$\int \left(d^2(fx)^m \left(a+bx^n+cx^{2n}\right)^p + 2dex^n(fx)^m \left(a+bx^n+cx^{2n}\right)^p + e^2x^{2n}(fx)^m \left(a+bx^n+cx^{2n}\right)^p\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{d^2(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\,\text{AppellF1}\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right.}{f(m+1)}$$

$$\frac{2dex^{n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\,\text{AppellF1}\left(\frac{m+n+1}{n},-p,-p,\frac{m+2n+1}{n},-\frac{}{b-}\right.}{m+n+1}$$

$$\frac{e^2x^{2n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\,\text{AppellF1}\left(\frac{m+2n+1}{n},-p,-p,\frac{m+3n+1}{n},-\frac{}{b-}\right.}{m+2n+1}$$

input
```
Int[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(d^2*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (
1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (2*d*e*x^(1 + n)*(f*x)^m*(a + b*x^n +
 c*x^(2*n))^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)
*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]))^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*AppellF1[
(1 + m + 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c
]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + 2*n)*(1 + (2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.152.3.1 Defintions of rubi rules used

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.152.4 Maple [F]

$$\int (fx)^m\,(d+e\,x^n)^2\,\left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

input
```
int((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

3.152.5 Fricas [F]

$$\int (fx)^m\,(d+ex^n)^2\,\left(a+bx^n+cx^{2n}\right)^p\,dx = \int (ex^n+d)^2\left(cx^{2\,n}+bx^n+a\right)^p(fx)^m\,dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m
, x)
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m\,(d+ex^n)^2\,\left(a+bx^n+cx^{2n}\right)^p\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.152.7 Maxima [F]

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p dx = \int (ex^n+d)^2 (cx^{2n}+bx^n+a)^p (fx)^m dx$$

input
```
integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

3.152.8 Giac [F(-2)]

Exception generated.

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-128,[1,0,5,3,0,6,4,1,6,0,2]%%%}+%%%{512,[1,0,5,3,0,6,4,0,
7,1,1]%%%
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p dx = \int (f\,x)^m\,(d+e\,x^n)^2\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p dx$$

input
```
int((f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int((f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p, x)
```

3.153 $\int (fx)^m (d+ex^n) (a+bx^n+cx^{2n})^p \, dx$

3.153.1 Optimal result

Integrand size = 29, antiderivative size = 323

$$\int (fx)^m (d+ex^n) (a+bx^n+cx^{2n})^p \, dx$$
$$= \frac{d(fx)^{1+m} \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2c}{b-\sqrt{b}}\right.}{f(1+m)}$$
$$+ \frac{ex^{1+n}(fx)^m \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m+n}{n},-p,-p,\frac{1+m+2n}{n}\right.}{1+m+n}$$

output
```
d*(f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*
x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+e*
x^(1+n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+n)/n,-p,-p,(1+m+2*n)/n
,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+m+n)/
((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))
^p)
```

3.153.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.85

$$\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$= \frac{x(fx)^m \left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+x^n(b+cx^n))^p \left(d(1+m+n)\operatorname{AppellF1}\left(\frac{1+m}{n},-p,-\right.\right.}{(1+m)}$$

input
```
Integrate[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d*(1 + m + n)*AppellF1[(1 + m)/n, -p,
-p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])] + e*(1 + m)*x^n*AppellF1[(1 + m + n)/n, -p, -p, (1 + m +
2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a
*c])]))/((1 + m)*(1 + m + n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
))^p)
```

3.153.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$\Big\downarrow \text{ 1884}$$
$$\int \left(d(fx)^m \left(a+bx^n+cx^{2n}\right)^p + ex^n(fx)^m \left(a+bx^n+cx^{2n}\right)^p\right) dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{d(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\operatorname{AppellF1}\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right.}{f(m+1)}$$
$$\frac{ex^{n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\operatorname{AppellF1}\left(\frac{m+n+1}{n},-p,-p,\frac{m+2n+1}{n},-\frac{2c}{b-\sqrt{b}}\right.}{m+n+1}$$

input
```
Int[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(d*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1
+ m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x^(
2*n))^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 +
(2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
))^p)
```

3.153.3.1 Defintions of rubi rules used

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.153.4 Maple [F]

$$\int (fx)^m (d+e\,x^n)\left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

input
```
int((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

3.153.5 Fricas [F]

$$\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)\left(cx^{2n}+bx^n+a\right)^p (fx)^m \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

3.153.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.153.7 Maxima [F]

$$\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)\left(cx^{2n}+bx^n+a\right)^p (fx)^m \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

3.153.8 Giac [F]

$$\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)\left(cx^{2n}+bx^n+a\right)^p (fx)^m \, dx$$

input
```
integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (f\,x)^m \left(d+e\,x^n\right) \left(a+b\,x^n+c\,x^{2\,n}\right)^p dx$$

input
```
int((f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int((f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p, x)
```

3.154 $\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$

3.154.1 Optimal result

Integrand size = 22, antiderivative size = 158

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$= \frac{(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-\cdots}}\cdots\right.}{f(1+m)}$$

output
```
(f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*c*
x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

3.154.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$= \frac{x(fx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+x^n(b+cx^n)\right)^p \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2\cdots}{b+\sqrt{\cdots}}\cdots\right.}{1+m}$$

input
```
Integrate[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(f*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)
/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
])/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*
((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.154.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \; 1721$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int (fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-}\right.$$

$$\Big\downarrow \; 1012$$

$$\frac{(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{AppellF1}\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},\right.}{f(m+1)}$$

input
```
Int[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
((f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 +
m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

3.154.3.1 Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

3.154.4 Maple [F]

$$\int (fx)^m \left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

input
```
int((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

3.154.5 Fricas [F]

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^p (fx)^m\,dx$$

input
```
integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

3.154.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

3.154.7 Maxima [F]

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2\,n}+bx^n+a\right)^p (fx)^m \, dx$$

input
```
integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

3.154.8 Giac [F]

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2\,n}+bx^n+a\right)^p (fx)^m \, dx$$

input
```
integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (f\,x)^m \left(a+b\,x^n+c\,x^{2\,n}\right)^p dx$$

input
```
int((f*x)^m*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int((f*x)^m*(a + b*x^n + c*x^(2*n))^p, x)
```

3.155 $\int \frac{(fx)^m(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$

3.155.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(fx)^m (a+bx^n+cx^{2n})^p}{d+ex^n}\,dx = \text{Int}\left(\frac{(fx)^m (a+bx^n+cx^{2n})^p}{d+ex^n},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

3.155.2 Mathematica [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a+bx^n+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(fx)^m (a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]
```

output
```
Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

3.155.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+bx^n+cx^{2n}\right)^p}{d+ex^n} \, dx$$

$$\Big\downarrow \text{ 1887}$$

$$\int \frac{(fx)^m \left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}dx$$

input
```
Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]
```

output
```
$Aborted
```

3.155.3.1 Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

3.155.4 Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\,x^n+c\,x^{2n}\right)^p}{d+e\,x^n}dx$$

input
```
int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

output
```
int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

3.155.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} \, dx = \int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{ex^n + d} \, dx$$

input
```
integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

3.155.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.155.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} \, dx = \int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{ex^n + d} \, dx$$

input
```
integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

3.155.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a+bx^n+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(cx^{2n}+bx^n+a)^p(fx)^m}{ex^n+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

3.155.9 Mupad [N/A]

Not integrable

Time = 8.70 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a+bx^n+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(f\,x)^m\,(a+b\,x^n+c\,x^{2\,n})^p}{d+e\,x^n}\,dx$$

input
```
int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x)
```

output
```
int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x)
```

3.156 $\int \frac{(fx)^m(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$

3.156.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

3.156.2 Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx=\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

output
```
Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

3.156.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2} \, dx$$

$$\Big\downarrow \text{1887}$$

$$\int \frac{(fx)^m \left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}dx$$

input
```
Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

output
```
$Aborted
```

3.156.3.1 Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

3.156.4 Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\,x^n+c\,x^{2n}\right)^p}{(d+e\,x^n)^2}dx$$

input
```
int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

output
```
int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

3.156.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \frac{(fx)^m \left(a+bx^n+cx^{2n}\right)^p}{\left(d+ex^n\right)^2}\,dx=\int \frac{\left(cx^{2\,n}+bx^n+a\right)^p (fx)^m}{\left(ex^n+d\right)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x^n + d^2)
, x)
```

3.156.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(fx)^m \left(a+bx^n+cx^{2n}\right)^p}{\left(d+ex^n\right)^2}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.156.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m \left(a+bx^n+cx^{2n}\right)^p}{\left(d+ex^n\right)^2}\,dx=\int \frac{\left(cx^{2\,n}+bx^n+a\right)^p (fx)^m}{\left(ex^n+d\right)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

3.156.8 Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} \, dx = \int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{(ex^n + d)^2} \, dx$$

input
```
integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

3.156.9 Mupad [N/A]

Not integrable

Time = 9.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} \, dx = \int \frac{(f\,x)^m \left(a + b\,x^n + c\,x^{2\,n}\right)^p}{(d + e\,x^n)^2} \, dx$$

input
```
int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x)
```

output
```
int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```